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ORIGINAL RESEARCH ARTICLE

COMPARATIVE ANALYSIS AND SIMULATION OF THE *d-q* REFERENCE FRAME MODEL OF THE THREE PHASE INDUCTION MOTOR AND ITS MODIFICATION

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ABSTRACT

Induction motors play an important role in residential, commercial, and industrial centers around the globe. They are rugged, reliable in operation and require low maintenance. The dynamic models of these motors are typically employed to understand their behavior in transient, steady state and under dynamic conditions. Several models of the three-phase induction motor exist in literature, but this paper proposes an alternative to the standard *d-q* reference frame model of the symmetrical three phase induction motor. The proposed model is obtained by modifying the standard model in a way in which the d-axis of the reference frame is aligned with the rotor flux vector. The alignment decouples the q-axis component from the d-axis component of the stator current using the vector control technique. The model is analyzed in the synchronous reference frame and then a dynamic simulation is performed. A comparison between the standard d-q reference frame model and the proposed model was performed. The simulation showed identical results of motor speed, torque and flux behavior for the two models. The difference between the two models manifests in their alternate characteristics for the d-q axes voltages and currents. It was observed that the magnitudes of the d-axis and q-axis voltages for the standard model are equal to the peak value of the applied phase voltages and zero respectively. However, the results of the proposed model revealed otherwise. Furthermore, it was observed that the d-q axis voltages and currents of the proposed model exhibit oscillations during motor acceleration time because of the dynamics of alignment but attain constant values during the time that motor speed is constant. The proposed model is easier to solve because the rotor dynamics are described by one equation instead of two equations as in the case of the standard *d-q* reference frame model.

1.0 Introduction

The three phase induction motor has often been described as the workhorse and race horse of the industry (Boldea and Nasar, 2002; Olarinoye et al., 2018). They are rugged, cheap, robust and have low maintenance requirement (Tamimi, 2018; Li *et al.,* 2021). Additionally, it is mostly used in commercial and industrial centers across the globe (Richardson *et al*., 2021). The three phase induction motor continues to receive wider attention because of advancements in semiconductor technology towards development of cheap and fast digital signal processors and control strategies that enable the motor to perform tasks in ways like never before (Hareesh and Jayanand, 2021; Abad, 2016; El-Zohri and Mosbah, 2019). They are now being increasingly used in industrial and transportation sectors of the economy and they are available in a wide variety of power range (Noor *et al.,* 2013; Menghal and Laxmi, 2020; Ke *et al*., 2020). The modeling of the symmetrical three phase induction motor has been extensively discussed in literature. Several models have been developed and applied by different authors to solve different problems. Some of these models include the equivalent circuit model, a-b-c reference frame models and the d-q reference frame model. The d-q model is the most popular because it not only allows for steady state, dynamic and transient studies but it also makes motor analysis easier and reduces computation time (Sanjay et al., 2015; Kuznyetsov, 2019). Some of the methods that have been applied to model the induction motor includes; numerical integration, finite element-based, complex vector representation etc. Kuznyetsov (2019) developed a numerical model of a three phase induction machine in its natural a-b-c reference frame utilizing the method of average voltages at the integration step (AVIS). The behavior of the model was compared for a set of conventional numerical methods alongside AVIS. Although the model was analyzed, the focus of the work was on the calculation speed of the model. Menghal and Laxmi (2020) implemented d–q model of the three phase induction motor in MATLAB/Simulink. The transient and steady state analysis of the motor were carried out under different load conditions. Open loop and closed loop modes of operation using artificial intelligence methods were also examined. A conventional three phase induction motor model derived in the arbitrary reference frame was presented, analyzed in the rotor reference frame and simulated in (Deb and Sarkar, 2016). The results of simulation were compared with the results of simulating an existing Simulink model of the three phase induction motor also in the rotor reference frame. An agreement in the results was reported.

This paper proposes a modification to the d-q reference frame model of the three phase induction motor in a way in which the d-axis of the reference frame is aligned with the rotor flux vector for the purpose of studying motor behavior. The alignment decouples the torque current or q-axis component of the stator current from the flux current otherwise known as the d-axis component of the stator current using the vector control technique. The model is analyzed in the synchronous reference frame and simulated. Simulation results are compared with the results of simulating the standard d-q reference frame model in the synchronous reference frame as well. The model is particularly useful in the analysis of motor behavior when vector control is not the objective.

