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ORIGINAL RESEARCH ARTICLE

PROBABILISTIC ASSESSMENT OF A TYPICAL REINFORCED CONCRETE GIRDER UNDER CRITICAL LOADING

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ABSTRACT

Reinforced concrete bridges are continuously subjected to increased traffic load and exposed to harsh environments causing bridge members to deteriorate, thus, affecting their durability, safety and performance. This work aims at a probabilistic assessment of a reinforced concrete bridge girder under critical loading considering the flexural and shear failure modes of the bridge. A limit state function for the failure modes was generated and, the stochastic variables and statistical parameters were determined. Additionally, the safety indices for each failure mode were determined using First Order Reliability Method (FORM) procedure. Mathematical models were developed and the uncertainties in structural resistance, applied loading as well as structural components were included using the probabilistic method. A computer program in FORTRAN language was developed and deployed for the reliability analysis of the bridge girder to ascertain the level of safety using First-Order Reliability Method. The results revealed that the safety index for shear and flexural failure mode decreases as the span is increased. For concrete strength of 30 N/mm^2 and a depth of 1000 mm, the safety index for shear and flexural failure reduced from 4.5 to 2.0 and 10.7 to 3.37 respectively as the span increases from 10 m to 19 m. This indicates that the shear and flexural failure modes are sensitive to increase in span. In order to obtain a minimum safety target of 3.8, it is recommended that the depth of the beam should not be less than 1200 mm while the span should not exceed 16 m, and concrete grade should not be less than 35 N/mm^3.

1.0 Introduction

Bridges constitute a significant portion of the transportation network of which an efficient and well-planned transportation infrastructure has significant impact on the socioeconomic and social development of any nation. For land transport, road transport in particular, highway bridges constitute a vital, critical and challenging component which make movement between two or more places or regions efficient (Mahboubi and Kioumarsi 2021). However, highway bridges in most parts of the world including developing countries are subjected to continuously increasing traffic from heavy truck-loads (Ajagbe et al., 2012), and exposure to harsh environments (Val et al., 2000). They likewise impair due to operating conditions, natural hazards, and other forms of physical attacks (Ren et al., 2021) as well as deteriorate due to ageing. To salvage bridges to serve their structural and functional performance (Lounis, 2004), a high proportion of highway bridges have been categorized as structurally deficient and functionally obsolete (Frangopol and Liu, 2007). Therefore, it is pertinent to allocate adequate resources for their maintenance, rehabilitation and possible replacement with time (Val et al., 2000). Nevertheless, the high cost of bridge maintenance, rehabilitation and renewal can seldom be accommodated by transport authorities saddled with such responsibilities. Therefore, transport authorities in many countries including developing countries have taken measures to create a number of bridge management programs (Liu and Frangopol, 2006). A variety of maintenance optimization including simple economic frameworks to complicated Markovian decision processes has been employed in different bridge management systems (Lounis and Zoubir, 2006). An effective and reliable bridge management system relies on the presence of an effective optimization algorithm and a reliable model for performance prediction.

An effective bridge management system involves taking competent decisions regarding bridges grounded on available data on resistance, loading, adverse environmental impact and possible results of bridge malfunctioning (Holicky et al., 2010). However, the material properties of bridges, its loading history together with mechanical and environmental conditions amongst others (Frangopol et al., 2008) are uncertain parameters which lead to developing reliability based approach to evaluate existing bridges. Probabilistic assessment, a measure of reliability of a system is a procedure to determine the probability that a system does not reach a defined limit state under a specified reference duration. Probabilistic assessment of structures can be regarded as a rational measure of performance since it is a function of uncertainties related to loads and load carrying capacity (Czarnecki and Nowak, 2006). Over time, an increasing trend of reliability has been observed in the design and assessment of bridges (Zona et al., 2010; Osumeje et al., 2016; Steel and Sorensen, 2014; Frangopol et al., 2008; Cheng 2014).

First Order Reliability Method (FORM) (Melchers and Beck 2018; Nikolaidis et al., 2004; Gong and Zhou 2017) is an effective approach in appraising the reliability of engineering systems. Its utility and precision have been ascertained in many use cases (Low and Tang 2007). FORM has been used in reliability analysis in various bridge components and its loading (Akgül and Frangopol 2004; Cheng and Li 2009; Li et al., 2013; Okeil et al., 2013; Osumeje et al., 2016). However, failure modes of bridge girder under critical loading have been neglected. Therefore, this work aim at the probabilistic assessment of a reinforced concrete bridge girder subjected to critical loading. This will be achieved by generating limit state function for the failure modes, identifying stochastic variables with their statistical parameters, and finally determining safety index for each failure mode using the (FORM).

