Automatic Optimal Thresholding Using Generalized Fuzzy Entropies and Genetic Algorithm

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Abstract

The use of fuzzy entropy for image segmentation is one of the most popular methods, which is used today. In a classical fuzzy entropy, using a fuzzy complement with an equilibrium point of 0.5 is a limitation, which reduces the chances of obtaining an optimal result. We use generalized fuzzy entropy phrases in this paper, which uses fuzzy complements of Sugeno and Yager, and corresponds the equilibrium point to the m parameter ($0<m<1$), and increases the chance of finding the optimal threshold. So, we will have many pictures depending on the points of balance, and by the genetic algorithm, we choose the best decision among them. The effect of this method have considered in medical images to find the brain tumors. Results have shown that the use of generalized fuzzy entropy and the genetic algorithm can greatly be used to find the optimal threshold. Presented method is very effective for reducing the number of intensity levels. Problems may cause images with height amount of unwanted information which is saved to the expanse of subjective more important information.

Keywords: Image Segmentation; Fuzzy Entropy; Generalized Fuzzy Entropy; Fuzzy Complement Operator; Genetic Algorithm.

1. Introduction

The fuzzy entropy determines the fuzziness level and ambiguity of a fuzzy set, it is also a basic concept for the theory of fuzzy sets. The Fuzzy entropy has been widely used for image segmentation and threshold selection (Huang, & Wang, 1995; Li, Zhao, & Cheng, 1995; Cheng, Chen, & Sun, 1999). The first definitions of fuzzy entropy were presented by Deluca and Termini (De Luca, & Termini, 1972). The basic functions of union, intersections, and fuzzy complementation were used by them in their definitions which were introduced by Zadeh (Zadeh, 1965). One of the important characteristics of using a fuzzy supplement in the definition of entropy is that there is the greatest ambiguity at the equilibrium point of $x = 0.5$. This issue in real applications always confronts us with a truth. If we use the concept of fuzzy uncertainty to calculate the darkness of an image, as Pal uses it (Pal, 1982). A single point of equilibrium for a basic fuzzy complement does not necessarily result in acceptable results. By generalizing the concept of fuzzy entropy, Zenzo (Zenzo, Cinque, & Levialdi, 1998) created a condition in which we have the highest amount of fuzziness and uncertainty for the equilibrium points of $x=m$ ($m\in(0,1)$) and the application of this generalized fuzzy entropy for threshold selection in image processing, increases the chances of obtaining an appropriate image.

But the fuzzy entropy generalized by Zenzo did not refuse the fourth condition of a fuzzy entropy, which includes a complement operator for a fuzzy set. Therefore, the generalized fuzzy entropies were then presented (Fan, & Zhao, 2008). in which the comprehensiveness of the four conditions of the fuzzy entropy definition was taken into account. To apply these generalized fuzzy entropy definitions was made to select the optimal threshold with m membership degree, (0<m<1).

But using these methods for image segmentation, provides m image which ultimately to choose the best picture is done by a person. Depending on the individual view, the selected final images may also be different. In this study we present a method in which a variety of fuzzy complements are used for fuzzy entropies, to improve the previous methods. in addition, at the end, the genetic algorithm is used to select the best image from the m image provided by the extended fuzzy entropy, to have the optimal image selection by an intelligent method.

The structure of this article is as follows: In the second section, we give an overview of the definition of fuzzy entropy, In Section 3 we examine the various definitions of generalized fuzzy entropy. The fourth part introduces the method of image segmentation using generalized fuzzy entropy and the use of various fuzzy supplements. In Section 5, an intelligent selection of optimal image based on genetic algorithm is proposed. The performance of the proposed method in images related to the identification of brain tumors has been done in section 6. Finally, the conclusions and recommendations are presented in Section 7.

2. Fuzzy Entropy

In this paper, we show the set of all fuzzy sets on universal set X by means of F(X), and the set of all crisp sets by means of P(X).

- μ_A (x) is the membership function of A∈F(X).
- [a] is the fuzzy set of x, so that $\mu_{[a]}(x) = a$.

Complement, union and intersection operators for these sets are defined as follows:

A^C Is the indicator of completes$A$, so that c(x) = 1− x (x∈ [0, 1]). The union of two fuzzy sets of A and B is shown as $A \cup B$ and defined as $\mu_{A\cup B}$(x) = max $(\mu_A(x))$, μB (x)) and intersection of two fuzzy sets of A and B is shown as $A \cap B$ and defined as $\mu_{A\cap B}$(x) =min$(\mu_A(x))$, μB (x)). A fuzzy set A∗ is called a sharpening of A. For the set of A* which is sharper than A, If μ_A (x) ≥0.5, then μ_{A^*}(x) ≥ μ_A (x) and if μ_A(x) ≤ 0.5 then μ_{A^*}(x) ≤ μ A (x). Deluca and Termini.4, defined the fuzzy entropy for the calculation of the uncertainty level of a fuzzy set, defined on the universal set X as follows:

e(A) = 0 if A∈P(X);
e(A) reaches to its maximum if A=[0.5];
If A∗ is a sharpening of A, then e(A*) ≤ e(A);
e(A^C) = e(A).