1.0 Materials and methods

2.1 The Standard D-Q Reference Frame Model

The voltage, flux and electromagnetic torque equations in the synchronous reference frame are provided in Equations (1) – (10) (Boldea and Nasar, 2002; El-Zohri and Mosbah, 2019). In the synchronous reference frame, all the voltages, currents and flux linkages associated with the stator and rotor d-q windings are DC in a balanced sinusoidal steady state (El-Zohri and Mosbah, 2019; Menghal and Laxmi, 2020). This is an advantage that makes the synchronous reference frame model attractive for motor control purposes.

Voltage Equations

$$V_{ds} = R_s i_{ds} + p\lambda_{ds} - \omega_s \lambda_{qs} \qquad (1)$$

$$V_{qs} = R_s i_{qs} + p\lambda_{qs} + \omega_s \lambda_{ds} \qquad (2)$$

$$V_{dr} = R_r i_{dr} + p\lambda_{dr} - \omega_{sl} \lambda_{qr} \qquad (3)$$

$$V_{qr} = R_r i_{qr} + p\lambda_{qr} + \omega_{sl} \lambda_{dr} \qquad (4)$$

Flux Linkage Equations

$$\lambda_{qs} = L_s i_{qs} + L_m i_{qr} \quad (5)$$

$$\lambda_{ds} = L_s i_{ds} + L_m i_{dr} \quad (6)$$

$$\lambda_{qr} = L_r i_{qr} + L_m i_{qs} \quad (7)$$

$$\lambda_{dr} = L_r i_{dr} + L_m i_{ds} \quad (8)$$

Electromagnetic Torque Equations

$$T_e = \frac{3P}{4}\frac{L_m}{L_r}\left(\lambda_{dr} i_{qs} - \lambda_{qr} i_{ds}\right) \quad (9)$$

$$T_e = J\left(\frac{2}{P}\right) p\omega_r + T_L \quad (10)$$

The model equations (1) – (10) were obtained from series of derivation based on the reference frame theory (Krause et al., 2013). V_{ds} and V_{qs} are the voltages applied to the stator windings in the d – q reference frame. i_{ds} and i_{qs} are the d – q axes currents in the stator windings. i'_{dr} and i'_{qr} are respectively the currents in the d – q axis of the rotor windings. λ_{qs} and λ_{ds} are the flux linkages of the stator windings respectively. λ'_{qr} and λ'_{dr} are the flux linkages of the rotor windings in the d – q axis. R_s and R_r are the stator and rotor winding resistances respectively. T_e is the electromagnetic torque produced in the motor while T_L is the load torque. p is a differential operator and it is defined as;

$$p = \frac{d}{dt} \quad (11)$$

$$L_s = L_{ls} + L_m \quad (12)$$

$$L_r = L_{lr} + L_m \quad (13)$$

$$\omega_{sl} = \omega_s - \omega_r \quad (14)$$

L_s , L_r are the self-inductances of the stator and rotor respectively. $L_{lr} L_{ls}$ are the leakage inductances of the rotor and stator while L_m is the mutual inductance between the rotor and stator circuits. ω_r is the rotor speed in rad/s, ω_s is the supply frequency or the synchronous speed in rad/s and ω_{sl} is the slip speed also in rad/s. parameter P is the number of poles on the stator. The equivalent circuit derived from Equations (1)-(8) is provided in Figure 1.