2. Materials and Method

2.1 Estimating Parameters for Reliability of Bridge Girder

According to Nowak et al. (2022), loads acting on a highway bridges consist of dead loads; live or traffic induced static and dynamic loads; load due to environmental factors including temperature, wind, snow and seismic actions; and other forms of loads like collision and braking loads. In formulating the reliability function, the components from loads are treated as random variables with their variation described by a cumulative distribution function (CDF), a mean value, and a coefficient of variation. However, in this study, the load combination used for the highway bridge is limited to dead load and dynamic load which are obtained using influence line for a loaded Sino truck (Osumeje et al., 2016).

2.2 The Bridge Model

The bridge model used in this research was a simply supported with deck of 13.0 m span situated in an exposed setting. The bridge section is composed of an open cross section of precast reinforced concrete longitudinal beam constantly spaced at 2060 mm between each other. The precast longitudinal beams upper flange is well coupled to an in-situ deck slab of 200 mm depth cast on a precast concrete slab formwork of 50 mm thick, giving a total of 250 mm depth of slab. The bridge substructure and superstructure are connected using bearing pads. In-situ reinforced concrete piles connected with pile caps is used as the bridge foundation. Length of 11.0 m was adopted as the span of bridge with a carriageway of 7.2 m and a walkway of 1.5 m at the two ends of the bridge.

2.3 Material Properties

Materials used in the design are specified in accordance with British Standards Institution (2004). The class of concrete is C40/50 whereas steel reinforcement is S500. The concrete used in the design has a unit weight of 25 kN/m^3; the unit weight of asphalt concrete is 22 kN/m^3; and weight of parapet wall was 0.5 kN/m all in accordance with the mentioned code.

2.4 Limit State Evaluation

Limit state refers to a situation of a probable failure of a structure after which it stops to achieve its design purpose reasonably both in the area of safety and serviceability. According to Ayyub and McCuen (2016), the capacity of a structural member to resist failure relies on not just the strength of the member but also the various load composition acting on the structural member. Measuring the risk of failure due to variety of load application on a structural member generates stresses and moments which causes Engineers and related professionals to evaluate the safety and reliability of structural members. Reliability based assessment method is a probabilistic method that involves loads and strengths of structural members' sections and material properties characterized with their identified and hypothesized formulations, expressed in relations to their formulation types, averages and variances Melchers and Beck (2018). In general, the resistant of a structural member is required to surpass the applied load to avoid failure and, in this case, the reliability analysis is used as a tool to evaluate the effect of uncertainties in structural design.

Herein, the reduction in strength of a reinforced concrete structural member as a result of shear and flexure is considered with their limit state functions formulated. Similar to the study conducted by (Osumeje et al., 2016), these limit state function were analyzed by means of first order reliability method.

2.6 Live Load from Moving Vehicles and Location of Resultant Load on Bridge

2.6.1 Total Live Load

Self-weight of vehicle (semi-articulated truck) = 18 tones = 180 kN.

Maximum carrying capacity of semi-trailer truck =1200 bags of cement, where each bag weight 50 kg.

Load carried by semi-trailer = 1200 x 50 kg = 60000 kg = 600 kN

Total live load = 180 + 600 = 780 kN

Figure 1 shows load distribution on a 6-axles Sinotruck according to (Eurocode 1, 2003). From the Figure, the 780 kN total live load is distributed to the axles in accordance to the Eurocode. From the figure, Q_{k1} = 150 kN, Q_{k2} = 120 kN, and Q_{k3} = 90 kN were adopted according to the code.

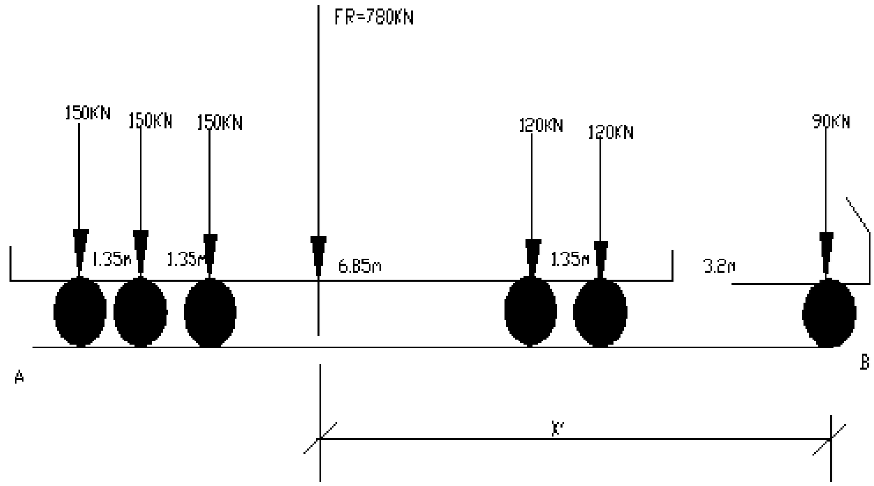

Figure 1: Load Distribution on 6-Axles (Sinotruck) according to Eurocode

2.6.2 Resultant Position of Load

From Figure 1, the location of the resultant force F_R can be obtained as follows;

