

## **FUNCTIONAL AND DYNAMIC ANALYSIS OF A 4 DOF PROTOTYPE ROBOT**

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*The article presents the elements of a 4 DOF prototype robot which consists of a base, a platform, arm 1, arm 2 and an effector. Laws governing the variation in time of rotational angles are presented, designed for controlling the four motors which actuate the robot. The geometrical and mass data of the robot's elements allow for cinematic and dynamic analysis of each element as well as for the entire system. In the technological applications that are analyzed in this article – applications which simulate a continuous welding process – the tip of the effector travels, at constant speed, in a straight line and in a semicircle.*

**Keywords:** industrial robots; work productivity; mechanical engineering; technological applications; cinematic and dynamic analysis.

The construction of robots, as a scientific domain, has based itself on Mechanics and Cybernetics. Concurrently development of constructive solutions for robots has been formulated in a series of neighboring areas which have favored this.

The practical objective of robot creation was to transfer upon them the types of activities which are difficult for a human operator – repetitive, monotonous or dangerous. Such activities are auxiliary production operations as:

- loading and unloading of facilities, machine tools, and automated production equipment;
- primary or main production operations (welding, cutting, painting, assembly etc.);
- extreme environment operations (underwater, outer space, radioactive and toxic environments).

Industrial robots are one of the most important means of complex production automation and increasing work productivity and quality. Compared to traditional automation solutions industrial robots present numerous advantages in terms of versatility and rapid configuration which

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allows for development of flexible automated production systems based on universal yet complex robotic equipment.

A manipulative robot contains two linked units: the control unit and the physical handling unit. The command unit includes the sensor subsystem, the data processing and storage subsystem (e. g. server or NAS-type unit) and the transmission subsystem.

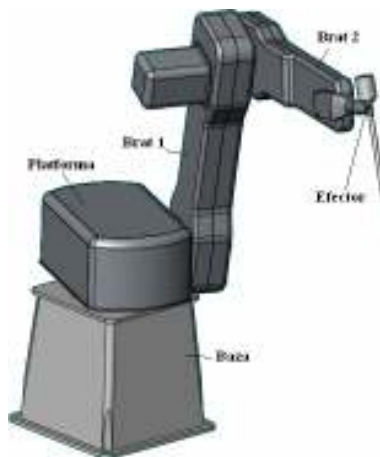
The handling unit is engaged by means of actuating motors, which can be mounted either on mobile or fixed elements of the assembly. The number of actuators is generally equal to the manipulative unit's number of degrees of freedom (DOF), although while carrying out certain operations supplemental links can be imposed. Motion transfer from the motors to the elements of the assembly is achieved by means of transmission mechanisms of various types.

There are high demands regarding the execution speed and motion precision of an industrial robot, thus the elasticity of all the elements involved need to be taken into account.

This article is based on the 4 DOF industrial robot prototype developed by the author during his doctoral studies.

### Description Of The 4 DOF Prototype Robot

The general configuration of the robot is presented in figure 1. The component parts of the robot are: the base, the platform, arm 1, arm 2 and effector.



**Fig. 1** Configuration of the 4 DOF prototype robot

#### The Base

Figure 2 represents the base of the robot, with a mass of 6,686 kg and a volume of  $5,105 \times 10^{-3} \text{ m}^3$ . On the base there is a fixed gear with 100 straight teeth with module 2, angle of obliquity  $\alpha = 20^\circ$  and tooth width 12 mm. There are two ball bearings mounted coaxially with the gear which form a cylindrical joint between the base and the platform.

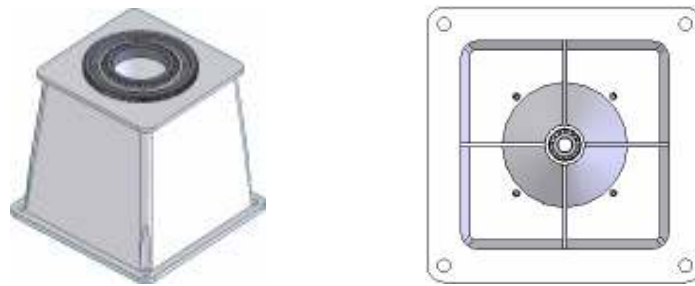


Fig. 2 Base of the prototype robot

In figure 3.a the platform and the spatial reference system to which the center of mass coordinates and moments of inertia are expressed.

Mass characteristics of the platform:

mass = 12,695 kg, volume =  $3,081 \cdot 10^{-3} \text{ m}^3$ .

