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On a Bivariate Poisson Negative Binomial Risk Process

Krasimira Kostadinova[1], Leda Minkova[2]
[1]Faculty of Mathematics and Informatics, Shumen University "K. Preslavsky"
Shumen, Bulgaria, Email: kostadinova@shu-bg.net
[2]Faculty of Mathematics and Informatics, Sofia University "St. Kl.Ohridski"
Sofia, Bulgaria, Email: leda@fmi.uni-sofia.bg

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***Abstract*—In this paper we define a bivariate counting process as a compound Poisson process with bivariate negative binomial compounding distribution. We investigate some of its basic properties, recursion formulas and probability mass function. Then we consider a risk model in which the claim counting process is the defined bivariate Poisson negative binomial process. For the defined risk model we derive the distribution of the time to ruin in two cases and the corresponding Laplace transforms. We discuss in detail the particular case of exponentially distributed claims.**

***Keywords*-bivariate negative binomial distribution; compound birth process; ruin probability**

I. Introduction

We consider the stochastic process $N(t),\ t > 0$ defined on a fixed probability space $(\Omega, \mathcal{F}, P)$ and given by

$$N(t) = X_1 + X_2 + \ldots + X_{N_1(t)}, \tag{1}$$

where $X_i,\ i = 1, 2, \ldots$ are independent, identically distributed (iid) as X random variables, independent of $N_1(t)$. We suppose that the counting process $N_1(t)$ is a Poisson process with intensity $\lambda > 0$ ($N_1(t) \sim Po(\lambda t)$). In this case $N(t)$ is a compound Poisson process. The probability mass function (PMF) and probability generating function (PGF) of $N_1(t)$ are given by

$$P(N_1(t) = i) = \frac{(\lambda t)^i e^{-\lambda t}}{i!}, \quad i = 0, 1, \ldots \tag{2}$$

and

$$\psi_{N_1(t)}(s) = e^{-\lambda t(1-s)}. \tag{3}$$

The compound Poisson distribution is analyzed by many authors; see Johnson et al. [3], Grandell, [2], Minkova [8]. The corresponding compound Poisson process is commonly used as a counting process in risk models; see for example Klugman et al. [5], Minkova [9].

In this paper we suppose that the compounding random variable X has a bivariate negative binomial distribution, given in the next Section II. In Section III we define a counting process with the Bivariate Poisson Negative Binomial distribution (BPNB). We derive the moments and the joint PMF. Then, in Section IV, two types of ruin probability are considered for the risk model with BPNB distributed counting process. We derive the Laplace transforms and analyze the case of exponentially distributed claims.

II. Bivariate Negative Binomial Distribution

Let us consider the bivariate negative binomial distribution, defined by the following PGF, given in Kocherlakota and Kocherlakota [6]

$$\psi_1(s_1, s_2) = \left(\frac{\gamma}{1 - \alpha s_1 - \beta s_2}\right)^r, \tag{4}$$

where $\gamma = 1 - \alpha - \beta$ and $r \geq 1$ is a given integer number. We use the notation $(X, Y) \sim BNB(r, \alpha, \beta)$. The PMF of (X, Y) is given by

$$\begin{aligned} &P(X = k, Y = l) \\ &= \binom{k+l}{l}\binom{r+k+l-1}{k+l}\alpha^k \beta^l \gamma^r \end{aligned} \tag{5}$$

for $k, l = 0, 1, \ldots,$ $(k, l) \neq (0, 0)$, and $P(X = 0, Y = 0) = \gamma^r$. The marginal distributions are again negative binomial with PGFs

$$\psi_1(s_1) = \psi_1(s_1, 1) = \left(\frac{\gamma}{1 - \beta - \alpha s_1}\right)^r \tag{6}$$

and

$$\psi_1(s_2) = \psi_1(1, s_2) = \left(\frac{\gamma}{1 - \alpha - \beta s_2}\right)^r. \tag{7}$$