3. Generalized Fuzzy Entropy

As mentioned, the equilibrium point of the base complement operator is only at x =0.5, and this is a limitation to use to select the threshold. Other complement operators have been proposed by researchers, which eliminate the limit of the unique equilibrium point (Sugeno, 1977; Lowen, 1978; Yager, 1980; Malinowski, 1993).
For example, Sugeno has presented a fuzzy complement function as follows:

$$C_\lambda(x) = \frac{1-x}{1+x}, (\lambda(-1,\infty)). \quad (1)$$

In which the equilibrium point of this function is equal to m = (-1±√(1+λ)) / λ.
yager has provided the following fuzzy complement function:

$$c_w(x) = (1-x^w)^{1/w}, w\in(0,+\infty) \quad (2)$$

Which has the equilibrium point of m = 1 / √(w&2). Based on the fuzzy complement with the non-unique equilibrium point, various functions for generalized fuzzy entropy (e_m: F(X) → R^+) are presented (Khotanloua et. al., 2009), which refuses the four conditions below:

(1) e m (A) =0 if A∈P(X);
(2) e m (A) attains its maximum if A=[m] ;
(3) If A* is shaper than A , then em (A*) ≤ em (A).
Here A* satisfies that μA (x) ≥ μA (x) when μA (x) ≥ m ; μA (x) ≤ μA (x) when μA (x) ≤ m ;
(4) e(Acm) = e(A).

But in all of them, only one fuzzy complement has been used. In this paper we use generalized fuzzy entropy functions with multiple complements to improve performance. Some of the generalized fuzzy entropy relations for a fuzzy set A on a finite x set are given below:

Where

$$MM(A) = \sum_{i=1}^{n} \mu_A(x_i) \quad (3)$$

$$e_m^1(A) = \frac{M(A \cap A^{c_m})}{M(A \cup A^{c_m})} \quad (4)$$

$$e_m^2(A) = \sum_{i=1}^{n} \frac{\min(\mu_i(x_i), \mu_{A^{c_m}}(x_i))}{\max(\mu_i(x_i), \mu_{A^{c_m}}(x_i))} \quad (5)$$

$$e_m^3(A) = M(A \cap A^{c_m})$$

$$e_m^1(A) = 1 - \frac{M(A \cup A^{c_m}) - M(A \cap A^{c_m})}{n} \quad (6)$$

Where

$$MM(A) = \sum_{i=1}^{n} \mu_A(x_i)$$

4. Fuzzy Entropy-Based Image Segmentation Method

To choose the threshold for image processing based on generalized fuzzy entropy has been one of the most commonly used methods in recent research. In this method we consider the parameter m in the interval (0,1) with steps 0.05, and we will find for each image 19 outcomes and then, by intelligent genetic algorithm, the best decision will be chosen from these results. We first consider the image of size M × N with a Matrix Q, where q xy shows the gray level of the pixel (x, y). If we look at the image as a fuzzy set, μ_Q q_{xy} represents the membership value, which indicates the brightness level of the pixels (x,y) in the Q matrix. To show the gray level brightness, we use the s function, which is defined as follows: (Li & Yang, 1989)

$$S(x,a,b,d) = \begin{cases} 0, & x \le a \\ \frac{(x-a)^2}{(b-a)(d-a)}, & a \le x \le b \\ 1 - \frac{(x-a)^2}{(b-a)(d-a)}, & b \le x \le d \\ 1, & x \ge d \end{cases} \quad (7)$$

In which x is the gray level in Q and a, b, d are the parameters that identifies the shape of S function, and a > b > d.