Center of mass position with respect to the platform's reference system (fig. 3.a):

$X_C = 1.87 \text{ mm}$ ,  $Y_C = 105 \text{ mm}$ ,  $Z_C = 275.76 \text{ mm}$ .

Moments of inertia ( $\text{kg} \times \text{m}^2$ ) with respect to the reference system's axes:

$J_{xx} = 1049,705 \cdot 10^{-3}$ ,  $J_{xy} = 11,259 \cdot 10^{-3}$ ,  $J_{xz} = -1,850 \cdot 10^{-3}$ ,  
 $J_{yx} = 11,259 \cdot 10^{-3}$ ,  $J_{yy} = 1128,946 \cdot 10^{-3}$ ,  $J_{yz} = 52,554 \cdot 10^{-3}$ ,  
 $J_{zx} = -1,850 \cdot 10^{-3}$ ,  $J_{zy} = 52,554 \cdot 10^{-3}$ ,  $J_{zz} = 152,449 \cdot 10^{-3}$ .

Moments of inertia ( $\text{kg} \times \text{m}^2$ ) calculated with respect to a central system with axes parallel to the axes of the reference system:

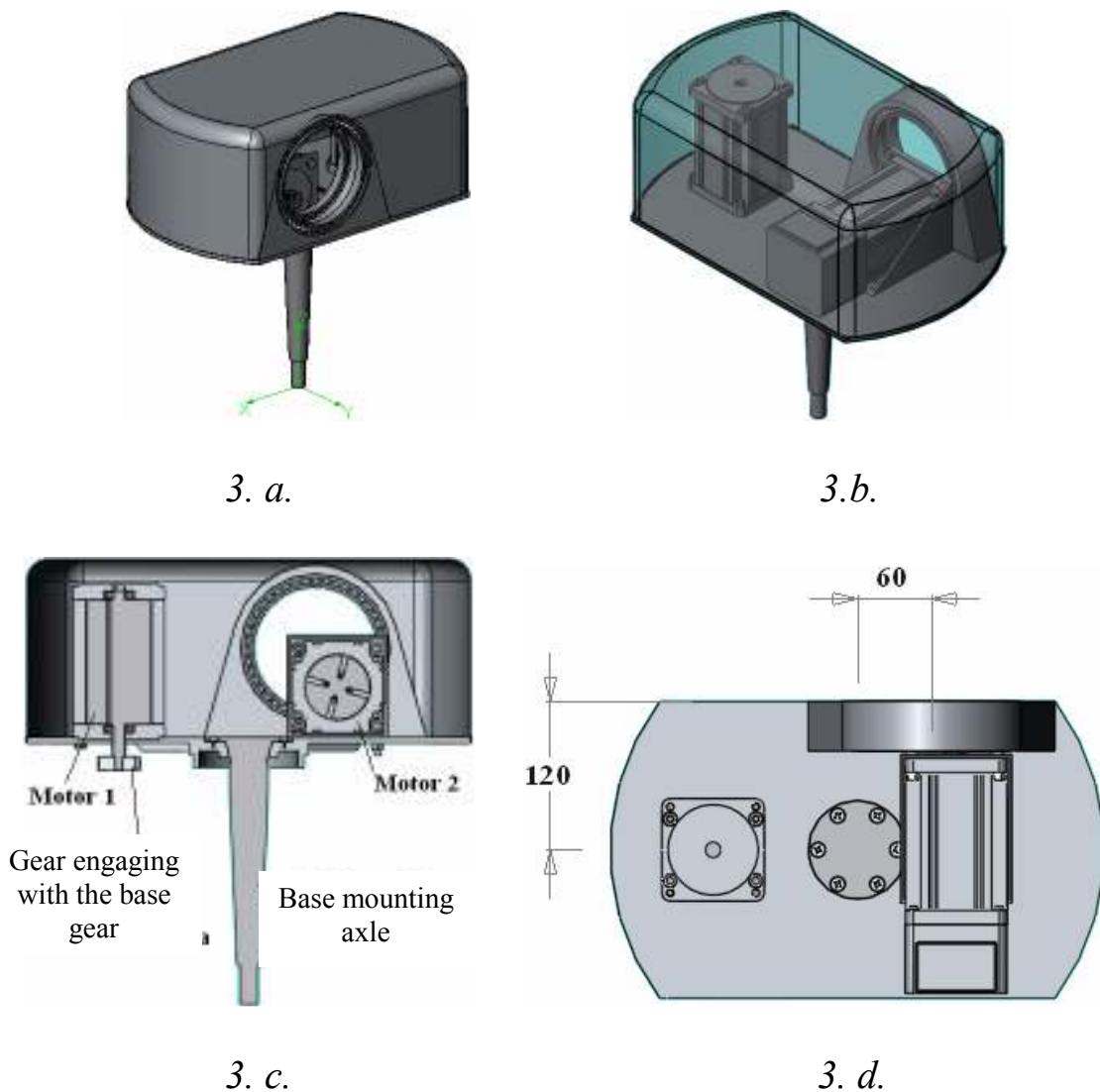
$J_{xx_C} = 81,820 \cdot 10^{-3}$ ,  $J_{xy_C} = 10,926 \cdot 10^{-3}$ ,  $J_{xz_C} = -8,380 \cdot 10^{-3}$ ,  
 $J_{yx_C} = 10,926 \cdot 10^{-3}$ ,  $J_{yy_C} = 163,522 \cdot 10^{-3}$ ,  $J_{yz_C} = 3,385 \cdot 10^{-3}$ ,  
 $J_{zx_C} = -8,380 \cdot 10^{-3}$ ,  $J_{zy_C} = 3,385 \cdot 10^{-3}$ ,  $J_{zz_C} = 149,901 \cdot 10^{-3}$ .