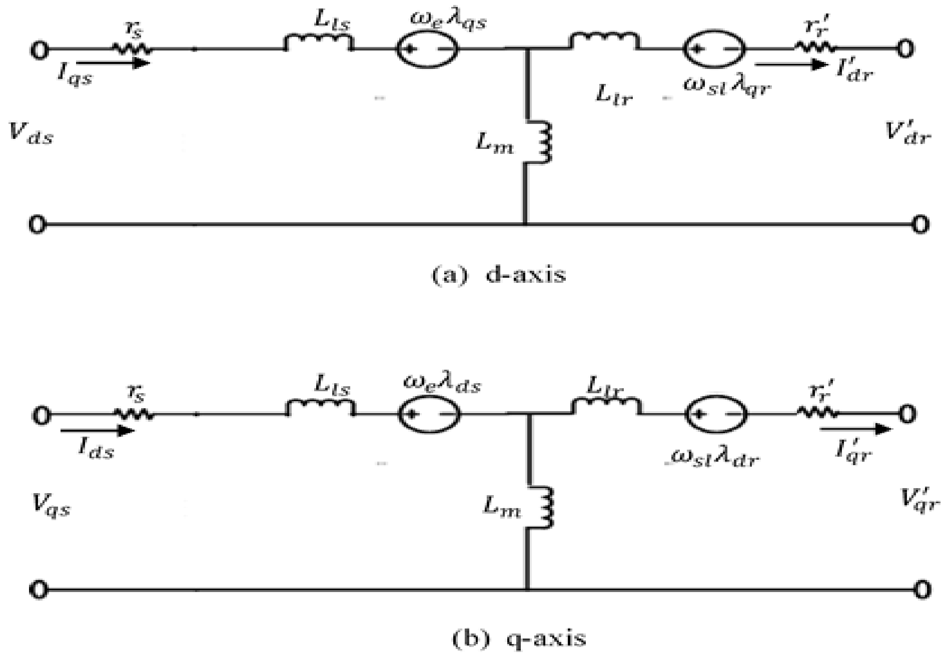

Figure 1: Equivalent circuit of the standard d-q reference frame model of the three phase Induction motor

2.2 Modifications to the d-q Reference Frame Model

In order to effectively decouple the flux current, $\mathrm{i_{ds}}$ from the torque current $\mathrm{i_{qs}}$ in the motor model, the d-axis is now aligned permanently with the rotor flux linkage space vector as shown in Figure 2.

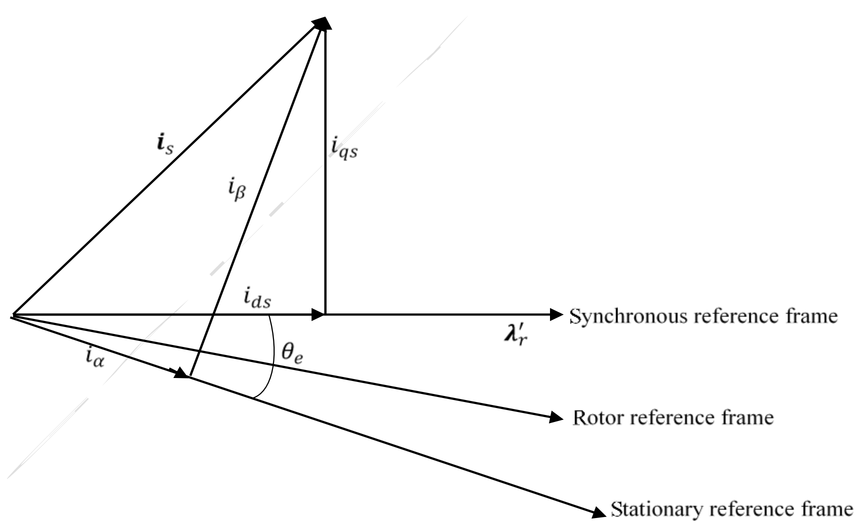

Figure 2: Rotor Flux Vector Alignment with the d-Axis

This means that the d-axis component of the rotor flux space vector, λ_{dr} is now equal to the rotor flux magnitude, λ_{r}. This cancels the q-axis component of the rotor flux linkage space vector, λ_{qr}. The rotor flux vector is computed as follows;

$$\lambda_{\mathrm{r}} = \sqrt{\lambda_{\mathrm{qr}}^2 + \lambda_{\mathrm{dr}}^2} \qquad (15)$$

Mathematically, aligning the d-axis with the rotor flux vector means equating λ_{qr} to zero in equations (3), (4), (7), (9) and (15).

Therefore the following modifications result;

$$\lambda_r = \lambda_{dr} \tag{16}$$

$$i_{qr} = -\frac{L_m}{L_r} i_{qs} \tag{17}$$

$$\omega_s - \omega_r = \frac{R_r L_m i_{qs}}{L_r \lambda_r} \tag{18}$$

With i_{qr} substituted as provided in (17), the torque produced in the motor represented by Equation (9) is modified because of the alignment of the rotor flux magnitude with the d-axis to the following

$$T_e = \frac{3P}{2}\frac{L_m}{L_r}\left(\lambda_r i_{qs}\right) \tag{19}$$

These modifications have been obtained considering that in the squirrel cage rotor, rotor voltages V_{dr} and V_{qr} are both equal to zero since the windings are short-circuited. The rotor flux dynamics is developed from (3) and (8) to give the following equation;

$$\tau_r \frac{d\lambda_r}{dt} + \lambda_r = L_m i_{ds} \tag{20}$$

Where $\tau_r = \frac{L_r}{R_r}$

τ_r is defined as the time constant of the rotor circuit. The rotor flux in now dependent on the d-axis component of the stator current as seen from equation (20). The dynamics are those of a first order system governed by the rotor time constant, τ_r. Based on the d-axis alignment, rotor flux linkage is now dependent solely on the d-axis component of the stator current and if it is maintained constant, then according to equation (19) the torque production will be dependent entirely on the q-axis component of the stator current. In this manner, the control of torque and flux may be achieved independently as in the case of the direct current (DC) motor. The model of the three-phase induction motor that achieves a decoupling of the control of torque and flux in the motor is therefore provided as follows in equations (22) – (30);