The image segmentation based on fuzzy entropy is based on the selection of maximum fuzzy entropy as threshold. The shape of the function s is determined by the parameters a, b, d, so the threshold selection problem is related to the combination of these parameters, So that maximum fuzzy entropy is obtained (Li, Zhao, & Cheng, 1995, Cheng, & Chen, 1997, Hrubes & Kozumplik, 2007). This optimal combination of parameters is represented by (a*, b*, d*), Then, using the generalized fuzzy entropy for image segmentation, the optimal threshold value is selected as follows:

$$(a^*, b^*, c^*) = Arg \max_{0 \le a < b < d} e_m(a,b,d). \quad (8)$$

$$T^* = t, if \mu_Q(t) = m. \quad (9)$$

$$g(x,y) = \begin{cases} 0 & q(x,y) \le T^* \\ 1 & q(x,y) > T^* \end{cases}. \quad (10)$$

When we consider the fuzzy complement of the Sugeno class instead of the base fuzzy complement, the relation between m, and ƛ is obtained as follow:

$$ƛ = \frac{1-2m}{m^2} \quad (11)$$

Which is a mapping of m∈(0,1) to ƛ∈(-1,∞).

If we use the fuzzy complement of the yager class, the relationship between m and w is as follow:

$$w = -\frac{1}{log_2 m} \quad (12)$$

which is a mapping of m∈(0,1) to w∈(-1,∞).

By respect to these relations, the relations introduced in (3-6) for generalized fuzzy entropy include parameters m (m∈(0,1)) . Then we divide m in the internal (0,1) with steps 0.05. 19 results are obtained for each image. the best visual image will be selected by a smart algorithm.

5. Genetic Algorithm

Genetic algorithm is an intelligent tool for optimization, In this method, the problem is transformed from decimal space to the binary space in which each row is a set of parameters as an answer to the problem. The quality of all the answers in each generation is evaluated by a proper function. The best answer is selected with the highest probability along with a random exchange of information in two parts for intersection. These solutions are then used with the genetic mutation used to maintain genetic diversity and avoid local extremes. The stimulus operator returns randomly selected bits to a genetic sequence. An example of a flowchart genetic algorithm is shown in Fig 1. In this case, an answer, containing the parameters a, b, c, will be the membership function of μ(a,b,c).

All of these parameters are integers from 0 to 255, similarly, an answer of the problem will be encrypted with 24 bits.

6. Experimental Results

In this section, we compare our method performance with that of the classical fuzzy entropy-based method and efficient methods used today for image tumor description. The generalized fuzzy entropy formulas mentioned in (3) ~ (6). The complement functions are segeno and Yager's complement.

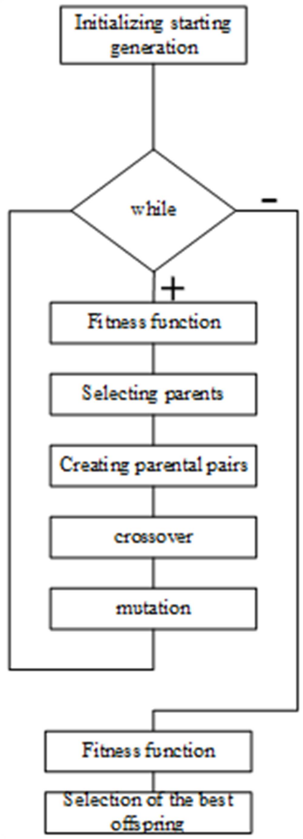

Figure 1. Flow Diagram of Genetic Algorithm

6.1. Compare with Traditional Fuzzy Entropy Based

The experimental images are meningioma and glioblastoma. For each of this images, first, we have extracted 19 Retrieved images with generalized fuzzy entropy and then with Genetic Algorithm we find the optimal image.

For meningioma (Figure 2), with the generalized fuzzy entropy, the threshold of fuzzy entropy is 104, and the final threshold of the generalized fuzzy entropy with (GA) is located at m = 0.75 (T=149). For glioblastoma (Figure 3), wuth the generalized fuzzy entropy, the threshold of fuzzy entropy is 117, and the final threshold of this method is located at m = 0.85 (T=157).