Directional cosines of the main inertia axes and main moments of inertia ( $\text{kg} \times \text{m}^2$ ) with respect to the center of mass:

$I_x = (0.99, 0.12, -0.11)$   $J_{px} = 79,502 \cdot 10^{-3}$ ,  
 $I_y = (0.07, 0.29, 0.96)$   $J_{py} = 149,515 \cdot 10^{-3}$ ,  
 $I_z = (0.15, -0.95, 0.27)$   $J_{pz} = 166,22 \cdot 10^{-3}$ .

### The Platform

The platform has two eccentric-mounted motors (figure 3.c) which provide movement of the platform with respect to the base (motor 1) and movement of arm 1 with respect to the platform (motor 2). Figure 3.d represents the positioning of the arm 1 mounting bearing with respect to the platform's rotation axis.



**Fig. 3** Prototype robot's platform

A gear with 17 teeth is mounted on the axle of motor 1 which engages the gear mounted on the base, ensuring movement of the platform.

The axis of the arm 1 mounting bearing is placed at  $z=320 \text{ mm}$  with respect to the spatial reference system. On the axle of motor 2 a gear with 20 straight teeth module 1,  $\alpha = 20^\circ$ , 12 mm wide is mounted. The gear engages on the inside with a crown gear with 100 teeth mounted on arm 1, ensuring rotational movement of arm 1 with respect to the platform.

### Arm 1

In figure 4.a the spatial reference system of arm 1 is presented as well as the distance between the axes of the bearings.

Mass characteristics of arm 1:

$$\text{mass} = 7,571 \text{ kg}, \text{ volume} = 2,263 \cdot 10^{-3} \text{ m}^3.$$

Center of mass position with respect to the reference system of the arm (fig. 4.a):

$$X_C = 0.00 \text{ mm}, Y_C = 300,15 \text{ mm}, Z_C = -18,46 \text{ mm}.$$

Moments of inertia ( $\text{kg}\cdot\text{m}^2$ ) with respect to the axes of the reference system:

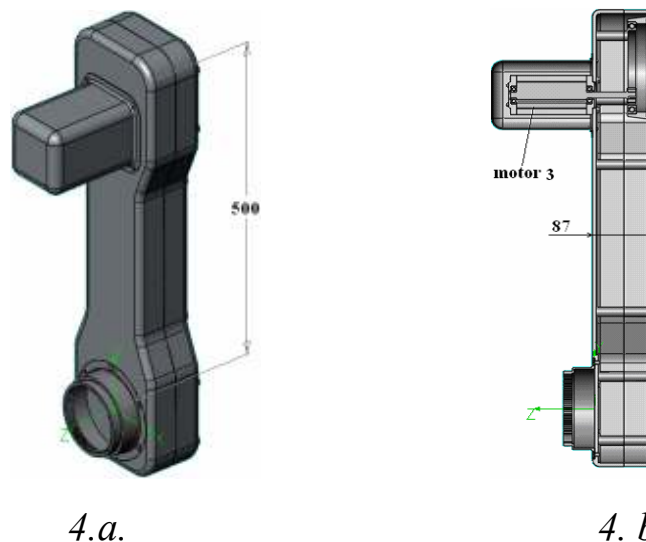
$$\begin{aligned} J_{xx} &= 1048,565 \cdot 10^{-3}, & J_{xy} &= 10,439 \cdot 10^{-6}, & J_{xz} &= 1,319 \cdot 10^{-6}, \\ J_{yx} &= 10,439 \cdot 10^{-6}, & J_{yy} &= 43,312 \cdot 10^{-3}, & J_{yz} &= -28,161 \cdot 10^{-3}, \\ J_{zx} &= 1,319 \cdot 10^{-6}, & J_{zy} &= -28,161 \cdot 10^{-3}, & J_{zz} &= 1040,733 \cdot 10^{-3}. \end{aligned}$$