$$F_R = 3Q_{k1} + 2Q_{k2} + Q_{k3} = 780\ kN \quad (1)$$

Where $Q_{k1} = 150kN,\ Q_{k2} = 120kN, and\ Q_{k3} = 90kN$

Taking moment about point B in the figure,

$$\sum M_B = 0 \quad (2)$$

$$(F_R \cdot x') - [120(3.2 + 4.55) + 150(11.4 + 12.75 + 14.1)] = 0 \quad (3)$$

$$(780 \cdot x') = 5737.5 + 930$$

$$x' = 8.55\ m$$

Therefore, the resultant force acts at a distance of 8.55 m away from point B.

2.7 Limit State Function for Shear in Interior Beam

Self-weight of asphalt = $A_{asph}\ x\ \gamma_{asph} = 2.27\ kN/m$
Self-weight of slab = $A_{slab}\ x\ \gamma_c = 12.88\ kN/m$
Self-weight of girder = $A_g\ x\ \gamma_c = 13.00\ kN/m$
where γ_{asph} and γ_c represents the unit weight of asphalt and concrete respectively and A_{asph}, A_{slab}, and A_g represents the areas of asphalt, slab and girder respectively.

$$G_k = (2.27 + 12.88 + 13.00)\ x\ 1.35 = 38\ kN/m \quad (4)$$

Where G_k is the dead load.

2.7.1 Shear of Dead Load

The shear force equation for the dead load is given by equation 5

$$v_{dl} = \frac{G_k l}{2} = \frac{38\ x\ L}{2} = \frac{38\ x\ 13}{2} = 247\ kN \quad (5)$$

v_{dl} is the shear force and L is the span of girder.

2.7.2 Load from Traffic Lane

Load from traffic lane (q_{kn}) is given by $q_{kn} = 336(\frac{1}{L})^{0.67}\ (per\ national\ lane)$ (6)

Adjustment factor for the lane is given by α_2:

$$\alpha_2 = 0.264 b_L = 0.274\ x\ 3.6 = 0.986. \quad (7)$$

Therefore, lane load per unit length is now given by;

$$q_k = \frac{q_{kn}\ x\ \alpha_2}{3.6} \quad (8)$$

2.7.3 Critical Shear for Live Load V_{ll}

For span L = 10.0 m, the critical shear for live load is given by the expression;

$$V_{LL} = \frac{Q_{k1}(3L-4.05) + Q_{k2}(L-9.55)}{L} + \frac{q_k L}{2} \quad (9)$$

For span L = 13.0 m, critical shear force for live load is given by;

$$V_{LL} = \frac{Q_{k1}(3L-4.05) + Q_{k2}(2L-20.45)}{L} + \frac{q_k L}{2} \quad (10)$$

Likewise, for L > 13.0 m;

$$V_{LL} = \frac{Q_{k1}(3L-4.05) + Q_{k2}(2L-20.45) + Q_{k3}(L-14.1)}{L} + \frac{q_k L}{2} \quad (11)$$

The applied shear stress $V_{applied}$ is given by;

$$V_{applied} = \left[\frac{V_{dl} + V_{LL}}{bd}\right] \quad (12)$$

V_{dl} is the shear force from dead load, b and d are the width and depth of girder.

Hence, the limit state function is given by;

$$G(x_1) = 0.138 f_{ck}\left(1 - \frac{f_{ck}}{250}\right) - V_{applied} \quad (13)$$

Where, f_{ck} is the characteristic strength of concrete.

In obtaining the critical shear for the live load, Load Model 1 (LM1), which according to Eurocode 2 is placed with point load over the support is as shown in Figure 2.

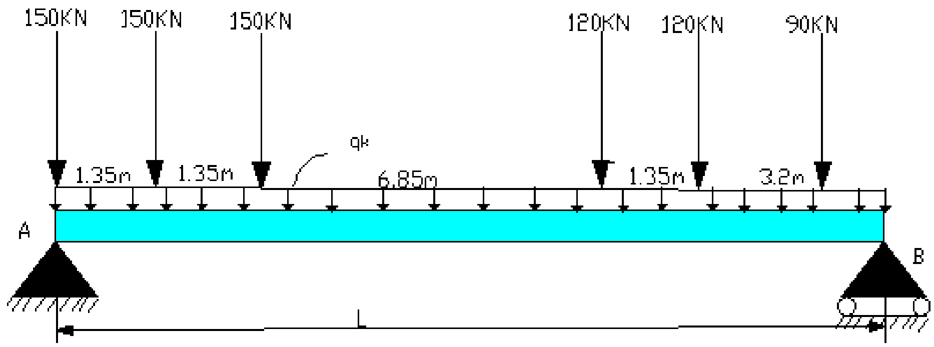

Figure 2: Arrangement of Load Model I for critical shear

2.8 Limit State Function for Flexure in Interior Beam

2.8.1 Resultant Moment Due to Dead Load

The resultant moment ((M_{dl}) due to the effect of dead load is given by the expression;

