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# UNDERSTANDING THE MIXING PHENOMENA-FROM STRUCTURAL STABILITY TO CHAOS

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Abstract. Nowadays, computational fluid dynamics (CFD) becomes more and more mature. In the same time, it becomes more and more difficult to contribute fundamental research to it. Although the software tools in this area are increasing in importance, the way how CFD develops remain unpredictable, and it is part of what makes it an exciting and attractive discipline.

The mixing phenomena - and the mixing theory – are using more and more CFD tools. This modern theory issued in the flow kinematics after hundred of years of stability study, has mathematical methods and techniques which are developing a continuous significant relation between turbulence and chaos.

The turbulence is an important feature of dynamic systems with few freedom degrees, the so-called far from equilibrium systems. These are widespread between the models of excitable media. The present paper exhibits some recent results of the turbulent mixing study, based on computational tools of MAPLE11 soft. The data would be statistically analyzed, in order to construct a significant guideline in understanding the transition from stability to chaos in these excitable media.

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## 1.Introduction. Flow, kinematics

The mixing theory appears in an area with far from complete solving problems: the flow kinematics. Its methods and techniques developed the significant relation between turbulence and chaos. The turbulence is an important feature of dynamic systems with few freedom degrees, the so-called "far from equilibrium systems". These are widespread between the models of excitable media, and a recent goal is to find a consistent and coherent theory to stand up that a mixing model in excitable media leads to a far from equilibrium model.

After a hundred years of stability study, the problems of flow kinematics are far from complete solving. Since the beginnings, considering the stability of laminar flows with infinitesimal turbulences was a fruitful investigation method. The non-linearity could operate in the sense of stabilizing the flow, and then the basic flow is replaced with a new stable flow, which is considered a secondary flow. This secondary flow could be further replaced by a tertiary flow, and so on. In fact, it is about a bifurcations sequence, and the Couette flow could be the best example in this sense.

This context becomes more difficult if the non-linearity is in the sense of increasing of the growing rate of linear unstable modes. In fact, we are talking about strong turbulence problems, an area which still needs a lot of substance.

The problems of turbulence were recently approached in a special manner, in [1,5,6]. It concerns, on one hand, a special vortex technology which offers a lot of applications in all fields of bio-engineering, especially in processing the polluted fluids, and on the other hand, the mathematical and computational models used for handling this phenomena.

Starting from the importance of implementation of some optimized technologies for processing the polluted fluids, the benefits of this technology are both of scientific and technologic type [6]. It concerns, one one-hand, finding new physic- mathematical models for describing at optimal parameters the turbulent mixing created by a vorticity structure, and from technological standpoint, developing the vortex technology for handling the polluting materials.

Generally, the statistical idea of a flow is represented by a map:

$$x = \Phi_t(X), \; withX = \Phi_t(t=0)(X) \tag{1}$$

We say that $X$ is mapped in $x$ after a time t. In the continuum mechanics the relation (1) is named *flow*, and it is a diffeomorphism of class $C^k$. Moreover, the dynamical system defined by (1) must satisfy the relation:

$$0 \langle J \langle \infty, J = \det \left( \frac{\partial x_i}{\partial X_j} \right), J = det(D\Phi_t(X)) \tag{2}$$

where $D$ denotes the derivation with respect to the reference configuration, in this case $X$. The relation (2) implies two particles, $X_1$ and $X_2$, which occupy the same position x at a moment. Non-topological behavior (like break up, for example) *is not allowed.*

With respect to $X$ it is defined the basic measure for the deformation, namely the *deformation gradient*, $\mathbf{F}$:

$$\mathbf{F} = (\nabla_X \Phi_t(\mathbf{X}))^T, \; F_{ij} = \left( \frac{\partial x_i}{\partial X_j} \right) \tag{3}$$

where $\nabla_X$ denotes differentiation with respect to $X$. According to (2), $\mathbf{F}$ is non singular. The basic measure for the deformation with respect to x is the *velocity gradient* ( $\nabla_x$ denote differentiation with respect to $x$).

By differentiation of $x$ with respect to $X$ there are obtained the basic deformation for a material filament, and for the area of an infinitesimal material surface, namely the *length and surface deformation,* with the relations

$$\lambda = (C : MM)^{\frac{1}{2}}, \; \eta = (\det F) \cdot \left( C^{-1} : NN \right)^{\frac{1}{2}} \tag{4}$$

with $\mathbf{C}(= \mathbf{F}^T \mathbf{F})$ the Cauchy-Green deformation tensor, and the length and surface vectors $M, N$ defined by

$$\mathbf{M} = d\mathbf{X}/ |d\mathbf{X}|, \; \mathbf{N} = d\mathbf{A}/ |d\mathbf{A}| \tag{5}$$

the length and surface deformations have a scalar form, used in calculus, namely

$$\lambda = C_{ij} \cdot M_i \cdot N_j, \;\; \eta = (\det F) \cdot \left( C_{ij}^{-1} \cdot N_i \cdot N_j \right), \; with \sum M_i^2 = 1, \; \sum N_j^2 = 1 \tag{6}$$

The deformation tensor $\mathbf{F}$ and the associated tensors $\mathbf{C}$, $\mathbf{C}^{-1}$ represent the basic quantities in the deformation analysis for the infinitesimal elements.