Moments of inertia ( $\text{kg}\cdot\text{m}^2$ ) calculated with respect to a center system with axes parallel to the axes of the reference system:

$$\begin{aligned} J_{xx_C} &= 363,932 \cdot 10^{-3}, & J_{xy_C} &= 3,505 \cdot 10^{-6}, & J_{xz_C} &= 1,746 \cdot 10^{-6}, \\ J_{yx_C} &= 3,505 \cdot 10^{-6}, & J_{yy_C} &= 40,731 \cdot 10^{-3}, & J_{yz_C} &= 13,797 \cdot 10^{-3}, \\ J_{zx_C} &= 1,746 \cdot 10^{-6}, & J_{zy_C} &= 13,797 \cdot 10^{-3}, & J_{zz_C} &= 358,681 \cdot 10^{-3}. \end{aligned}$$

Directional cosines of main inertia axes and main moments of inertia ( $\text{kg}\cdot\text{m}^2$ ) with respect to the center of mass:

$$\begin{aligned} I_x &= (0.00, 1.00, 0.04) & J_{px} &= 40,134 \cdot 10^{-3}, \\ I_y &= (0.00, -0.04, 1.00) & J_{py} &= 359,278 \cdot 10^{-3}, \\ I_z &= (1.00, 0.00, -0.00) & J_{pz} &= 363,932 \cdot 10^{-3}. \end{aligned}$$



**Fig. 4** Arm 1 of the prototype robot

An electric motor is mounted inside arm 1 (fig. 4.b) with the purpose of providing rotational movement to arm 2 by means of an interior cylindrical engagement. A gear with 20 teeth module 1,  $\alpha = 20^\circ$ , 12 mm wide is mounted on the axis of the motor.

## Arm 2

In figure 5.a the spatial reference system of arm 2 is presented, as well as the distance between the axes of the bearings.

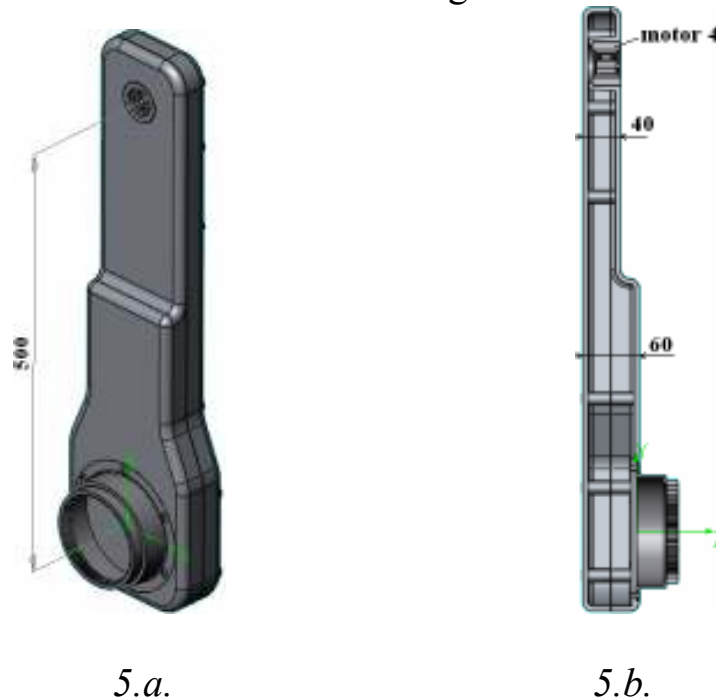


Fig. 5. Arm 2 of the prototype robot

Mass characteristics of the arm:

mass = 4,157 kg, volume =  $1,321 \cdot 10^{-3} \text{ m}^3$

Center of mass coordinates with respect to the reference system:

$X = 0.0 \text{ mm}$ ,  $Y = 201.94 \text{ mm}$ ,  $Z = -296 \text{ mm}$ .

