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COVID-19 RESEARCH COMMUNICATION

Modelling the impacts of lockdown and isolation on the eradication of COVID-19

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***Abstract*—A model describing the dynamics of COVID-19 is formulated and examined. The model is meant to address the impacts of lockdown and social isolation as strategies for the eradication of the pandemic. Local stability analysis indicate that the equilibria are locally-asymptotically stable for $\mathscr{R}_0 < 1$ and $\mathscr{R}_0 > 1$ for the disease-free equilibrium and the endemic equilibrium respectively. Numerical simulations of the model equations show that lockdown is a more effective strategy in the eradication of the disease than social isolation. However, strict enforcement of both strategies is the most effective means that could end the disease within a shorter period of time.**

***Keywords*-COVID-19; isolation; lockdown; disease-free equilibrium; endemic equilibrium**

I. Introduction

The coronavirus disease, otherwise known as COVID-19, has been ravaging the world since the beginning of the year 2020. The disease which started in Wuhan, China in late December 2019, rapidly spread to almost all parts of the world by March 2020. Consequently, it was declared a global health emergency by the World Health Organisation (WHO) on March 11, 2020 ([1], [2]). Some of the immediate non-pharmaceutical measures taken in order to contain the spread of the pandemic includes quarantine, isolation, lockdown, border closure, wearing of face masks and other sanitary measures, including regular washing of hands and use of hand sanitizers ([4], [7]). Medical experts have acknowledged the effectiveness of these measures in curbing the spread of the disease ([6], [3]). However, the question remains as to how we can quantify the effectiveness of these non-pharmaceutical measures. Some mathematical models have been used to address some of these strategies, including the work of Frost et al [5] who examined the effects of lockdown on the control of COVID-19 in some African countries and predicted when the pandemic would end under different lockdown regimes. Anguelov et al [9] also examined the effects of lockdown on the control of the disease, especially the different levels of lockdown intensity in South Africa. Lockdowns and social isolation have been widely used as

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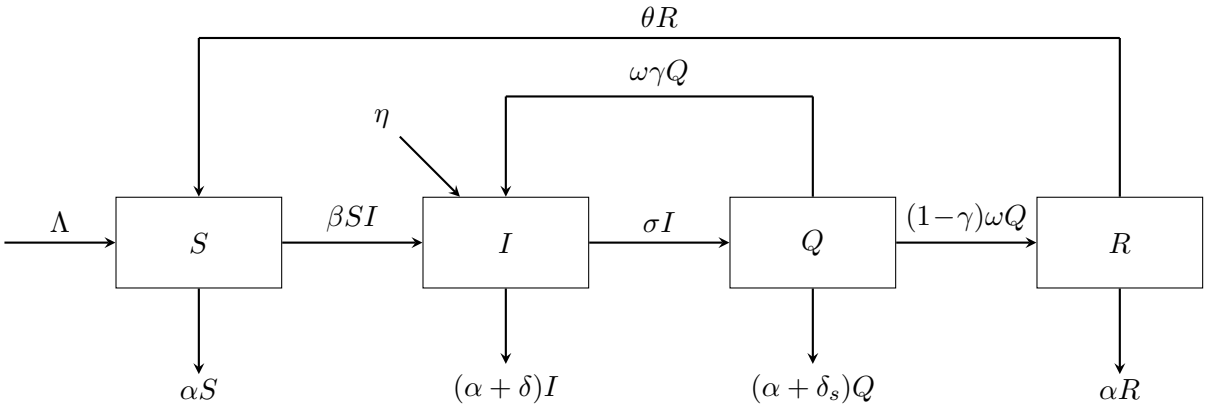

Fig. 1. Flow diagram for the transmission of the pandemic.

means of curbing the rapid spread of the pandemic [10], as well as curtailing the importation of the disease [11]. In the current work, we intend to examine the impacts of lockdown and social isolation in the eradication of the pandemic. We shall also try to establish which strategy is more effective in containing the disease in comparison with others. This, we believe, can guide the implementation of policies for optimal results. The remaining part of this paper are organised as follows: Section 2 is dedicated to the mathematical formulation of the model, stability analysis of equilibria will be the subject of section 3, while numerical simulations and discussion will be considered in section 4 and the conclusion will be done in section 5.