Denote by $\rho_1 = \frac{\alpha}{1-\beta}$ and $\rho_2 = \frac{\beta}{1-\alpha}$ the corresponding parameters of X and Y. In terms of ρ_1 and ρ_2, the PGFs (6) and (7) have the form

$$\psi_1(s_1) = \left(\frac{1 - \rho_1}{1 - \rho_1 s_1}\right)^r$$

and

$$\psi_1(s_2) = \left(\frac{1 - \rho_2}{1 - \rho_2 s_2}\right)^r.$$

The PGF of the sum $X + Y$ is given by

$$\psi_1(s, s) = \left(\frac{\gamma}{1 - (\alpha + \beta)s}\right)^r = \left(\frac{\gamma}{1 - (1 - \gamma)s}\right)^r. \tag{8}$$

From (8) it follows that $X + Y$ has a negative binomial distribution with parameters r and γ, say $X + Y \sim NB(r, \gamma)$, and for $j = 0, 1, \ldots,$ PMF

$$P(X + Y = j) = \binom{r+j-1}{j}\gamma^r (1 - \gamma)^j.$$

Denote the PGF in (8) by $\psi_1(s) = \psi_1(s, s)$.

III. The bivariate counting process

In this section we consider a compound Poisson process with bivariate negative binomial compounding distribution. The resulting process is a bivariate counting process $(N_1(t), N_2(t))$, defined by the PGF

$$\psi(s_1, s_2) = e^{-\lambda t(1 - \psi_1(s_1, s_2))}, \tag{9}$$

where $\psi_1(s_1, s_2)$ is the PGF of the compounding distribution, given in (4). We say that the counting process defined by (9) has a bivariate Poisson Negative binomial distribution with parameters $\lambda t, \alpha$ and β, and use the notation $(N_1(t), N_2(t)) \sim BPNB(\lambda t, \alpha, \beta)$. The marginal distributions are defined by the following PGFs

$$\psi(s_1) = e^{-\lambda t(1 - \psi_1(s_1))} \quad \text{and} \quad \psi(s_2) = e^{-\lambda t(1 - \psi_1(s_2))},$$

where $\psi_1(s_1)$ and $\psi_1(s_2)$ are given by (6) and (7).

The means are given by $E(N_1(t)) = \frac{r\alpha\lambda t}{\gamma}$ and $E(N_2(t)) = \frac{r\beta\lambda t}{\gamma}$, while the variances are $Var(N_1(t)) = \frac{\alpha r}{\gamma^2}[1 + r\alpha - \beta]\lambda t$ and $Var(N_2(t)) = \frac{\beta r}{\gamma^2}[1 - \alpha + r\beta]\lambda t$.

From (9) we obtain

$$\begin{aligned} &\frac{\partial^2 \psi(s_1, s_2)}{\partial s_1 \partial s_2} = \psi(s_1, s_2) r\alpha\beta\lambda t \\ &\times \left[\frac{r\gamma^{2r}\lambda t}{(1 - \alpha s_1 - \beta s_2)^{2r+2}} + \frac{(r + 1)\gamma^r}{(1 - \alpha s_1 - \beta s_2)^{r+2}}\right]. \end{aligned}$$

Upon setting $s_1 = s_2 = 1$ we obtain the product moment of $N_1(t)$ and $N_2(t)$ to be

$$E(N_1(t)N_2(t)) = \frac{r\alpha\beta}{\gamma^2}(r\lambda t + r + 1)\lambda t,$$

which yields the covariance between $N_1(t)$ and $N_2(t)$ as

$$Cov(N_1(t), N_2(t)) = \frac{r(r + 1)\alpha\beta}{\gamma^2}\lambda t.$$

For the correlation coefficient we have

$$\begin{aligned} &Corr(N_1(t), N_2(t)) \\ &= (r + 1)\sqrt{\frac{\alpha\beta}{(1+r\alpha-\beta)(1+r\beta-\alpha)}}. \end{aligned}$$

In terms of ρ_1 and ρ_2, the covariance and the correlation coefficient have the forms

$$Cov(N_1(t), N_2(t)) = \frac{r(r+1)\rho_1\rho_2}{(1-\rho_1)(1-\rho_2)}\lambda t$$

and

$$Corr(N_1(t), N_2(t)) = (r+1)\sqrt{\frac{\rho_1\rho_2}{(1+r\rho_1)(1+r\rho_2)}}.$$