$$M_{dl} = \frac{G_k L^2}{8} = \frac{38L^2}{8} \quad (14)$$

2.8.2 Resultant Moment Due to Live Load

Here, the resultant moment (M_{LL}) due to live load is given by the expression;

$$M_{LL} = [M_{Qk} + M_{qk}] \quad (15)$$

Where $M_{qk} = \frac{q_k L^2}{8}$, (16)

$$A_y = \frac{1}{L}(F_R)\left[\frac{L}{2} - (x' - x)\right], \quad (17)$$

$$M_{Qk} = A_y\left[\frac{L}{2} - x\right] - 4.05\,(Q_{k1}), \text{ and} \quad (18)$$

$$M_{applied} = M_{dl} + M_{ll}$$

Therefore, the limit state function is given by;

$$G(x_2) = 0.167 f_{ck} b d^2 - M_{applied} \quad (19)$$

To obtain M_{Qk} from LM1, the maximum bending moment occurs at the location where one of the tandem axles is applied at a distance from the mid-point of the beam equal to half of that between the aforesaid axle and the resultant of the moving load train as shown in Figure 3.

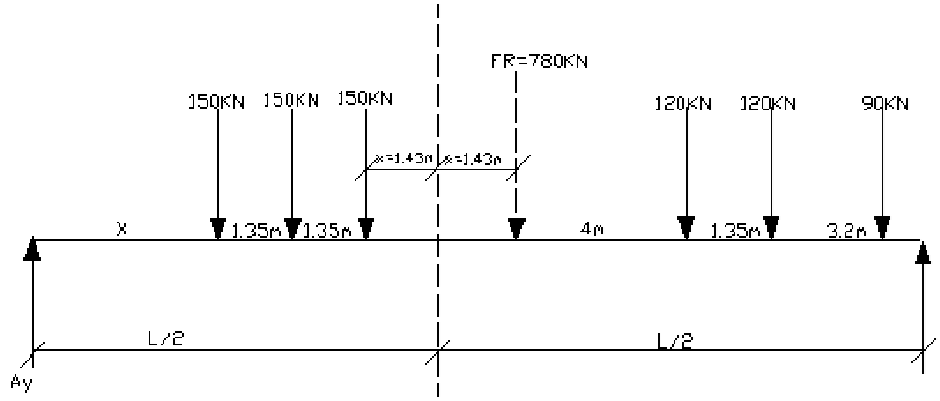

Figure 3: Arrangement of maximum bending of Load Model 1

From the above equations, $G(x_1)$ is the expression for failure model one under shear occurring in the beam, $G(x_2)$ is that for failure model two under flexure occurring in the deck beam, f_{yk} is the strength of steel, f_{ck} is strength of concrete, A_{st} is area of steel reinforcement, b is the width of section, and d is the effective depth. Table 1 shows the stochastic parameters generated for the limit state function in the program.

Table 1. Stochastic Parameters for Limit State Function (Osumeje et al. 2016)

S/No	Design variables	Notation	Unit	Variable Type	Mean	Coefficient of variation
1	Traffic Load	w_Q		Normal	1.0	0.05
2	Concrete compressive Strength	f_{ck}	N/mm^2	Lognormal	40	0.15
3	In-situ slab thickness	h	mm	Normal	250	0.05
4	Effective depth	d	mm	Normal	219	0.05
5	Width of beam	b	mm	Normal	400	0.05
6	Beam height	h	mm	Normal	1300	0.05
7	Effective depth	d	mm	Normal	1237	0.05

3.Results and Discussion

3.1 Effect of Span Length Variation on the Safety Index Under Flexure Condition

This section analyzes the impact of varying span of bridge girder on the safety index under the condition of flexure. Figures 4 to 7 are plots showing the relationship between safety indices and span of beams at various depth in the girder under the region of flexure. From Figures 4 to 7, it can be observed that as the span of girders increases, the safety index decreases in each of the varied beam depth. From Figure 4, which is for concrete strength of 30 N/mm^2, it can be observed that for the 1000 mm depth of beam, the safety index for flexural failure was found to reduce drastically from 20.70 to 3.37 as the span increases from 10 m to 19 m. This situation is similar with other plots of different concrete strength. This is because the applied moment which is a function of span, increases with span and therefore reduces the resistance of the section to flexural failure (Ibrahim 2014). To remedy this situation, suitable section with increased depth and reduced span should be selected.