Moments of inertia ( $\text{kg} \cdot \text{m}^2$ ) with respect to the reference system:

$J_{xx} = 349,131 \cdot 10^{-3}$ ,  $J_{xy} = -18,115 \cdot 10^{-6}$ ,  $J_{xz} = 1,067 \cdot 10^{-6}$ ,  
 $J_{yx} = -1,811 \cdot 10^{-6}$ ,  $J_{yy} = 13,850 \cdot 10^{-3}$ ,  $J_{yz} = -29,030 \cdot 10^{-3}$ ,  
 $J_{zx} = 1,067 \cdot 10^{-6}$ ,  $J_{zy} = -29,030 \cdot 10^{-3}$ ,  $J_{zz} = 352,211 \cdot 10^{-3}$ .

Moments of inertia ( $\text{kg} \cdot \text{m}^2$ ) calculated with respect to a center system with axes parallel to the axes of the spatial reference system:

$J_{xx_c} = 177,032 \cdot 10^{-3}$ ,  $J_{xy_c} = -11,163 \cdot 10^{-6}$ ,  $J_{yx_c} = 0,207 \cdot 10^{-6}$ ,  
 $J_{yx_c} = -11,163 \cdot 10^{-6}$ ,  $J_{yy_c} = 11,260 \cdot 10^{-3}$ ,  $J_{yz_c} = -8,076 \cdot 10^{-3}$ ,  
 $J_{zx_c} = 0,207 \cdot 10^{-6}$ ,  $J_{zy_c} = -8,076 \cdot 10^{-3}$ ,  $J_{zz_c} = 182,703 \cdot 10^{-3}$ .

Directional cosines of main axes of inertia and main moments of inertia ( $\text{kg} \cdot \text{m}^2$ ) with respect to the center of mass:

$$\begin{aligned} I_x &= (-0.00, 1.00, -0.05) & J_{px} &= 10,880 \cdot 10^{-3}, \\ I_y &= (1.00, 0.00, -0.00) & J_{py} &= 177,032 \cdot 10^{-3}, \\ I_z &= (-0.00, -0.05, -1.00) & J_{pz} &= 183,082 \cdot 10^{-3}. \end{aligned}$$

Arm 2 is equipped with an electrical motor (fig. 5.b) which engages in rotational movement the small arm which holds the effector.

### The Effector

In figure 6 the spatial reference system of the effector's arm is presented as well as the distance between the bearings axes.

Mass characteristics of the effector arm:

$$\text{mass} = 0,533 \text{ kg}, \text{ volume} = 0,141 \cdot 10^{-6}.$$

Coordinates for the center of mass with respect to the spatial reference system:

$$X = 125.11 \text{ mm}, Y = 8.51 \text{ mm}, Z = 34 \text{ mm}.$$

Moments of inertia ( $\text{kg} \times \text{m}^2$ ) with respect to the axes of the reference system:

$$\begin{aligned} J_{xx} &= 1,492 \cdot 10^{-3}, & J_{xy} &= 1,125 \cdot 10^{-3}, & J_{xz} &= 0,786 \cdot 10^{-3}, \\ J_{yx} &= 1,125 \cdot 10^{-3}, & J_{yy} &= 13,702 \cdot 10^{-3}, & J_{yz} &= 67,993 \cdot 10^{-6}, \\ J_{zx} &= 0,786 \cdot 10^{-3}, & J_{zy} &= 67,993 \cdot 10^{-6}, & J_{zz} &= 14,979 \cdot 10^{-3}. \end{aligned}$$

Moments of inertia ( $\text{kg} \times \text{m}^2$ ) calculated with respect to a central system with it's axes parallel to the axes of the reference system:

$$\begin{aligned} J_{xx_c} &= 1,444 \cdot 10^{-3}, & J_{xy_c} &= 0,558 \cdot 10^{-3}, & J_{xz_c} &= 0,497 \cdot 10^{-3}, \\ J_{yx_c} &= 0,558 \cdot 10^{-3}, & J_{yy_c} &= 5,352 \cdot 10^{-3}, & J_{yz_c} &= 0,048 \cdot 10^{-3}, \\ J_{zx_c} &= 0,497 \cdot 10^{-3}, & J_{zy_c} &= 48,307 \cdot 10^{-6}, & J_{zz_c} &= 6,600 \cdot 10^{-3}. \end{aligned}$$