In this framework, the mixing concept implies the *stretching* and *folding* of the material elements. If in an initial location $P$ there is a material filament $dX$ and an area element $dA$, there is defined the *deformation efficiency* in length and surface by the relations:

$$e_\lambda = \frac{D(\ln \lambda)}{Dt} = \mathbf{D} : \mathbf{mm}, \quad e_\eta = \frac{D(\ln \eta)}{Dt} = \nabla \mathbf{v} - \mathbf{D} : \mathbf{nn} \tag{7}$$

where $D$ is the deformation tensor, obtained by decomposing the velocity gradient in its symmetric and non-symmetric part [1,5].

In most cases the flow $x = \Phi_t(x)$ is unknown and has to be obtained by integration from the Eulerian velocity field. If this can be done analytically, then $\mathbf{F}$ can be obtained by differentiation of the flow with respect to the material coordinates $\mathbf{X}$. The flows of interest belong to two classes: i) flows with a special form of $\nabla v$ and ii) flows with a special form of $\mathbf{F}$. The second class is of very large interest, as it contains the so-called Constant Stretch History Motion – CSHM flows.

## 2.The analysis of the mixing behavior in excitable media

### 2.1. Computational features

The mathematical modeling has matched the experiments [1,4].

Numeric simulation of 3D multiphase flows is currently in study, approaching different computational implements. In the mathematical framework, the flow complexity implies the following three stages:

- modeling the global swirling streamlines;
- local modeling of the concentrated vorticity structure;
- introducing the elements of chaotic turbulence.

The mathematical model associated to the vortex phenomena is the 3D version of the widespread isochoric two-dimensional flow [5]

$$\begin{gathered} \dot{x_1} = G \cdot x_2 \\ \dot{x_2} = K \cdot G \cdot x_1, \ -1 < K < 1, \, G > 0 \end{gathered} \tag{8}$$

namely the following model is studied [1]:

$$\begin{gathered}\dot{x_1} = G \cdot x_2 \\ \dot{x_2} = K \cdot G \cdot x_1, \ -1 < K < 1, \, G > 0 \\ \dot{x_3} = c, \ c = const.\end{gathered} \tag{9}$$

where for the third component, the “z-axis” it was taken the rotation velocity, with a constant value, c.

A lot of tests were realized for this model, both analytical and numerical. The statistical cases were very few, about 60. There were approached various computational standpoints for the mixing phenomena, starting with the analysis of the mixing efficiency [1,2] for 3D model, and continuing with some phase-portrait analysis for 2D models [3].

In what follows it is approached another computational standpoint, in order to achieve more features for unifying this mixing theory. Namely, using specific tools of MAPLE11 soft, it is developed a phase-portrait analysis for 3D model in order to compare the performance of specific but different tools: the “phase-portrait” plot builder and the discrete – numeric plot builder.

The phase-portrait builder is a plot builder which realizes the phase-portrait for a differential equations system [7]. It is a fast procedure, based on specific numeric methods for approximating the solution of the studied differential system. For the present aim it was chosen the classical method of Euler, in fact “forward Euler” method.

From the parameters standpoint, there were chosen approximately the same values like in the analysis with discrete-numeric plot builder. The difference consists in choosing some classical initial conditions and varying the parameter KG. In what follows, there are presented some significant of the main studied cases. The cases are labeled on the figures.

## 2.2. The classical analysis case

In order to assign some basic features of different specific qualitative analysis, let us present in what follows some basic cases of the classic analysis of the 3D mixing model associated to the vortex technology presented above [4]. Since in the classic analysis the target is the study of the efficiencies $e_\lambda$ and $e_\eta$ at *successive* moments, the analysis is a discrete one, which aims testing the special events that could appear at various, random, values of the versors $M_i, \ N_j$. Therefore, the factor $(D:D)^{1/2}$ which in concrete cases has numerical values, has not an important significance in the numeric analysis.

