Isomorphism Between Estes' Stimulus Fluctuation Model and a Physical-Chemical System

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Abstract

Although Estes' Stimulus Sampling Theory has almost completely lost its influence, its theoretical framework has not been disproved. Particularly, one theory in that framework, Stimulus Fluctuation Model, is still important because it explains spontaneous recovery. In this short note, the process of the theory is shown to be isomorphic to the diffusion of solution between compartments. Envisioning the theory as diffusion will make it appear less artificial and suggest natural extensions.

Currently Estes' theoretical framework, Stimulus Sampling Theory, will seem to have completely lost its influence. Recent textbooks on animal learning rarely mention it. If it is ever mentioned, it will be lumped together as "before Rescorla-Wagner" (Rescorla & Wagner, 1972) along with other old theories, and only the name will be mentioned without any mathematical details.

However, this picture misses the subtle nuance of the current research. Although Estes' specific theory was completely replaced by the Rescorla-Wagner model and other newer theories, the general framework of stimulus sampling has not been disproved, and indeed some newer theories still use this framework. (See Raaijmakers & Shiffrin, 2002, for a similar view).

The Stimulus Sampling Theory is a set of theories, not a single theory. One of them that has not lost relevance today is the Stimulus Fluctuation Model (Estes, 1955). This theory is still important because it explains spontaneous recovery while many current theories (e.g., the Rescorla-Wagner model) fail to do so.

To briefly summarize, spontaneous recovery is explained as follows. Stimulus elements are either conditioned or unconditioned. They are in either of two sets; the available set S, and the rest, the unavailable set S'. Only elements in the available set elicit response and undergo learning. Elements are exchanged with time between the two sets (stimulus fluctuation) with probability j and j'. After extensive conditioning, conditioned elements prevail both in S and S'. And then after extinction, responding gets weak as most elements in S became unconditioned ones as a result of learning. However, as time passes, elements in S and S' are gradually exchanged, resulting in restoration of conditioned elements into S. Therefore responding is somewhat restored (spontaneous recovery). Although a relatively ancient theory, this explanation has not been disproved and continues to be cited in the present day articles.

Formally, Estes derived

$$p(t)=p(0)[J-(J-1)a^t]+p'(0)(1-a^t)(1-J)$$

where $J=j'/(j+j')$ and $a=1-j-j'$.

$p(t)$ and $p'(t)$ represent proportion of conditioned elements in S and S' respectively at time t. Also see Mensink and Raaijmakers (1989) for a corresponding formulation in the continuous-time case.

In this brief note, this process is pointed out to be isomorphic to the physical-chemical process of diffusion. There are many forms of diffusion. Among them, the following type exactly corresponds to the Stimulus Fluctuation Model. Consider two compartments with volume V_1 and V_2 separated by a semipermeable membrane (Figure 1). Each compartment initially contains solution of concentration $C_1(0)$ and $C_2(0)$. And each compartment is well stirred, so that there are no concentration gradients within the compartment. With time, diffusion occurs across the membrane, and concentrations approach each other. This process is described by the same differential equations as in Mensink and Raaijmakers (1989). The solution is of course identical, although it can take quite different appearances depending on whether one expresses it in terms of the amount of substance or concentration. In terms of concentration, the following is the solution:

$$C_1 = \frac{C_2(0)V_2 + C_1(0)V_1 + V_2(C_1(0) - C_2(0))e^{\frac{-D(V_1+V_2)t}{V_1V_2}}}{V_1 + V_2}$$

$$C_2 = \frac{C_2(0)V_2 + C_1(0)V_1 - V_1(C_1(0) - C_2(0))e^{\frac{-D(V_1+V_2)t}{V_1V_2}}}{V_1 + V_2}$$

where D represents the rate of diffusion. The amount of substance corresponds to the number of conditioned elements, the concentration corresponds to the proportion of conditioned elements (which determines responding), and the volume corresponds to j and j'.

Of course, it will be meaningless to note the equivalence of a psychological model to a physical system if there are no merits to do so. Here there seem to be several clear merits. First, one can envision natural images. Without such an image, one is left wondering why one should consider an artificial setting in which there are balls that move between two boxes for an unknown reason with certain probabilities. Also one can seek natural extensions or variations of the theory. As the stimulus fluctuation process has been shown to be essentially diffusion, one can also consider other forms of diffusion; e.g.,

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2}.$$

In fact, this equation will be found first if one consults physical or chemical textbooks for diffusion. Also one may be able to find already-existing diffusion simulators to see vivid images of the process.

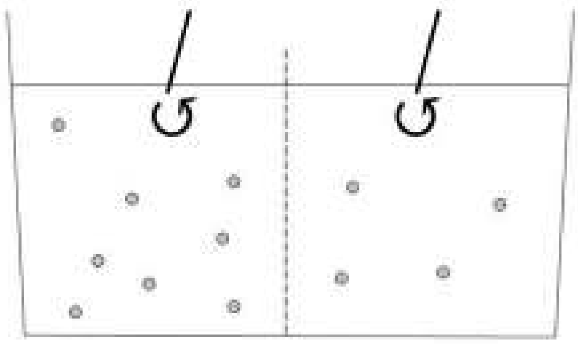

Figure 1. Two compartments containing solution separated by a membrane.

References

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