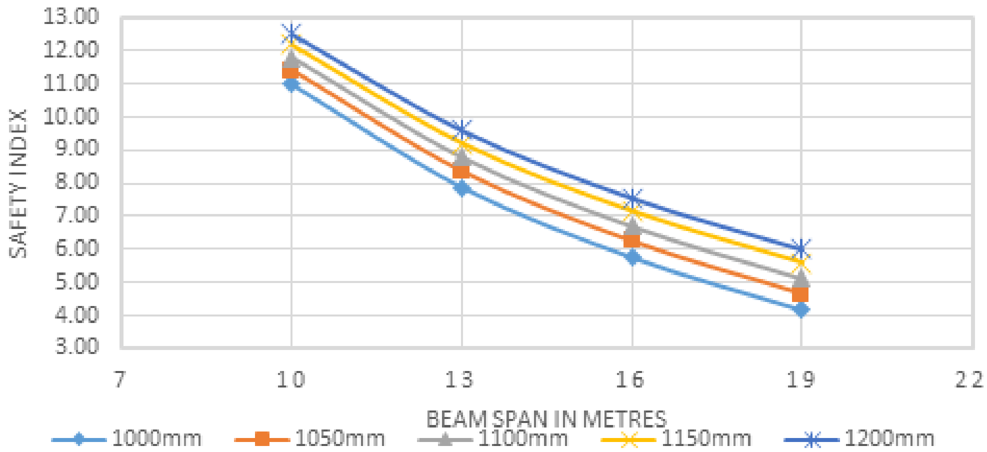

Figure 4: Flexural safety index Vs Beam span for various beam depth for 30 N/mm^2 characteristic strength of concrete

Figure 5, shows the plot of the relationship between safety index and span of beams at various depth in the girder under the region of flexure for 35 N/mm^2 characteristics strength of concrete. From the Figure, it can be observed that as the span of girders increases, the safety index decreases in each beam depth. This is because the moment acting on a member is a function of span. Likewise, it can be seen that the higher the depth of girder, the higher the safety index.

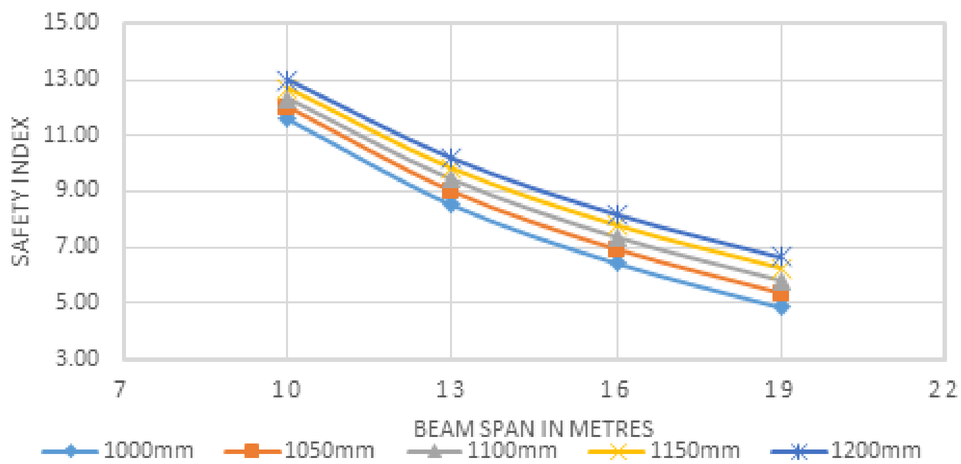

Figure 5: Flexural safety index Vs Beam span for various beam depth for 35 N/mm^2 characteristic strength of concrete

Figure 6, shows the relationship between safety index and span of beams at various depth in the girder under the region of flexure for 40 N/mm^2 characteristics strength of concrete. Similarly, from the Figure, it can be observed that as the span of girders increases, the safety index decreases in each beam depth. This is because the moment acting on a member is a function of span. Likewise, it can be seen that the higher the depth of girder, the higher the safety index.