$$V_{ds} = R_s i_{ds} + p\lambda_{ds} - \omega_e \lambda_{qs} \tag{22}$$

$$V_{qs} = R_s i_{qs} + p\lambda_{qs} + \omega_e \lambda_{ds} \tag{23}$$

$$\tau_r \frac{d\lambda_r}{dt} + \lambda_r = L_m i_{ds} \tag{24}$$

$$\omega_s = \omega_r + \frac{R_r L_m i_{qs}}{L_r \lambda_r} \tag{25}$$

$$T_e = \frac{3P}{2}\frac{L_m}{L_r}\left(\lambda_r i_{qs}\right) \tag{26}$$

$$\lambda_{ds} = L_s i_{ds} + L_m i_{dr}\ , \tag{27}$$

$$\lambda_{qs} = L_s i_{qs} + L_m i_{qr}\ , \tag{28}$$

$$\lambda_r = L_r i_{dr} + L_m i_{ds}\ , \tag{29}$$

$$0 = L_r i_{qr} + L_m i_{qs}, \tag{30}$$

It is important to note that because of the alignment, the rotor flux linkage vector rotates at the same speed as the d-q windings which in this case is the synchronous speed and therefore the rotor flux angle is calculated as;

$$\theta_s = \int_0^t \omega_s(\tau)d\tau \quad (31)$$

where τ is the variable of integration. The flux angle has to be calculated at all times and used to transform the a-b-c variables to d-q quantities at every instant according to equations (32) – (33) in order to maintain the alignment of the rotor flux vector with the d-axis.

$$\begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad (32)$$

$$\begin{bmatrix} V_{ds} \\ V_{qs} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix} \quad (33)$$

The choice of the synchronous reference frame for the analysis of the induction motor ensures that all voltages, currents and flux linkages are constants (or DC) in balanced sinusoidal steady state. The equivalent circuit derived from Equations (22)-(30) is provided in Figure 3. This model can be used to study the behavior of the induction motor where vector control is not the objective.

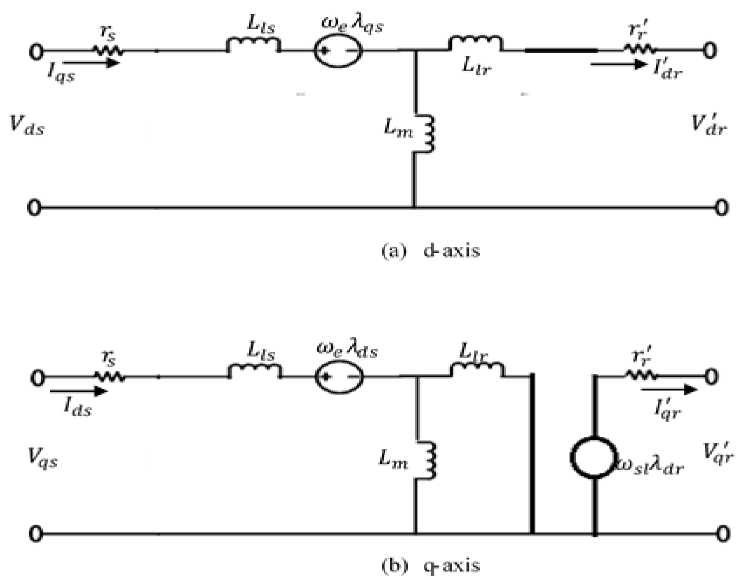

Figure 3: Equivalent circuit of the standard d-q reference frame model of the three phase Induction motor

2.3 Solutions to both the d-q Reference Frame Model and the Modified Model

In solving the d-q reference frame model of equations (1) - (11), the stator and rotor flux linkages λ_{ds}, λ_{qs}, λ_{dr} and λ_{qr} are often considered as state variables. Voltages serve as inputs and the currents i_{ds}, i_{qs}, i_{dr} and i_{qr} are then calculated from equations (5) - (8) as follows;

$$\begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{qr} \\ i_{dr} \end{bmatrix} = \begin{bmatrix} L_s & 0 & L_m & 0 \\ 0 & L_s & 0 & L_m \\ L_m & 0 & L_r & 0 \\ 0 & L_m & 0 & L_r \end{bmatrix}^{-1} \begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{qr} \\ \lambda_{dr} \end{bmatrix} \tag{34}$$

The electromagnetic torque produced in the motor is calculated from equations (9) and (10). The solution diagram to the standard d-q reference frame Model is provided in Figure 4. The block diagram of the solution to the modified d-q reference frame model in which d-axis is aligned with the rotor flux linkage vector is provided in Figure 5.