Directional cosines of the main axes of inertia and main moments of inertia ( $\text{kg} \times \text{m}^2$ ) with respect to the center of mass:

$$\begin{aligned} I_x &= (0.99, 0.14, 0.09) & J_{px} &= 1,319 \cdot 10^{-3}, \\ I_y &= (0.14, -0.99, 0.02) & J_{py} &= 5,430 \cdot 10^{-3}, \\ I_z &= (0.10, -0.00, -1.00) & J_{pz} &= 6,648 \cdot 10^{-3}. \end{aligned}$$

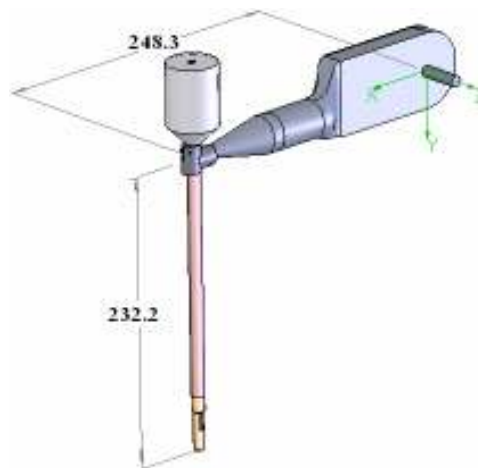


Fig. 6 The effector of the prototype robot

### Rotational Angles Variation With Time

In order to determine the variation of rotational angles for all four motors, as a function of time, the following notations are used:

- Specific moments of time:  $t_0 = 0$  s.,  $t_1 = 0.2$  s.,  $t_3 = 1.8$  s.  $t_3 = 2.0$  s.;
- Maximum values of rotation angles  $Y_{3_1}, Y_{3_2}, Y_{3_3}, Y_{3_4}$  at the four motors, corresponding to the moment  $t_3$ ;
- $Y_{1_1}, Y_{1_2}, Y_{1_3}, Y_{1_4}$  values of the rotation angles at the four motors corresponding to the moment  $t_1$ ;
- $Y_{2_1}, Y_{2_2}, Y_{2_3}, Y_{2_4}$  values of the rotation angles at the four motors, corresponding to the moment  $t_2$ .

We adopt:

$$j = 1, 2, 3, 4; \quad Y_3 = \begin{cases} p \\ \frac{p}{2} \\ 1,5 \cdot p \\ \frac{p}{4} \end{cases} \quad Y_2 = 0.95 \times Y_3, \quad Y_1 = 0.05 \times Y_3$$

For motor 1, mounted to the platform – which rotates the platform with respect to the base – we consider:

$$Y_1(t) = \begin{cases} 0 & \text{if } t < t_0 \\ a_{11}t^2 + b_{11}t + c_{11} & \text{if } (t < t_1) \dot{U}(t > t_0) \\ a_{21}t + b_{21} & \text{if } (t > t_1) \dot{U}(t < t_2) \\ a_{31}t^2 + b_{31}t + c_{31} & \text{if } (t > t_2) \dot{U}(t < t_3) \\ Y_1(t_3) & \text{if } t > t_3 \end{cases}, \quad (1)$$

Where coefficients of variable  $t$  are:



Motor 1	Coefficient	$a_{11} = 3.92699$		$b_{11} = 0$		$c_{11} = 0$	
		$a_{21} = 1.76715$		$b_{21} = - 0.19535$		--	
		$a_{31} = - 4.41786$		$b_{31} = 17.67148$		$c_{31} = - 14.51023$	
	Time (s)	$t_0 = 0$	$t_1 = 0.20$	$t_2 = 1.80$	$t_3 = 2.00$		

For motors 2, 3 și 4, laws  $Y_2(t)$ ,  $Y_3(t)$ ,  $Y_4(t)$  are adopted, similar to expression (1) with the following coefficients:

Motor 2	Coefficient	$a_{12} = 1.9635$		$b_{12} = 0$		$c_{12} = 0$	
		$a_{22} = 0.88357$		$b_{22} = - 0.09817$		--	
		$a_{32} = - 2.20893$		$b_{32} = 8.83573$		$c_{32} = - 7.25512$	
	Time (s)	$t_0 = 0$	$t_1 = 0.20$	$t_2 = 1.80$	$t_3 = 2.00$		