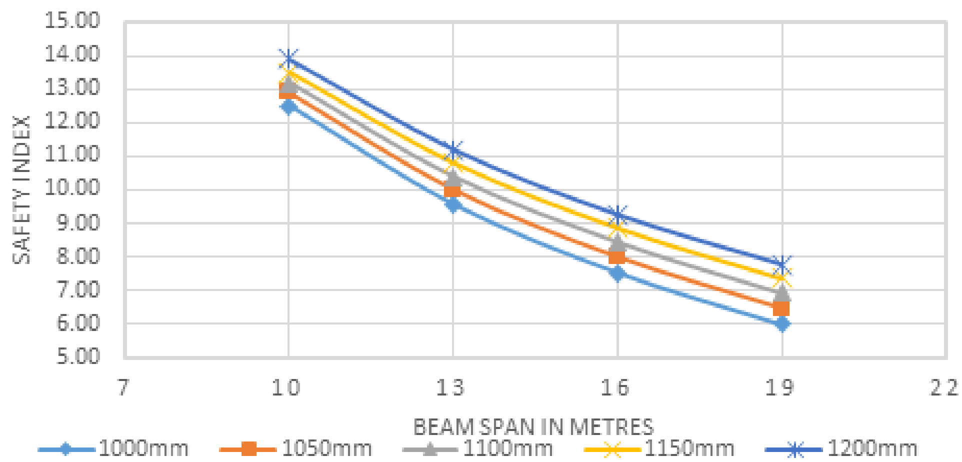

Figure 6: Flexural safety index Vs Beam span for various beam depth for 40 N/mm^2 characteristic strength of concrete

Figure 7, depicts the relationship between safety index and span of beams at various depth in the girder under the region of flexure for 50 N/mm^2 characteristics strength of concrete. Similarly, from the Figure, it can be observed that as the span of girders increases, the safety index decreases in each beam depth. This is because the moment acting on a member is a function of span. Likewise, it can be seen that the higher the depth of girder, the higher the safety index.

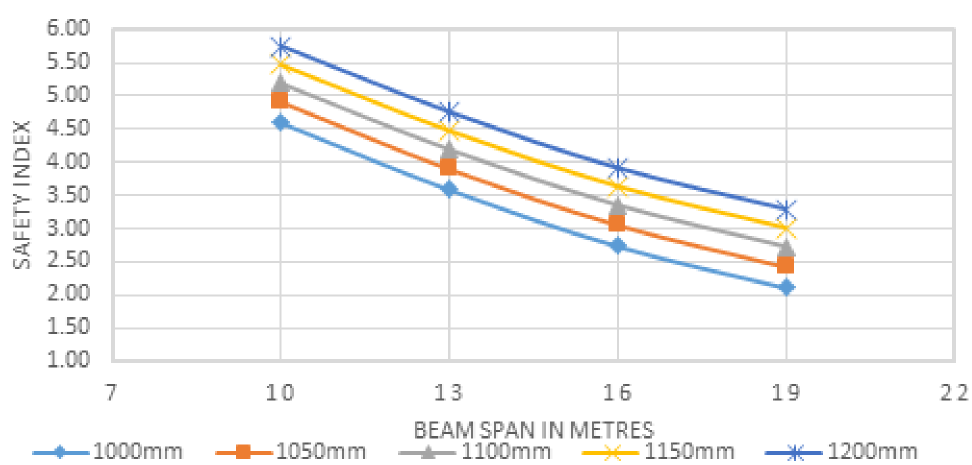

Figure 7: Flexural safety index Vs Beam span for various beam depth for 50 N/mm^2 characteristic strength of concrete

3.2 Effect of Span Length Variation on the Safety Index Under Shear Stress Condition

In this section, the effects of varying span of the bridge girder on the safety index with reference to shear stress are analyzed. Figures 8 to 11 shows the relationship existing between safety index and span of beams for different of girder under shear. Figure 8 shows the results for concrete strength of 30 N/mm^2, and for 1000 mm depth of beam. It can be seen that the safety index for shear failure reduce drastically from 4.50 to 2.0 as the span increases. This condition is similar for other concrete strength. This situation clearly indicates that the longer the span, the lower the resistance of the member to shear and likewise, the larger the section, the more the resistance of the member to shear Aguwa (2013).