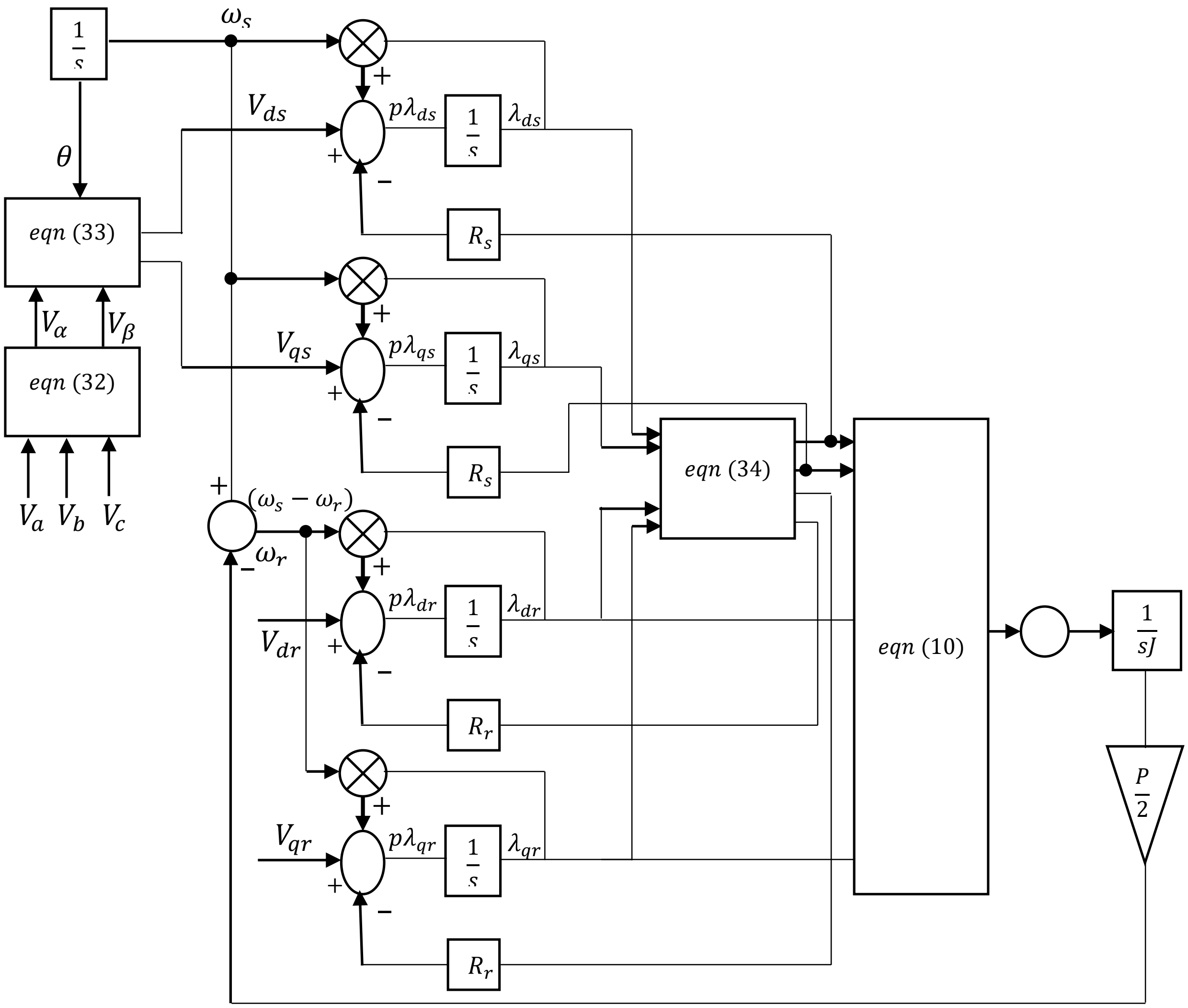

Figure 4: Block Diagram of the Solution to the Standard d-q Reference Frame Model (Model 1)