II. Formulation of the Model

We construct a mathematical model which focuses on the investigation of the impacts of lockdown and isolation as strategies for containing the transmission of COVID-19. The population is divided into the following compartments: the susceptibles $S(t)$, the infected $I(t)$, the isolated $Q(t)$ and the recovered $R(t)$, with the total population given by

$$N(t) = S(t) + I(t) + Q(t) + R(t).$$

The flow diagram of the transmission dynamics based on these compartments is shown on Figure 1. From the flow diagram, some isolated persons may escape, and thus move back into the infected compartment at the rate $\omega\gamma Q$ and there is importation of infections at the rate η.

Following the flow chart in Figure 1, the governing equations for the dynamics of the disease become

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda + \theta R - \beta SI - \alpha S \\
\frac{dI}{dt} &= \eta + (\beta S - \alpha - \delta - \sigma)\, I + \omega\gamma Q \\
\frac{dQ}{dt} &= \sigma I - (\alpha + \delta_s + \omega) Q \\
\frac{dR}{dt} &= (1-\gamma)\omega Q - (\alpha + \theta) R
\end{aligned} \tag{1}$$

with the initial conditions

$$S(0) > 0, I(0) > 0, Q(0) \geq 0, R(0) \geq 0.$$

The parameter Λ, is the recruitment rate of susceptible individuals (taken as the average birth rate per 1,000 in Nigeria), η is the per capita rate of importation of infected persons into the population, α is the natural death rate of individuals in all the compartments, β is the effective rate of infection, σ is the isolation effort, while $0 < \gamma < 1$ is the proportion of isolated individuals who escape from isolation at the rate ω, θ is the rate at which the recovered lose immunity and become susceptible again, δ and δ_s are the disease induced death rates of the infected and the isolated respectively. All the parameters are positive and each of them is taken as a rate per day.

The feasible region for the system (1) is given by

$$\Psi = \left\{ (S, I, Q, R)\, \epsilon R_+^4 : S + I + Q + R \leq \frac{\Lambda}{\alpha} \right\},$$

which is positively invariant. The basic reproduction ratio $\mathscr{R}_0$ of the model is obtained using the next generation matrix procedure. Hence, we express the governing equations of the infected compartments as

$$\frac{dX_i}{dt} = F_i(X) - V_i(X), \tag{2}$$

where F_i are the new infections in compartment i and V_i are the rates of transfer of infections in and out of the compartments. From (1), we obtain

$$F_1 = \beta SI, F_2 = 0$$

and

$$\begin{aligned} V_1 &= (\alpha + \delta + \sigma)I - \omega\gamma Q - \eta, \\ V_2 &= (\alpha + \delta_s + \omega)Q - \sigma I. \end{aligned}$$

Hence, at the DFE, we have

$$F = \begin{pmatrix} \frac{\beta\Lambda}{\alpha} & 0 \\ 0 & 0 \end{pmatrix}, V = \begin{pmatrix} \alpha + \delta + \sigma & -\omega\gamma \\ -\sigma & \alpha + \delta_s + \omega \end{pmatrix}$$

and

$$|V| = (\alpha + \delta + \sigma)(\alpha + \delta_s + \omega) - \sigma\omega\gamma.$$

The basic reproduction number, $\mathscr{R}_0$ is the spectral radius of the matrix (FV^{-1}), given by

$$\mathscr{R}_0 = \frac{\beta\Lambda(\alpha + \delta_s + \omega)}{\alpha[(\alpha + \delta + \sigma)(\alpha + \delta_s + \omega) - \sigma\omega\gamma]} \tag{3}$$

provided $(\alpha + \delta + \sigma)(\alpha + \delta_s + \omega) > \sigma\omega\gamma$.