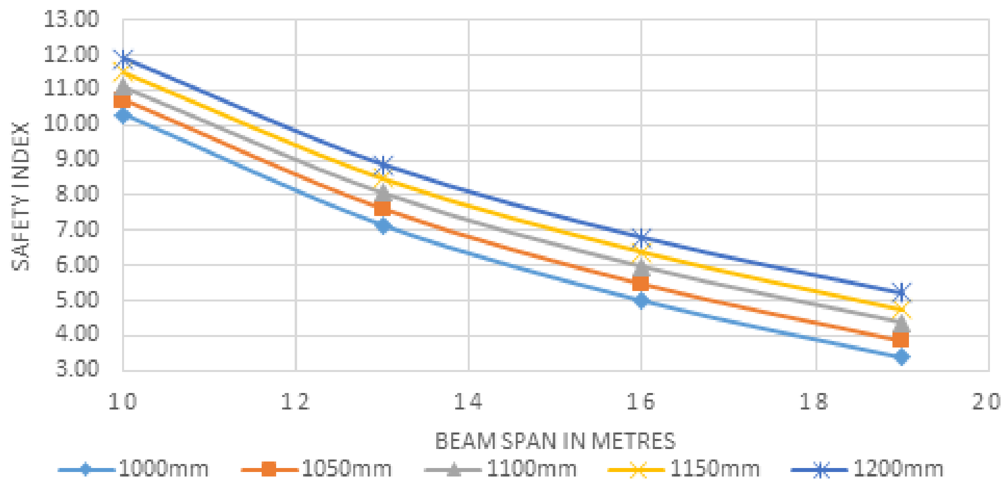

Figure 8: Shear safety index Vs Beam span for various beam depth for 30 N/mm^2 characteristic strength of concrete

Figure 9 shows the relationship between safety index and span of beams for different sizes of girder under shear for 35N/mm^2 characteristic strength of concrete. It can be seen for each depth of girder that the safety index reduces as the span increases. This is because shear for increases with span on structural members. Furthermore, taking a constant span, the safety index is observed to increase with the depth of the girder. This indicates that the longer the span, the lower the resistance of the member to shear. likewise, the larger the section, the more the resistance of the member to shear.

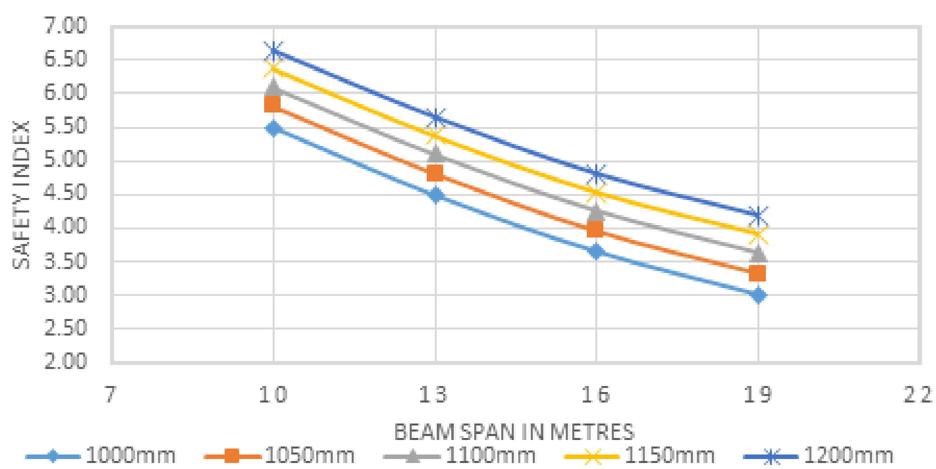

Figure 9: Shear safety index Vs Beam span for various beam depth for 35 N/mm^2 characteristic strength of concrete

Similarly, Figure 10 depicts the relationship between safety index and span of beams for different sizes of girder under shear for 40 N/mm^2 characteristic strength of concrete. It can be seen for each depth of girder that the safety index reduces as the span increases. This is because shear for increases with span on structural members. Furthermore, taking a constant span, the safety index is observed to increase with the depth of the girder. This indicates that the longer the span, the lower the resistance of the member to shear. likewise, the larger the section, the more the resistance of the member to shear.

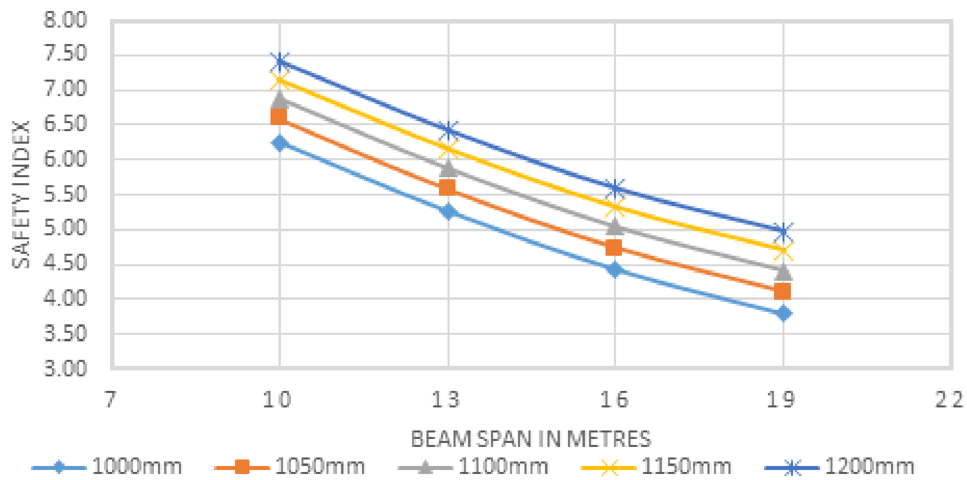

Figure 10: Shear safety index Vs Beam span for various beam depth for 40 N/mm^2 characteristic strength of concrete