A. Joint Probability Mass Function

The probability function of the joint distribution of $(N_1(t), N_2(t))$ is given by expanding the PGF $\psi(s_1, s_2)$ in powers of s_1 and s_2. Denote by $f(i, j) = P(N_1(t) = i, N_2(t) = j)$, $i, j = 0, 1, 2, \ldots$, the joint probability mass function of $(N_1(t), N_2(t))$. We rewrite the PGF of (9) in the form

$$\begin{aligned}\psi(s_1, s_2) &= e^{-\lambda t}\sum_{m=0}^{\infty}\frac{(\lambda t)^m}{m!}\psi_1^m(s_1, s_2)\\ &= e^{-\lambda t}\sum_{m=0}^{\infty}\frac{(\lambda t\gamma^r)^m}{m!}\frac{1}{(1-\alpha s_1-\beta s_2)^{rm}}.\end{aligned} \tag{10}$$

Denote by $\psi^{(i,j)}(s_1, s_2) = \dfrac{\partial^{i+j}\psi(s_1, s_2)}{\partial s_1^i \partial s_2^j}$, for $i, j = 0, 1, \ldots$, and $(i, j) \neq (0, 0)$, the derivatives of $\psi(s_1, s_2)$. From (10) we get the following:

$$\begin{aligned}&\psi^{(i,j)}(s_1, s_2) = e^{-\lambda t}\alpha^i\beta^j\textstyle\sum_{m=1}^{\infty}\frac{(\lambda t\gamma^r)^m}{m!}\\ &\times\frac{rm(rm+1)\ldots(rm+i-1)(rm+i)\ldots(rm+i+j-1)}{(1-\alpha s_1-\beta s_2)^{rm+i+j}}.\end{aligned} \tag{11}$$

From Johnson et al. [4], it is known that for $i, j = 0, 1, \ldots,\ \ (i, j) \neq (0, 0)$,

$$f(i, j) = \left.\frac{\psi^{(i,j)}(s_1, s_2)}{i!j!}\right|_{s_1=s_2=0}. \tag{12}$$

The result is given in the next theorem.

Theorem 1. *The probability mass function of $(N_1(t), N_2(t))$ is given by*

$$\begin{aligned}f(i, j) &= \binom{i+j}{j}\alpha^i\beta^j\\ &\times\sum_{m=1}^{\infty}\binom{rm+i+j-1}{i+j}\frac{(\lambda t\gamma^r)^m}{m!}e^{-\lambda t},\\ &i, j = 0, 1, \ldots,\ (i, j) \neq (0, 0),\end{aligned} \tag{13}$$

and $f(0, 0) = e^{-\lambda t(1-\gamma^r)}$.

Proof. The initial value $f(0, 0) = e^{-\lambda t(1-\gamma^r)}$ follows simply from the PGF $\psi(0, 0) = f(0, 0)$. Then (13) follows from (11) and (12).

□

IV. Bivariate risk model

Consider the following bivariate surplus process

$$\begin{aligned}U_1(t) &= u_1 + c_1 t - \textstyle\sum_{j=1}^{N_1(t)} Z_j^1\\ U_2(t) &= u_2 + c_2 t - \textstyle\sum_{j=1}^{N_2(t)} Z_j^2\end{aligned}$$

for two lines of business. Here u_1 and u_2 are the initial capitals, c_1, c_2 represent the premium incomes per unit time and $Z^1, Z_1^1, Z_2^1, \ldots,$ and $Z^2, Z_1^2, Z_2^2, \ldots$ are two independent sequences of independent random variables, independent of the counting processes $N_1(t)$ and $N_2(t)$, representing the corresponding claim sizes. The univariate case of this model was analyzed in Kostadinova [7]. Let $\mu_1 = E(Z^1)$ and $\mu_2 = E(Z^2)$ be the means of the claims. Denote by $S_1(t) = \sum_{j=1}^{N_1(t)} Z_j^1$ and $S_2(t) = \sum_{j=1}^{N_2(t)} Z_j^2$ the corresponding accumulated claim processes. The model, analyzed in Chan et al., [1] is the case when $N_1(t) = N_2(t) = N(t)$. Here we consider two possible times to ruin