III. Existence and Local Stability of Equilibria

Here, we examine the local stability of the disease-free equilibrium (DFE) as well as determine the existence and stability of the endemic equilibrium.

A. Disease-free Equilibrium

The Jacobian of the system (1) evaluated at the DFE yields the matrix

$$J_{E_0} = \begin{pmatrix} -\alpha & -\frac{\beta\Lambda}{\alpha} & 0 & \theta \\ 0 & \frac{\beta\Lambda}{\alpha} - (\alpha+\delta+\sigma) & \omega\gamma & 0 \\ 0 & \sigma & -(\alpha+\delta_s+\omega) & 0 \\ 0 & 0 & (1-\gamma)\omega & -(\alpha+\theta) \end{pmatrix}$$

with eigenvalues

$$\begin{aligned} &\lambda_1 = -\alpha,\ \lambda_2 = -(\alpha + \theta), \\ &\lambda_3 = \frac{1}{2}\left[(A-B-C) + \sqrt{(A-B+C)^2 + 4\sigma\omega\gamma}\right], \\ &\lambda_4 = \frac{1}{2}\left[(A-B-C) - \sqrt{(A-B+C)^2 + 4\sigma\omega\gamma}\right], \end{aligned}$$

where $A = \frac{\beta\Lambda}{\alpha}, B = \alpha + \delta + \sigma$ and $C = \alpha + \delta_s + \omega$. Hence the DFE, E_0, is locally-asymptotically stable if

$$B > A + \frac{\sigma\omega\gamma}{C} \Rightarrow BC > AC + \sigma\omega\gamma.$$

$$\therefore (\alpha+\delta+\sigma)(\alpha+\delta_s+\omega) - \sigma\omega\gamma > \frac{\beta\Lambda}{\alpha}(\alpha+\delta_s+\omega)$$

$$\Rightarrow \frac{\beta\Lambda(\alpha + \delta_s + \omega)}{\alpha[(\alpha + \delta + \sigma)(\alpha + \delta_s + \omega) - \sigma\omega\gamma]} = \mathscr{R}_0 < 1,$$

and we have the following result:

Theorem 1. *The disease-free equilibrium (DFE) of the model (1) is locally-asymptotically stable for $\mathscr{R}_0 < 1$, that is, $B > A + \frac{\sigma\omega\gamma}{C}$ and unstable otherwise.*

B. Endemic Equilibrium

The endemic equilibrium (S^*, I^*, Q^*, R^*) is obtained in terms of I^* as

$$\begin{aligned} S^* &= \frac{\Lambda}{\alpha + \beta I^*} + \frac{\sigma\theta(1-\gamma)\omega I^*}{C(\alpha + \theta)(\alpha + \beta I^*)}, \\ Q^* &= \frac{\sigma I^*}{C}, \\ R^* &= \frac{\sigma(1-\gamma)\omega I^*}{C(\alpha + \theta)}. \end{aligned}$$

Substituting the values of S^*, Q^* and R^* in the equation for I in (1), we obtain

$$(a\mathscr{R}_0 - \beta)I^{*2} + (b\mathscr{R}_0 - \alpha)I^* + c\mathscr{R}_0 = 0, \tag{4}$$

Parameter	Description	Estimated value
Λ	Recruitment rate of susceptible individuals	0.0375 estimated [14]
θ	Rate at which the recovered lose immunity	0.2 assumed
β	Effective rate of infection	0.14 estimated[15]
α	Natural death rate of susceptible individuals	0.015 assumed
σ	Isolation efforts	0.4 [3]
ω	Rate of recovery of isolated infectives	0.12 assumed
γ	Proportion of isolated individuals that recovered	0.27 estimated [8]
δ	Disease induced death rate of the infectives	0.021 estimated [15]
δ_s	Disease induced death rate of the isolated infectives	0.02 assumed
η	Rate of importation of infected individuals	0.1 assumed