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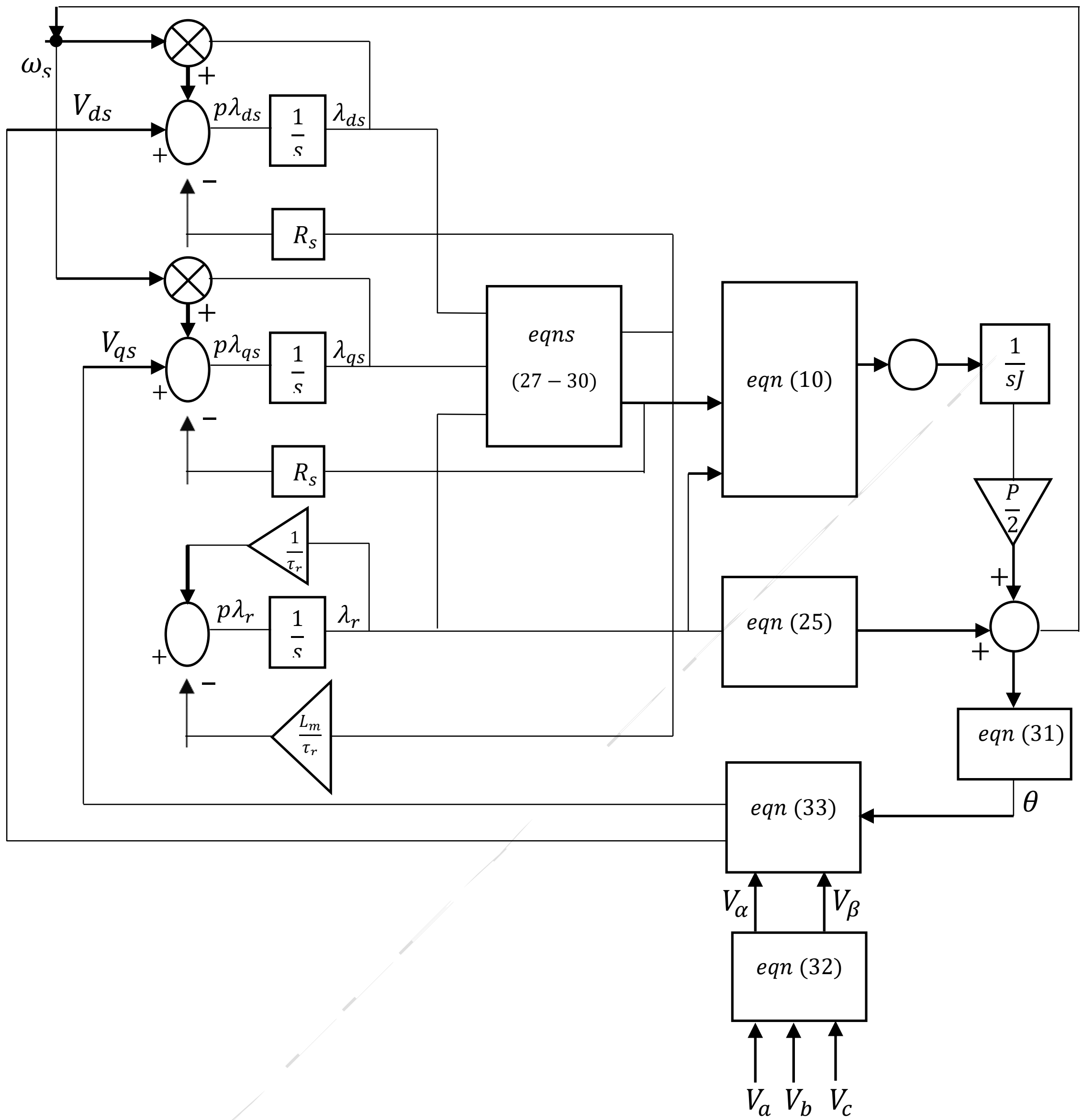

Figure 5: Block Diagram of the Solution to the Modified d-q Reference Frame Model (Model 2)

Figures 4 and 5 show that in the modified model, the synchronous speed or speed of the rotor flux has to be calculated using equation (25) and the rotor flux angle is then calculated from equation (31). The solution diagram of Figure 5 is clearly simpler than that of Figure 4. The parameters of the three phase induction motor applied in the simulation are provided in Table 1.

Table 1: Three-phase Induction Motor Parameters

Parameters	Values
Rated power	2.4 kW
Voltage	460V (L-L)
Rated speed	1750 rpm
Frequency	60Hz
Number of Poles	4
Stator resistance	1.77 Ω
Rotor resistance	1.34 Ω
Stator leakage reactance	5.25 Ω
Rotor leakage reactance	4.57 Ω
Mutual reactance	139 Ω

2.0 Results and Discussion

The solution diagrams of Figures 4 and 5 were implemented in MATLAB/Simulink and the results are presented and discussed in this section. Figure 6 (a) shows the load torque applied on the three phase induction motor models.