Motor 3	Coefficient	$a_{13} = 5.89049$		$b_{13} = 0$		$c_{13} = 0$	
		$a_{23} = 2.65072$		$b_{23} = - 0.29452$		--	
		$a_{33} = - 6.6268$		$b_{33} = 26.50719$		$c_{33} = - 21.76535$	
	Time (s)	$t_0 = 0$	$t_1 = 0.20$	$t_2 = 1.80$	$t_3 = 2.00$		

Motor 4	Coefficient	$a_{14} = 0.98175$		$b_{14} = 0$		$c_{14} = 0$	
		$a_{24} = 0.44179$		$b_{24} = - 0.04909$		--	
		$a_{34} = - 1.10447$		$b_{34} = 4.41786$		$c_{34} = - 3.62756$	
	Time (s)	$t_0 = 0$	$t_1 = 0.20$	$t_2 = 1.80$	$t_3 = 2.00$		

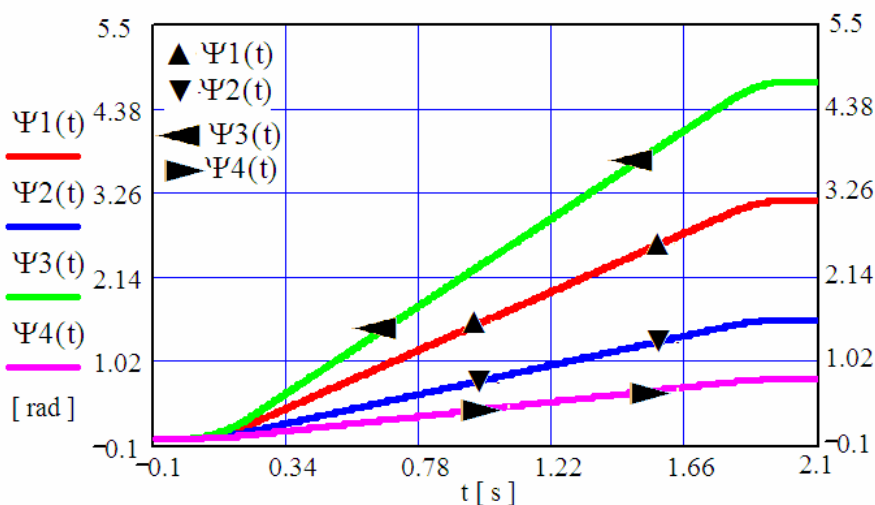


Fig. 7 Graphical representation of motor 1-4 axes rotation angles variation with time

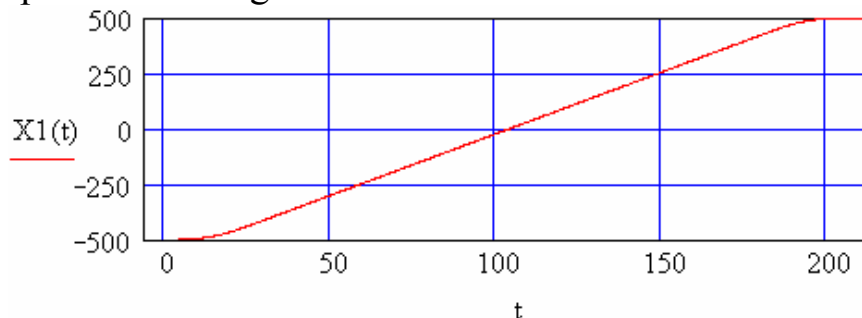
## Conclusions

With aid of the 4 DOF prototype robot's geometrical and mass characteristics and angular speeds of the four motors axes which actuate the robot we can determine:

- Variation with time of speed and angular acceleration of the platform's center of mass;
- Variation with time of the platform's kinetical energy;
- Components of the angular speeds and accelerations for arms 1 and 2;
- Variation with time of the robot's arms kinetical energy;
- Components of speed and acceleration of the effector's center of mass.

For processes requiring a straight line movement of the effector, in the horizontal plane of the workbench, at constant speed, rotation laws of the motors are to be determined.