Likewise, Figure 11 depicts the relationship between safety index and span of beams for different sizes of girder under shear for 50 N/mm^2 characteristic strength of concrete. It can be seen for each depth of girder that the safety index reduces as the span increases. This is because shear for increases with span on structural members. Furthermore, taking a constant span, the safety index is observed to increase with the depth of the girder. This indicates that the longer the span, the lower the resistance of the member to shear. likewise, the larger the section, the more the resistance of the member to shear.

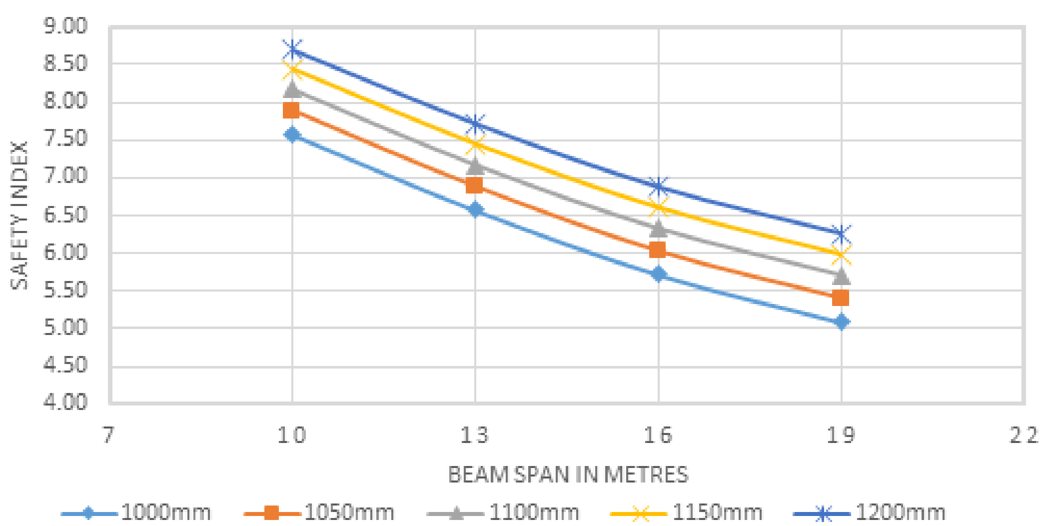

Figure 11: Shear safety index Vs Beam span for various beam depth for 50 N/mm^2 characteristic strength of concrete

4. Conclusion

This research carried out a deterministic and stochastic analysis on a reinforced concrete bridge girder under critical loading considering flexural and shear failure modes of the bridge. A limit state function for the shear and flexure failure modes was generated according to Eurocode 2 for critical load model. Influence line analysis was used to obtain the critical bending moments and shear forces.

Flexural and shear failure modes were determined using a safety index of 3.8. A minimum safety index of 2.11 was obtained for shear failure mode of the bridge whereas 3.37 was obtained as a minimum safety index for flexural failure mode of the bridge. In reinforced concrete girder, the shear failure mode is usually sudden with brittle nature. Therefore, shear failure is more critical than flexural failure which is ductile. This is in agreement with the finding of Eskenati and Pour (2015).

The analysis carried out evaluated the effect of some basic parameters for materials and loads in for the designing if reinforced concrete bridge girder under critical loading. The analysis showed that concrete grade, traffic load, and the section parameters of the girder have significant effect on the safety index of the bridge. Therefore, to increase the reliability of the girders; the design parameters should be selected systematically through a reliability-based approach.

From the analysis carried out, it showed that the safety index for the shear and flexural failure mode reduces as the span increases. For instance, for concrete strength of 30 N/mm^2 and a depth of 1000 mm, the safety index for shear failure was found to reduce from 4.5 to 2.0 as the span is increased from 10 m to 18.5 m. Similar trend also occurred with other concrete strength. Likewise, for flexural failure mode, the safety index reduced from 10.70 to 3.37 as the span is increased from 10 m to 19 m for concrete strength of 30 N/mm^2. Also, similar trend was also obtained for other concrete strength.

From this study to obtain a minimum safety index of 3.8 was obtained. Therefore, the depth of beam should be at least 1200 mm and span should not exceed 16 m. The concrete grade should not be less than 35 N/mm^2. Also, it is suggested that s reliability-based approach should be incorporated into Eurocode 2 design criteria for reinforced concrete strictures to accommodate uncertainties in structural design parameters.

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