$$\tau_{max} = \inf\{t|\max(U_1(t), U_2(t)) < 0\}$$

and

$$\tau_{sum} = \inf\{t|U_1(t) + U_2(t) < 0\},$$

and the corresponding ruin probabilities $\Psi_{\max}(u_1, u_2) = P(\tau_{max} < \infty)$ and $\Psi_{sum}(u_1, u_2) = P(\tau_{sum} < \infty)$. For the event of τ_{max} we have the following:

$$\{\max(U_1(t), U_2(t)) < 0\} = \{U_1(t) < 0, U_2(t) < 0\}$$

$$= \{u_1 + c_1 t - S_1(t) < 0, u_2 + c_2 t - S_2(t) < 0\}$$

$$= \{S_1(t) > u_1 + c_1 t, S_2(t) > u_2 + c_2 t\}.$$

It follows that the ruin probability $\psi_{\max}(u_1, u_2)$ is the joint survival function of $(S_1(t), S_2(t))$.

In a similar way, we obtain the event for the τ_{sum} :

$$\{U_1(t) + U_2(t) < 0\}$$

$$= \{S_1(t) + S_2(t) > u_1 + u_2 + (c_1 + c_2)t\},$$

i.e., the ruin probability $\psi_{sum}(u_1, u_2)$ is the survival function of the sum $S_1(t) + S_2(t)$.

According to the definition of $\Psi_{max}(u_1, u_2)$, for the no initial capitals, we have the following

$$\Psi_1(0)\Psi_2(0) \le \Psi_{max}(0, 0) \le \min\{\Psi_1(0), \Psi_2(0)\},$$

where $\Psi_1(0)$ and $\Psi_2(0)$ are the ruin probabilities of the models $U_1(t)$ and $U_2(t)$ with no initial capitals. The univariate Poisson Negative binomial risk model is analyzed in [7], where it is given that

$$\Psi_1(0) = \frac{\rho_1}{c_1(1 - \rho_1)}\lambda\mu_1 r$$

and

$$\Psi_2(0) = \frac{\rho_2}{c_2(1 - \rho_2)}\lambda\mu_2 r.$$

It follows that the upper bound of the ruin probability is given by

$$\Psi_{max}(0, 0) \le \min\{\frac{\rho_1}{c_1(1 - \rho_1)}\lambda\mu_1 r, \frac{\rho_2}{c_2(1 - \rho_2)}\lambda\mu_2 r\}.$$

The lower bound has the form

$$\Psi_{max}(0, 0) \ge \frac{\rho_1\rho_2}{c_1 c_2(1 - \rho_1)(1 - \rho_2)}\lambda^2\mu_1\mu_2 r^2.$$

A. Laplace transforms

Denote by $LT_{Z^1}(s_1)$ and $LT_{Z^2}(s_2)$ the Laplace transforms of the random variables Z^1 and Z^2. Then, the Laplace transform of $(S_1(t), S_2(t))$ is given by

$$LT_{(S_1(t),S_2(t))}(s_1, s_2) = E[e^{-s_1 S_1(t) - s_2 S_2(t)}]$$

$$= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} E[e^{-s_1(Z_1^1+\ldots+Z_i^1) - s_2(Z_1^2+\ldots+Z_j^2)}]$$

$$\times P(N_1(t) = i, N_2(t) = j).$$

According to the construction of the counting process, for the Laplace transform of $(S_1(t), S_2(t))$ we have

$$LT_{(S_1(t),S_2(t))}(s_1, s_2)$$

$$= \psi_{(N_1(t),N_2(t))}(LT_{Z^1}(s_1), LT_{Z^2}(s_2))$$

$$= e^{-\lambda t\left[1-\left(\frac{\gamma}{1-\alpha LT_{Z^1}(s_1)-\beta LT_{Z^2}(s_2)}\right)^r\right]}.$$

Using the parameters

$$\frac{\gamma}{1 - \beta} = 1 - \rho_1 \quad \text{and} \quad \frac{\gamma}{1 - \alpha} = 1 - \rho_2$$

of the marginal compounding distributions, we obtain the Laplace transforms of the marginal compound distributions to be

$$LT_{S_1(t)}(s_1) = LT_{(S_1(t),S_2(t))}(s_1, 0)$$

$$= e^{-\lambda t\left[1-\left(\frac{1-\rho_1}{1-\rho_1 LT_{Z^1}(s_1)}\right)^r\right]}$$

and

$$LT_{S_2(t)}(s_2) = LT_{(S_1(t),S_2(t))}(0, s_2)$$

$$= e^{-\lambda t\left[1-\left(\frac{1-\rho_2}{1-\rho_2 LT_{Z^2}(s_2)}\right)^r\right]}.$$