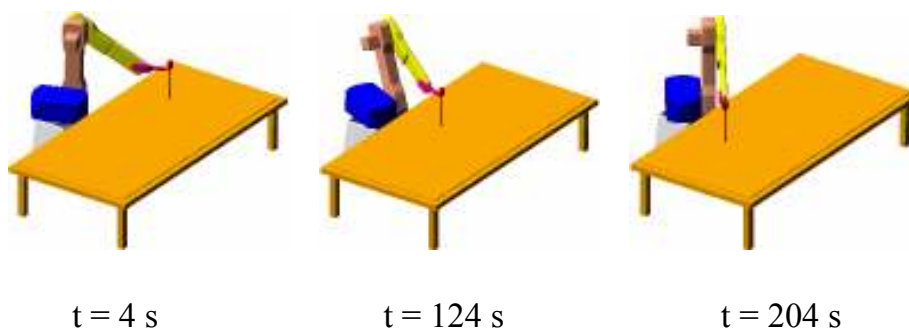
It is considered that the effector changes it's  $Ox$  axis coordinate the other coordinates remaining constant. The movement starts at the initial moment  $t = 4$  s, period  $[0,4]$ s which corresponds to effector positioning (positioning movement ends after 3,34928 s). For the  $Ox$  coordinate variation the law presented in figure 8 is chosen.



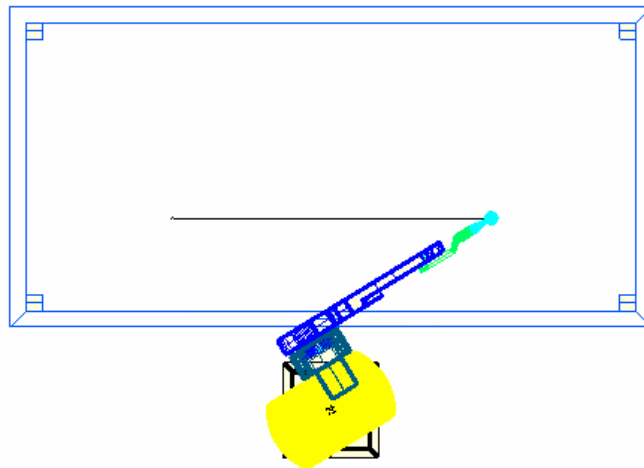
**Fig. 8** Law of the effector's tip  $Ox$  coordinate variation

The final position of the effector's tip is a point with coordinates:  $x = 500$  mm,  $y = 600$  mm și  $z = 407$  mm, final time - 204s.

In figure 9 the movement of the robot with the effector tip travelling in a straight line is presented.



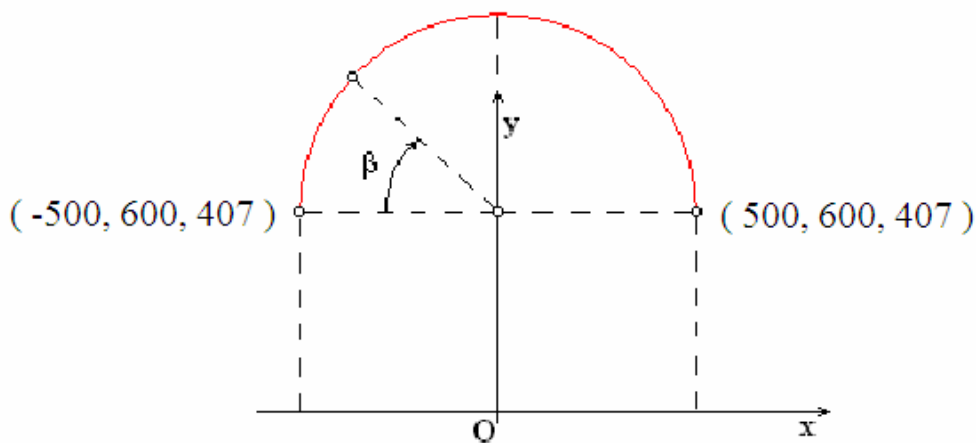
**Fig. 9** Robot's positions for different moments in time



**Fig. 10** Effector's tip trajectory

For a process requiring the effector's tip to move about a circular arc in the horizontal plane of the workbench, at constant speed, the laws of motor rotation are to be determined.

During the movement the tip of the effector changes its coordinates with respect to  $Ox$  and  $Oy$  axes,  $z$  coordinate remaining constant. The circular arc has the opening  $\pi$ , the center of its circle is placed on the  $Oy$  axis.



**Fig. 11** Position of the effector's tip

Motion starts at the moment  $t = 4s$ , when the tip is at coordinates  $(-500, 600, 407)$  and ends when time is  $t = 318.159s$  and the tip is at coordinates  $(500, 600, 407)$ . Variation with time of these coordinates is given by:

$$x(t) = -500 \cdot \cos(\beta(t))$$

$$y(t) = 600 + 500 \cdot \sin(\beta(t))$$

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