TABLE I
DESCRIPTION OF BASELINE PARAMETERS FOR MODEL (1).

where

$$a=\frac{\sigma\theta(1-\gamma)\omega}{\Lambda(\alpha+\theta)(\alpha+\delta_s+\omega)},\ b=1+\frac{\eta}{\Lambda},\ c=\frac{\alpha\eta}{\beta\Lambda}.$$

Based on the properties of roots of equation (4), as discussed extensively by Ouifki and Banasiak [13], we have the following results:

Theorem 2. *The model equation (1) has*

(i) *no endemic equilibrium when*

$$\mathscr{R}_0 > \max\left\{\frac{\alpha}{b},\frac{\beta}{a}\right\}.$$

(ii) *a unique endemic equilibrium when*

$$\frac{\alpha}{b}<\mathscr{R}_0<\frac{\beta}{a}\ \textit{or}\ \mathscr{R}_0<\min\left\{\frac{\alpha}{b},\frac{\beta}{a}\right\}.$$

(iii) *two endemic equilibria when*

$$\frac{\beta}{a}<\mathscr{R}_0<\frac{\alpha}{b}.$$

We now establish the stability of the endemic equilibrium using the eigenvalues of the Jacobian matrix evaluated at (S^*,I^*,Q^*,R^*), given by

$$J_{E^*}=\begin{pmatrix}-(\alpha+\beta I^*) & -\beta S^* & 0 & \theta\\ \beta I^* & \beta S^*-B & \omega\gamma & 0\\ 0 & \sigma & -C & 0\\ 0 & 0 & (1-\gamma)\omega & -(\alpha+\theta)\end{pmatrix} \tag{5}$$

Letting $H=\alpha+\beta I^*$ and $\Phi=\alpha+\theta$, we obtain the characteristic polynomial of the matrix (5) as

$$\lambda^4+a_1\lambda^3+a_2\lambda^2+a_3\lambda+a_4=0, \tag{6}$$

where

$$\begin{aligned}a_1&=B+C+H+\Phi-\beta S^*,\\ a_2&=\beta^2S^*I^*+(B+H)C+(H+\Phi)B\\ &\quad+(C+H)\Phi-\beta(C+H+\Phi)S^*-\sigma\omega\gamma,\\ a_3&=(BH+\beta^2S^*I^*)(C+\Phi)+(B+H)C\Phi\\ &\quad-\beta(CH+H\Phi+C\Phi)S^*-(H+\Phi)\sigma\omega\gamma\\ a_4&=\left(BH+\beta^2S^*I^*\right)C\Phi+\sigma\theta\omega\gamma\beta I^*\\ &\quad-(\beta CS^*+\sigma\omega\gamma)H\Phi-\beta\sigma\omega\theta I^*.\end{aligned}$$

From (6), the necessary and sufficient conditions for the stability of an endemic equilibrium can be determined by the nature of roots of the quartic equation, based on the Routh-Hurwitz criteria as summarised in the theorem below:

Theorem 3. *An endemic equilibrium is locally-asymptotically stable if*
(i) $B+C+H+\Phi>\beta S^*$
(ii) $\beta^2S^*I^*+(B+H)C+(H+\Phi)B+(C+H)\Phi>\beta(C+H+\Phi)S^*+\sigma\omega\gamma$
(iii) $\left(BH+\beta^2S^*I^*\right)C\Phi+\sigma\theta\omega\gamma\beta I^*>(\beta CS^*+\sigma\omega\gamma)H\Phi+\beta\sigma\omega\theta I^*$ *and*
(iv) $a_1a_2a_3>a_3^2+a_1^2a_4$.