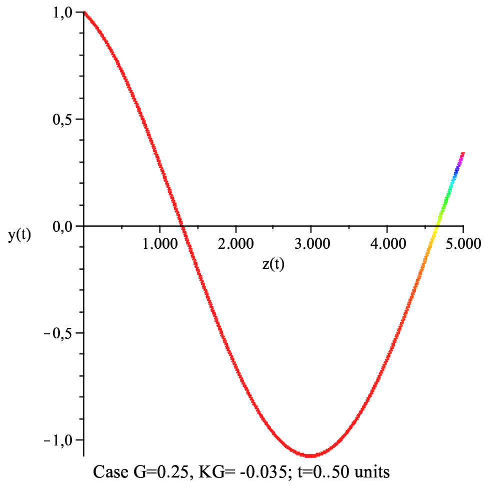
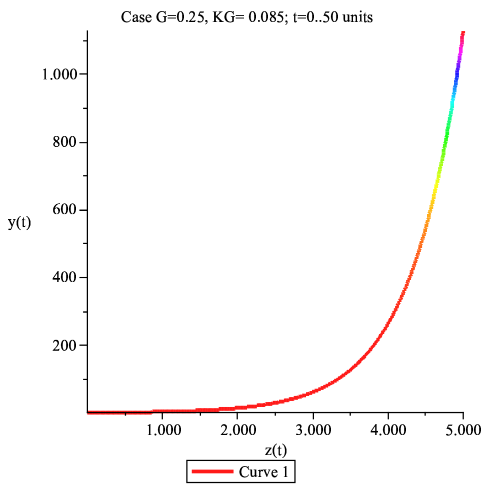



Figure 1:

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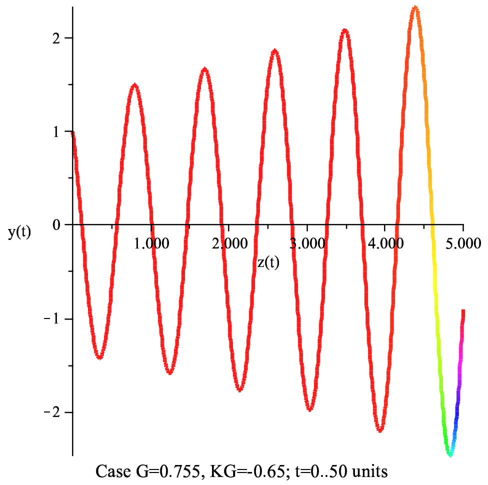


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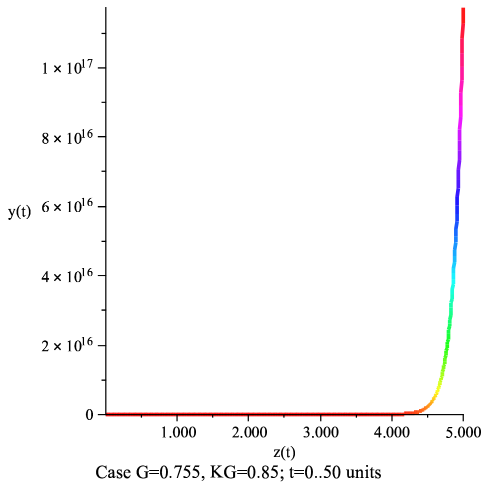


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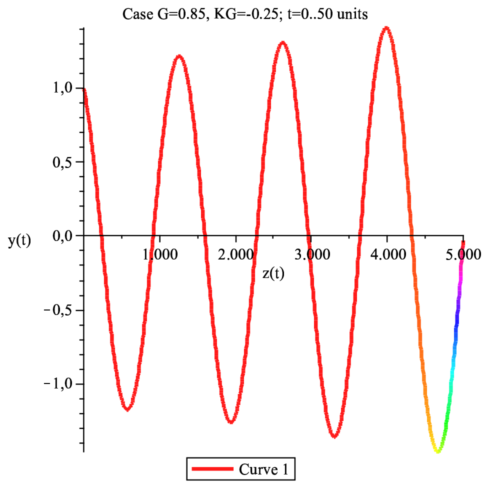


Figure 5:

With the numeric soft MAPLE11, the analysis has two parts. First, the following Cauchy problems are solved:

$$e_{\lambda}(t) = 0,\ x(0) = 0, e_{\eta}(t) = 0,\ \ x(0) = 0 \tag{10}$$

using a discrete numeric calculus procedure. The output of the procedure is a listing of the form $(t_i,\ x(t_i)), i = 0..25$ . In the second part there are realized discrete time plots, in 25 time units, following the listings obtained in the first part. The plots represent in fact *the image of the length and surface deformations*, in the established scale time.

Very few irrational value sets were chosen for the length, respectively surface versors [1,2,3]. It was taken into account the versor condition: $\sum M_i = 1$, $\sum N_j = 1$. Also, for the parameter K there were chosen few values, between them let us notice $K = 0.25$, regained also in the discrete-numeric plot builder analysis.

In what follows, some of significant graphical simulations are exhibited. Since the surface deformation is very significant, there are exhibited significant cases of this process.