We need the following result about Laplace transforms, given in Omey and Minkova ([10])

Lemma 1. *For the joint survival function $P(S_1(t) > x, S_2(t) > y)$ we have*

$$\int_0^\infty\int_0^\infty e^{-s_1 x - s_2 y} P(S_1(t) > x, S_2(t) > y)dxdy$$

$$= \frac{1 - LT_{S_1(t)}(s_1) - LT_{S_2(t)}(s_2) + LT_{(S_1(t),S_2(t))}(s_1, s_2)}{s_1 s_2}. \tag{14}$$

In our case we have

$$\int_0^\infty \int_0^\infty e^{-s_1 x - s_2 y} P(S_1(t) > x, S_2(t) > y) dxdy$$

$$= \frac{1}{s_1 s_2}\left[1 - e^{-\lambda t\left[1-\left(\frac{1-\rho_1}{1-\rho_1 LT_{Z^1}(s_1)}\right)^r\right]}\right.$$

$$-e^{-\lambda t\left[1-\left(\frac{1-\rho_2}{1-\rho_2 LT_{Z^2}(s_2)}\right)^r\right]}$$

$$\left. + e^{-\lambda t\left[1-\left(\frac{\gamma}{1-\alpha LT_{Z^1}(s_1)-\beta LT_{Z^2}(s_2)}\right)^r\right]}\right].$$

Lemma 2. *For the survival function $P(S_1(t) + S_2(t) > x)$ we have*

$$\int_0^\infty e^{-sx} P(S_1(t) + S_2(t) > x) dx = \frac{1}{s}[1 - LT_{S_1(t)+S_2(t)}(s)]. \qquad (15)$$

Then, for the ruin probability ψ_{sum} we have:

$$LT_{S_1(t)+S_2(t)}(s) = LT_{S_1(t),S_2(t)}(s, s)$$

$$= e^{-\lambda t\left[1-\left(\frac{\gamma}{1-\alpha LT_{Z^1}(s)-\beta LT_{Z^2}(s)}\right)^r\right]}.$$

and hence

$$\int_0^\infty e^{-sx} P(S_1(t) + S_2(t) > x) dx$$

$$= \frac{1}{s}\left[1 - e^{-\lambda t\left[1-\left(\frac{\gamma}{1-\alpha LT_{Z^1}(s)-\beta LT_{Z^2}(s)}\right)^r\right]}\right].$$

B. Exponentially distributed claims

Let us consider the case of exponentially distributed claim sizes, i.e. $F_{Z^1}(x) = 1 - e^{-\frac{x}{\mu_1}},\ x \geq 0$ and $G_{Z^2}(y) = 1 - e^{-\frac{y}{\mu_2}},\ y \geq 0$, and $\mu_1, \mu_2 > 0$.

Denote by

$$e(n, x) = \sum_{k=0}^{n} \frac{x^k}{k!} = \frac{e^x \Gamma(n + 1, x)}{\Gamma(n + 1)},$$

where $\Gamma(n)$ is a Gamma function and $\Gamma(a, x) = \int_x^\infty t^{a-1} e^{-t} dt$ is the incomplete Gamma function, the truncated exponential sum function.

For the ruin probability ψ_{max} we have

$$P(S_1(t) > x, S_2(t) > y)$$

$$= \sum_{i,j=0}^\infty \overline{F}^{*i}(x)\overline{G}^{*j}(y)P(N_1(t) = i, N_2(t) = j),$$

where $\overline{F}^{*i}(x) = e^{-\frac{x}{\mu_1}} e(i-1, \frac{x}{\mu_1}), i = 1, 2, \dots$ is the tail distribution of $Z_1^1 + \dots + Z_i^1$ and $\overline{G}^{*j}(y) = e^{-\frac{y}{\mu_2}} e(j-1, \frac{y}{\mu_2}), j = 1, 2, \dots$ is the tail distribution of $Z_1^2 + \dots + Z_j^2$.