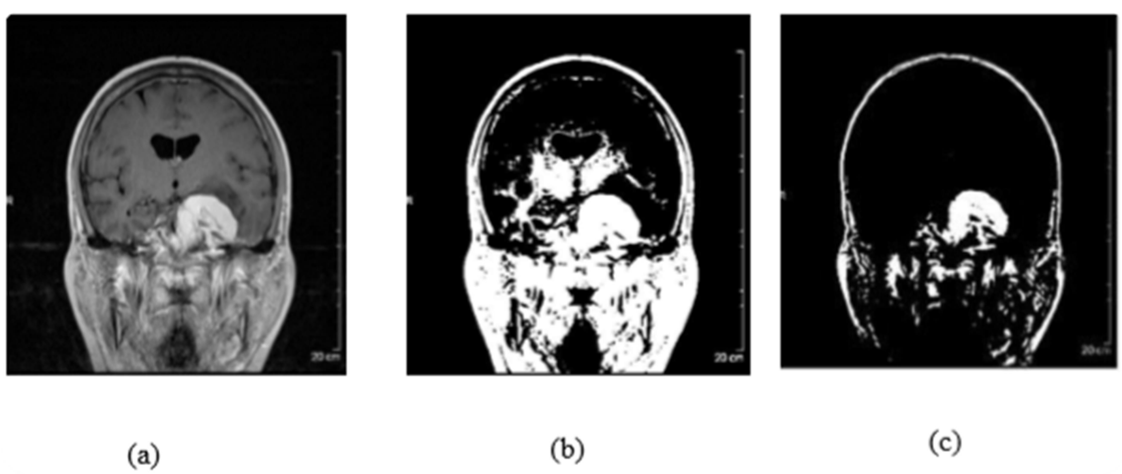

Figure 2. Meningioma image: (a) Prime image; (b) Processed with fuzzy entropy; (c) Processed with generalized entropy and (GA)

6.2. Compare with a Fuzzy Neural Network

In this part we compare the results obtained from this method with a fuzzy kohonen clustering network (N.I.Jabbar & M.Mehrotra, 2008). The result of the segmentation with our method obtained in m = 0.9 (T=200) (Figure 4).

Figure 4. The image results : (a) prime image; (b) processed with a fuzzy kohonen clustering network; (c) processed with generalized entropy and (GA)

6.3. Compare with a 3D Brain Tumor Segmentation in MRI Using Fuzzy Classification, Symmetry Analysis and Spatially Constrained Deformable Models

Based on this method, first we will have a segmentation in the presence of tumors. Then a tumor detection stage will be implemented with selection asymmetric regions, considering the approximate brain symmetry and fuzzy classification (H. Khotanloua, O.Colliotb, J.Atifc & I.Blocha, 2009, Moumen T El-Melegy & Hashim M Mokhtar, 2014, Chalumuri Revathi & B. Jagadeesh, 2017).

Compare this method with the method that we presented, gives the following result in m =0.7 (T=151) (Figure 5).

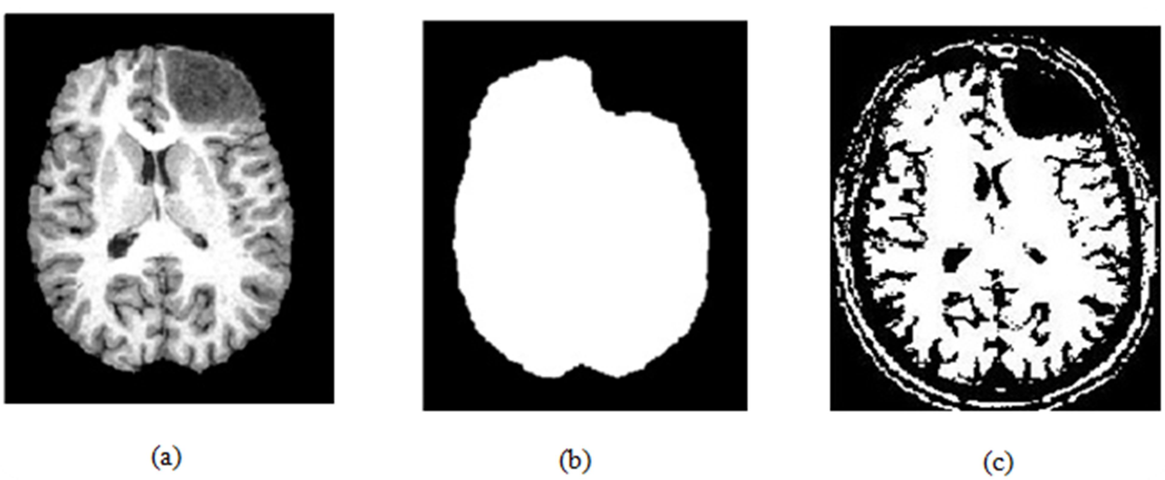

Figure 5. The image results: (a) prime image; (b) processed with a fuzzy classification, symmetry analysis and spatially constrained deformable models; (c) processed with generalized entropy and (GA)

7. Conclusions

With generalized fuzzy entropy and (GA)-based method we have segmentation for an image that the threshold can accept the membership value m. This method increases the chance of choosing optimal thresholds, so it provides better performance than the fuzzy entropy-based method.

Presented method is very effective for reducing the number of intensity levels. Problems may cause images with height amount of unwanted information which is saved to the expanse of subjective more important information. Results have shown that the use of generalized fuzzy entropy and the genetic algorithm can greatly be used to find the optimal threshold.

However, doing this method with other well-known algorithms such as ant colony optimization, particle swarm optimization and imperialist competitive algorithm, worth to research further.

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