## 3.Qualitative results. Remarks

At a first sight of the graphics, it is obvious that both in discrete and

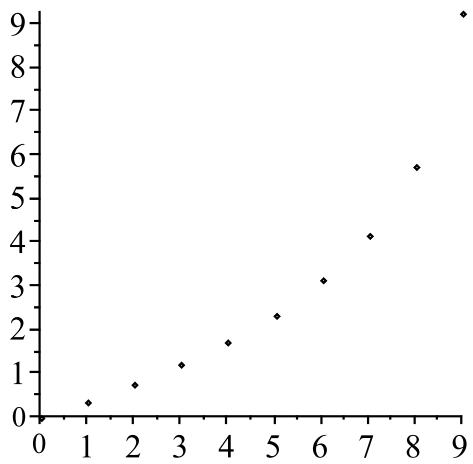


Figure 6:

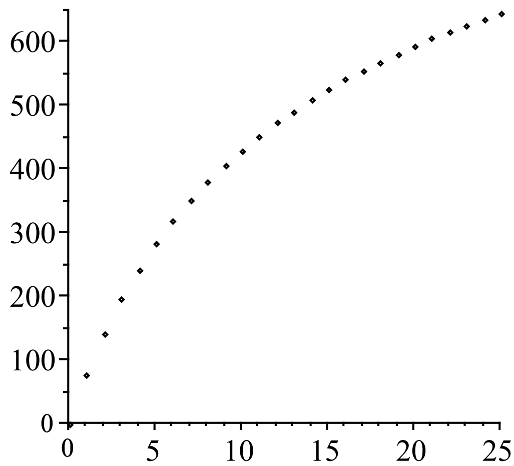


Figure 7:

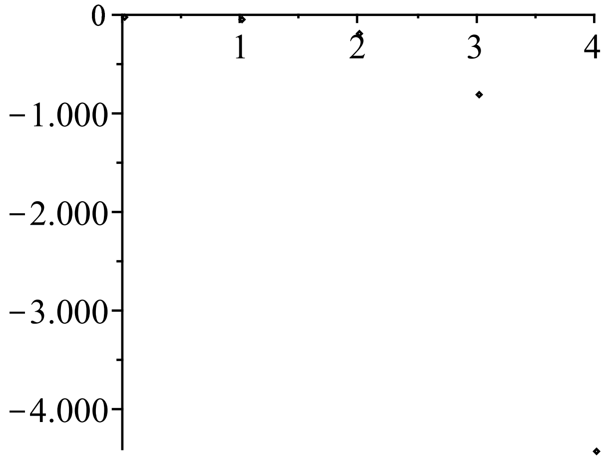


Figure 8:

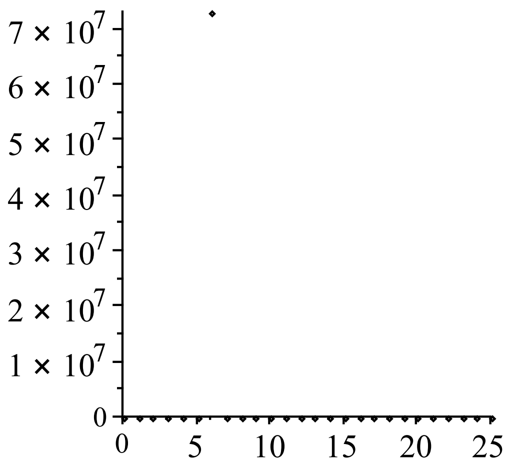


Figure 9:

continuous case, the phenomena is not linear; there are relatively linear cases – fig. 4,6,7 – but especially non-linear cases – the other pictures;

In fact it is about two types of phenomena for the same mixing model, and this is extremely important: on one hand, in the phase-portrait analysis it is very important to notice that when modifying the parameters the behavior is going to be periodic – fig.3,5 – for the same time units, and on the other hand, in the classical (discrete) analysis, when modifying the parameters there are involved the so-called "rare events" [1,4], corresponding to the breakup of the simulation – fig.8;

Thus, for the non-periodic flow, it must be noticed that a little perturbation has a consistent influence on the model, going into a far from equilibrium model. If there is annexed the irrationality of the length / surface versors values (an appliance used since the beginning of the mixing study [1,2,3,4]), a globally panel is obtained, with random distributed events. This space-time context consolidates the basic statement that the turbulent mixing flows must be approached as chaotic systems. This is in fact regaining the idea of a system / model high sensitive to initial conditions.

It is obvious that the testing of more and more values / parameters sets remains basic. If for 3D case, 60 statistical cases suffice for proving the special rare events, it is expected an acceptance testing of the consistency with the interdisciplinary area. Thus, the issues of repetitive phenomena (we have these phenomena in both analysis types above), give rise to achieve some appliances of chaotic dynamical systems. Also a next aim is testing more MAPLE11 appliances, both from numeric and graphic standpoint. Cumulating these features would produce numeric models, giving new research directions on the behavior in excitable media. This would be a next aim.

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