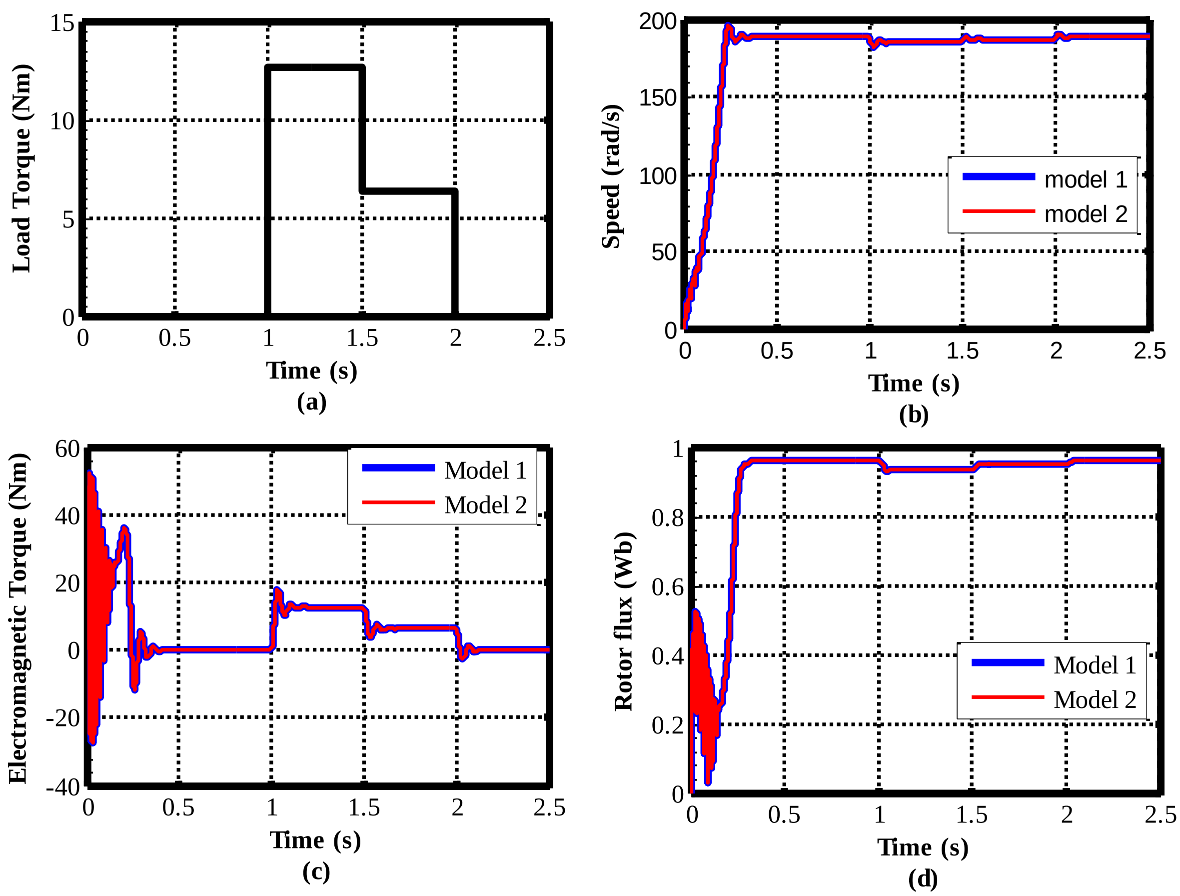

Figure 6: Simulation Results of Load Torque, Speed, Electromagnetic Torque and Rotor Flux

It can be seen that the motor was initially unloaded. A rated load of 12.644Nm was applied on the machine at steady state at a time of 1s and this load was stepped down to half its value at 1.5s. The load torque was further stepped down to 0 Nm at 2s up till the end of simulation time of 2.5s. The speed of the motor is seen in Figure 6 (b) to rise from zero to a steady state value of 185.5 mechanical rad/s in about 0.33 secs. Oscillations in response to the sudden application and reductions in load torque can be observed. The steady state speed rises slightly with each reduction in load. It can be observed that the speed profile is the same for both the standard and the modified d – q reference frame models. The electromagnetic torque produced in the motor is provided in Figure 6 (c). It can be seen to oscillate during the acceleration time and it becomes 0 Nm at steady state under no-load conditions. It equals the load torque in steady state after experiencing transients during changes in load torque in accordance with the motor dynamics given by equation (10). The electromagnetic torque is identical for the two models. The rotor flux profile is observed to be identical as well for the two models as shown in Figure 6 (d). It is constant at a value of 0.96 Wb in steady state at no load. The rotor flux can be observed to be fairly constant during to changes in load. The d – q voltages and currents are presented in Figure 7 (a) – (d).