Using the approach of Wangari, et al [12], we take the effective rate of infection, β, as the bifurcation parameter, and obtain

$$\beta_c=\frac{a\mathscr{R}_0}{1+\frac{\Lambda}{4\alpha\eta\mathscr{R}_0}(b\mathscr{R}_0-\alpha)^2} \tag{7}$$

Replacing β with β_c in (3) and solving for $\mathscr{R}_0$ yields the expression for the critical reproduction

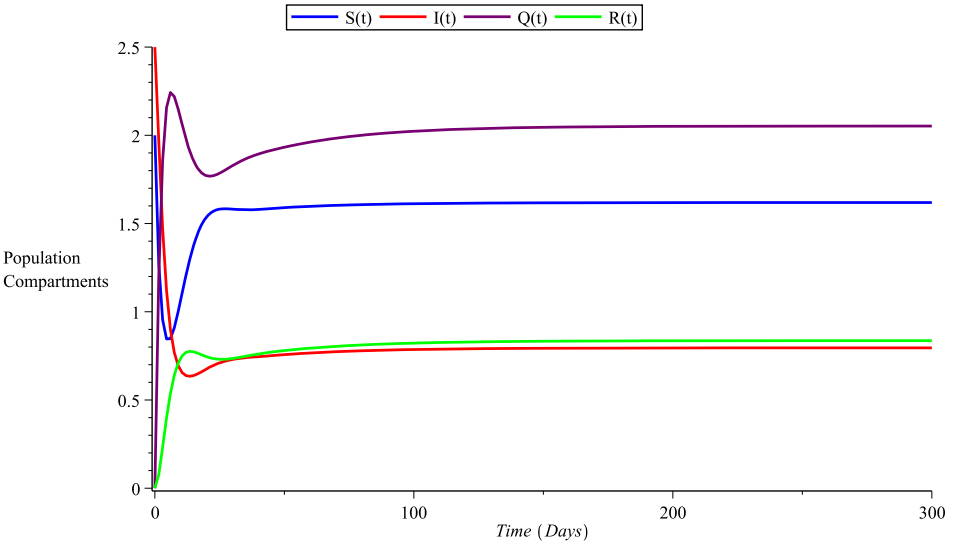

Fig. 2. Time-Course solution of model equations with relaxed lockdown.

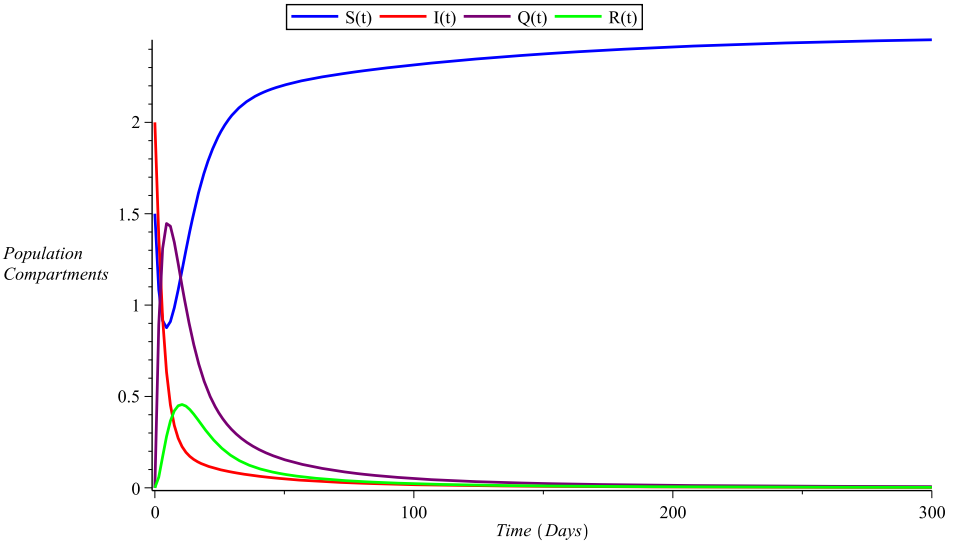

Fig. 3. Time-course solution with total lockdown and moderate isolation, $\eta = 0.0$.