In this case we have

$$P(S_1(t) > x, S_2(t) > y) = e^{-\lambda t(1-\gamma^r)}$$

$$+ \sum_{i=1}^\infty \alpha^i e(i-1, \tfrac{x}{\mu_1})$$

$$\times \sum_{m=1}^\infty \binom{rm+i-1}{i} \frac{(\lambda t \gamma^r)^m}{m!} e^{-\frac{x}{\mu_1}} e^{-\lambda t}$$

$$+ \sum_{j=1}^\infty \beta^j e(j-1, \tfrac{y}{\mu_2})$$

$$\times \sum_{m=1}^\infty \binom{rm+j-1}{j} \frac{(\lambda t \gamma^r)^m}{m!} e^{-\frac{y}{\mu_2}} e^{-\lambda t}$$

$$+ \sum_{i=1}^\infty \sum_{j=1}^\infty \alpha^i \beta^j e(i-1, \tfrac{x}{\mu_1}) e(j-1, \tfrac{y}{\mu_2}) \binom{i+j}{j}$$

$$\times \sum_{m=1}^\infty \binom{rm+i+j-1}{i+j} \frac{(\lambda t \gamma^r)^m}{m!} e^{-\frac{x}{\mu_1}-\frac{y}{\mu_2}} e^{-\lambda t}.$$

Substituting $x = u_1 + c_1 t$ and $y = u_2 + c_2 t$ in the last expression, we obtain the ruin probability $\psi_{max}(u_1, u_2)$, as it was shown in the previous section.

For the ruin probability $\psi_{sum}(u_1, u_2)$ in the case $Z^1 = Z^2 = Z$, and hence $\mu_1 = \mu_2 = \mu$, we obtain for the Laplace transform of the sum $S_1(t)+S_2(t)$:

$$LT_{S_1(t)+S_2(t)}(s) = e^{-\lambda t\left[1-\left(\frac{\gamma}{1-(\alpha+\beta)LT_Z(s)}\right)^r\right]}.$$

This means that

$$S_1(t) + S_2(t) = Z_1 + \dots + Z_{N(t)},$$

where $N(t) = X_1 + \dots + X_{N_1(t)},\ \ N_1(t) \sim Po(\lambda t)$, and $X_i \sim NB(r, \gamma)$.

The survival function of the sum $S_1(t) + S_2(t)$ is the survival function of the sum of claims, i.e.

$$P(S_1(t) + S_2(t) > x) = P(Z_1 + \dots + Z_{N(t)} > x)$$

$$= \sum_{i=0}^\infty \overline{F}^{*i}(x) P(N(t) = i),$$

where $\overline{F}^{*i}(x) = e^{-\frac{x}{\mu}} e(i-1, \frac{x}{\mu}), i = 1, 2, \dots$ is the tail distribution of $Z_1 + \ldots + Z_i$. Hence we have

$$P(S_1(t) + S_2(t) > x) = e^{-\lambda t(1-\gamma^r)}$$

$$+ \sum_{i=1}^{\infty}(1-\gamma)^i e(i-1, \tfrac{x}{\mu})$$

$$\times \sum_{m=1}^{\infty} \binom{rm+i-1}{i} \frac{(\lambda t \gamma^r)^m}{m!} e^{-\frac{x}{\mu}} e^{-\lambda t}$$

$$= e^{-\lambda t(1-\gamma^r)} + \sum_{i=1}^{\infty}(\alpha+\beta)^i e(i-1, \tfrac{x}{\mu})$$

$$\times \sum_{m=1}^{\infty} \binom{rm+i-1}{i} \frac{(\lambda t \gamma^r)^m}{m!} e^{-\frac{x}{\mu}} e^{-\lambda t}$$

and for $x = u_1 + u_2 + (c_1 + c_2)t$, we obtained the ruin probabilities ψ_{sum}.

V. Conclusion

In this study we introduce a compound Poisson process with bivariate negative binomial compounding distribution. Also, we find the moments and the joint probability mass function. Then we define the bivariate risk model with bivariate Poisson negative binomial counting process. We find the Laplace transform of the ruin probability and investigate a special case of exponentially distributed claims.

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