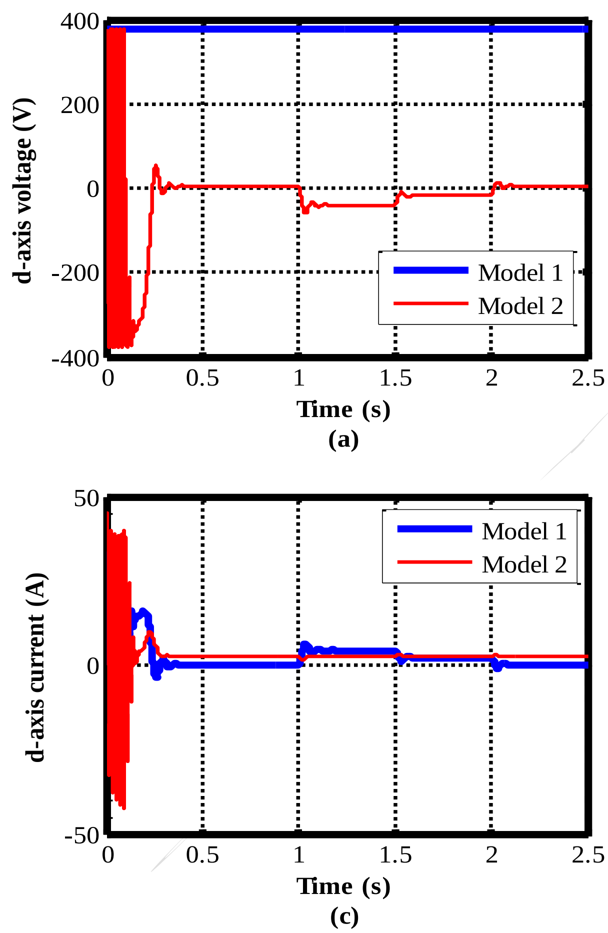

Figure 7: Simulation Results of d-q Axes Voltages and Currents

The d – q axes voltages and currents are DC voltages and currents in steady state. It can be observed that the d-axis voltage, in the case of the standard d-q reference frame, is equal to a constant value of 376V which is the peak value of the applied phase voltages in the a-b-c reference frame while the d-axis voltage is equal to 0V in the case of the modified model when no load is applied on the motor. Its magnitude, however, depends on the load applied to the motor. The q-axis voltage of the modified motor model, on the other hand, is equal to the peak value of the phase voltage applied as input to the model while the q-axis voltage of the standard motor model is equal to 0V and does not depend on the load. The magnitude of the d-axis current of the modified model is a constant value of 2.6A, regardless of load changes which is expected because it is proportional to the d-axis aligned rotor flux according to equation (24). The magnitude of the d-axis stator current of the standard motor model has a value that changes in direct proportion to the load applied on the motor. Its value is 0V when there is no load applied. The magnitude of the q-axis stator current of the modified motor model changes with load applied on the motor which is expected as well because it is proportional to the electromagnetic torque produced in the motor by design according to equation (26). This results demonstrate the decoupling of the d-axis and q-axis currents from each other in the case of the modified d-q reference frame model as predicted in the analysis. The magnitude of the q-axis current of the standard motor model has a constant value of 2.6A which does not depend on load changes to the motor. It is interesting to note that the axis in which voltage is equal to the peak value of the applied voltages is the axis in which the current is sensitive to load changes in both cases. Oscillations are present in the d-q axes voltages of the modified motor model because of the dynamics of alignment in accordance to equation (25).

3.0 Conclusion

This paper proposed an alternative to the standard d-q reference frame model of the symmetrical three phase induction motor. The two models were found to produce identical results for speed, torque and flux in the motor. The steady state rated motor speed was obtained as 185.5 mechanical rad/s which increased slightly with each reduction in load torque. The induced torque profile showed torque oscillations during changes in load torque. The rotor flux magnitude was fairly constant at a value of 0.96 Wb at steady state while the d-q axes voltages and currents exhibited alternate characteristics. It is noteworthy that the axis in which the voltage is equal to the peak value of the applied phase voltages is the axis in which the current is sensitive to load changes in both cases. This model is simpler and computationally inexpensive which should be recommended for use in studying the behavior of the induction motor where vector control is not necessary.

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