number as

$$\mathscr{R}_c = \frac{\alpha}{b^2\Lambda}[b\Lambda + 2\eta(ak-1) + 2\sqrt{\eta(ak-1)(a\eta k + b\Lambda - \eta)}], \quad (8)$$

where $k = \frac{\Lambda(\alpha+\delta_s+\omega)}{\alpha[(\alpha+\delta+\sigma)(\alpha+\delta_s+\omega)-\sigma\omega\gamma]}$, and a and b are as previously defined. Hence the critical reproduction number is obtained as $\mathscr{R}_c = 0.47$, based on the parameters defined in Table I. Consequently, the disease can be controlled, when $\mathscr{R}_0 < \mathscr{R}_c < 0.47$.

IV. Simulations and Discussion

An SIQRS model has been constructed and analysed theoretically. In this section, we carry out the numerical simulation of the model equations to confirm the theoretical results. The simulation is carried out based on the parameter values in Table I. The results are depicted on Figures 2 to

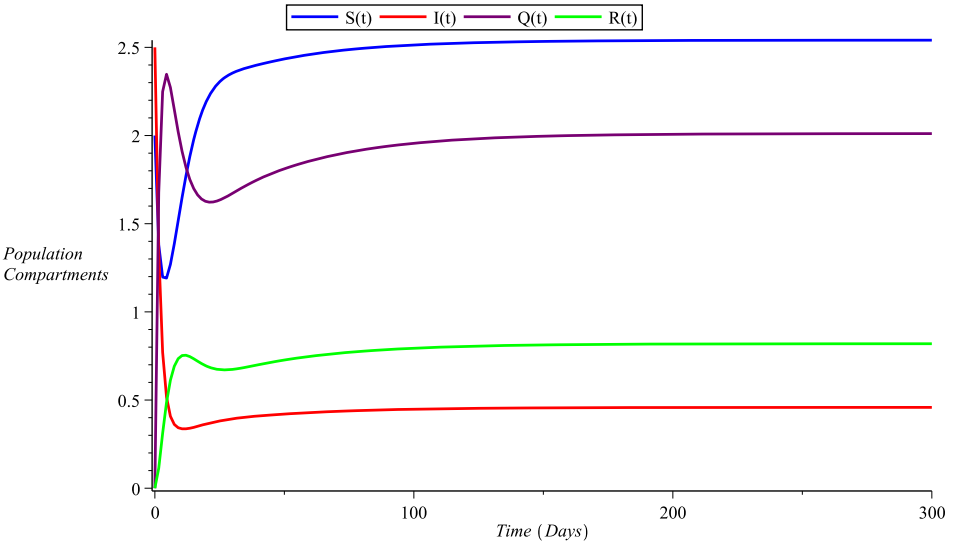

Fig. 4. Time-course solution with relaxed lockdown and strict social isolation, $\sigma = 0.70, \eta = 0.1$.

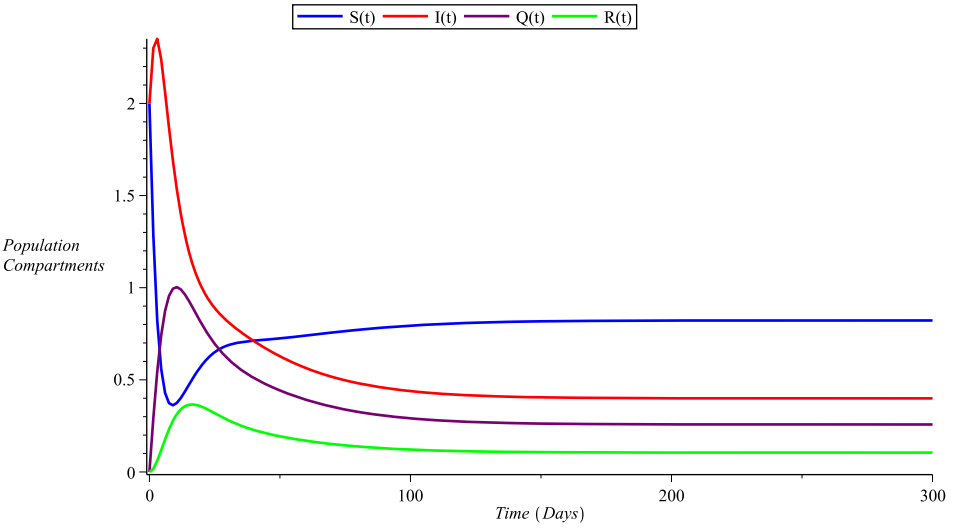

Fig. 5. Time-course solution with relaxed social isolation and total lockdown , $\sigma = 0.1, \eta = 0.0$.

6 below. Figure 2 depicts the case of enforcement of social isolation with relaxed lockdown, while Figure 3 shows the effects of a total lockdown and social isolation, hence the disease can be eradicated. The scenario depicted in Figure 4 is that of strict enforcement of social isolation and relaxed lockdown. Under this situation, incidence of the disease reduces in the population with lower risk than the case depicted in Figure 2.Figure 5 on the other hand, shows the effect of relaxing social isolation while observing total lockdown. Under this scenario, cases of the disease decline considerably, but persists in the population for a long period of time. The results shown in Figures 2 to 6 indicate that the disease can only be eradicated under a total lockdown regime. However, a strict enforcement of both strategies could lead to the eradication of the disease within a shorter period of time as shown in Figure 6.

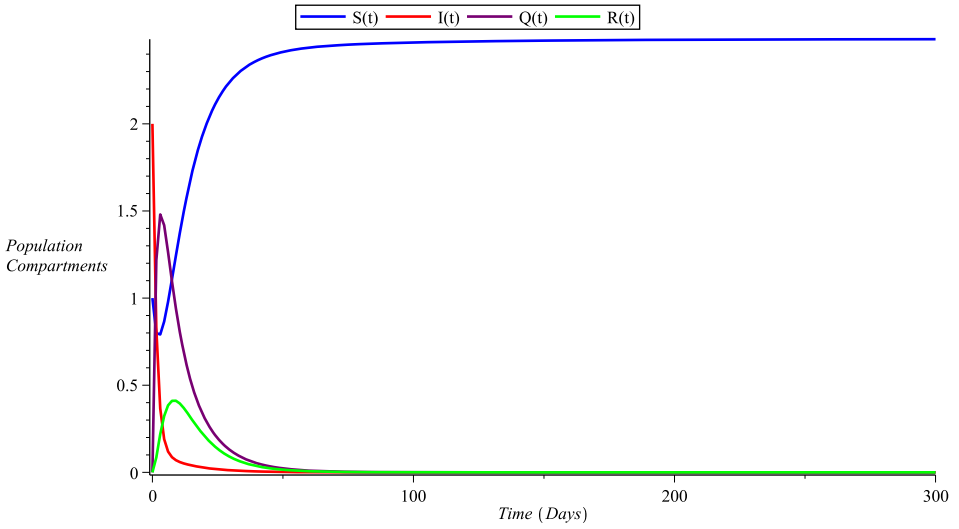

Fig. 6. Time-course solution with strict social isolation and total lockdown , $\sigma = 0.70, \eta = 0.0$.

V. Conclusion

An SIQRS model for the transmission of COVID-19 is investigated with a view to determining the impacts of lockdown and social isolation on the eradication of the disease. Conditions for the existence and local stability of the equilibria were determined.The results indicate that the model has two endemic equilibrium points, thus indicating the possibility of occurrence of a backward bifurcation. Numerical results show that lockdown is more effective than social isolation in the containment of the pandemic. However, when lockdown is enforced for a long time, it could affect the economy of the nation adversely, but it can still be used effectively by implementing in segments. That is, enforce the lockdown only in disease endemic areas. Nevertheless, enforcing both strategies together yield the best result as can be seen in Figure 6.

Declaration of Competing Interest

The author declares that they have no known competing interest or personal relationship that could have influenced the work reported in this paper.

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