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LABORATORY EVALUATION OF FREE OVERFALL EQUATIONS FOR PREDICTING WATER PROFILE OVER WEIR

M. K. Othman[1], S. Z. Abubakar[1], X. Litrico[2] and G. Belaud[3]

Abstract

Laboratory evaluation of three free overfall equations for predicting water profile over hydraulic structure – weir was conducted in Irrigation laboratory of Higher National College of Agriculture of Montpellier (ENSAM) , France. A broad-crested weir was constructed as a reduced physical model to simulate the gated-structure (Begemann gate) for upstream water control in the Main Canal of Hadejia Valley Irrigation Project (HVIP), Northern Nigeria. Instrumentation in the ENSAM laboratory allows precise variation of channel flows with incremental changes of 5 l/s. Three recently developed models in a progressive succession were used for comparison between measured and predicted data to establish degree of accuracy in the prediction of water profile passing over weir in open channel.

Results of the evaluation show that the three equations performed differently in predicting water profile over weir at the selected channel flows of 60 l/s, 30 l/s and 10 l/s respectively. These stream sizes were chosen because they were the maximum, medium and minimum for the canal capacity. Although the stream size directly influenced the water profile as a result of internal pressure due to convergence of stream filaments between upper and lower laminar flow. Longer velocity jumps were recorded at the higher stream size (60 l/s) of 35 cm from weir crest end while less than 25 cm distance was covered by the least stream size of 10 l/s. Generally, based on the R^2 values and percentage errors for the three models at all the three selected stream sizes, Cavailhé model predicted the water profile with high degree of accuracy followed by that of Davis and lastly the Hager model. The same order was observed on ease of manipulation and user friendliness. The choice of water profile prediction model with high degree of accuracy is important in the design of hydraulic structures for water conveyance, diversion and measurements in irrigation canal network.

1. Introduction

In the design of water control and measurement structures, the concept of creating critical flow by inducing a reduction of specific energy through raising of channel bed or constricting the canal cross sectional area, thereby increasing the discharge per unit width, or combination of the two, are mostly adopted. This is done to make the discharge a single valued function of the up stream stage because it was found that when a structure is introduced into a sub-critical flow causing it to pass through the critical to the super critical, the stage of upstream is independent of the down stream stage (Withers, 1974). This concept gives birth to the creation of critical-depth meters such as broad-crested weir, venturimeter, spillway and recently automatic control gates that have combination of broad-crested weir and gate structure. The hydraulic problems of the free overfall structures are concerned with characteristics of the control and dissipation of flow in downstream basin. Ordinarily, flow over the structure is free discharging where air is admitted to the under side of the flow nappe to avoid the jet being depressed by reduced underneath pressure. Dissipation of the flow in the downstream basin may be obtained by hydraulic jump, impact and turbulence induced in the basin where erosion and down stream flood may be the results. To address this issue, USDBR (1977) developed a relationship between drop distance Y, unit discharge passing over the crest Q and critical

[1] National Agricultural Extension and Research Liaison Services, Ahmadu Bello University, Zaria (e-mail : mkothman@yahoo.co.uk)
[2] CEMAGREF (Centre National du Machinisme Agricole, du Génie Rural, des Eaux et des Forêts) Montpellier, France (xavier.litrico.@montpellier.cemagref.fr)
[3] ENSAM (Ecole Nationale Supérieur d'Agronomie de Montpellier), France

depth h_c. To reinforce this effort, several researchers attempted to determine the point of critical depth where a single measurement at established channel section might permit a direct computation of the discharge passing over the structure (Rouse, 1936; Hager, 1983; Davis *et al.,* 1999). Part of this attempt was the development of mathematical equations to predict water flow profile falling over the hydraulic structures. Among these equations were those of Hager, Davis and Cavailhé. The three equations developed by these researchers could apply to both sub and super critical flows conditions. In using these equations, the designers may have difficulty of choosing the most efficient equation that can give a better prediction of the water profile at the same time links the water depth and discharge at each section of the overfall structure most accurately. It is pertinent to note that the overfall condition is of distinct importance in hydraulic and irrigation engineering for design and use of the structures to control or divert water in irrigation system. Beside its close relation to broad crested weir, it forms the starting point in computations of the surface curve in non-uniform channel flow in which the discharge spill into an open reservoir at the downstream end. Similarly, the equation of such condition is needed in the design of automatic control gates such as Begemann and Vlughter gates for determining the depth needed to be maintained at the upstream level. This paper presents the laboratory work and result of comparable accuracies of three selected equations in predicting the flow profile over the broad crested weir. The paper further highlighted the applicability of such principle in real-life situation where such weirs play dual functions of water diversion and upstream control/measurement structure as in the case of North Main Canal of Hadejia Valley Irrigation Project (Othman *et al*., 2003).

2. Background study of free overfall equations

There is limited effort on hydraulic studies of free overfall structures adaptable to open canal irrigation system, the world over. However, few structures (weir, flumes, orifice, inverted siphon and gates) attracted researchers' attention for more than seven decades across Europe, America, Asia and North Africa. The work on hydraulics of free overfall structure was one of such studies which could be dated back to sixty years ago. The applicability of weir under wide range of field situations made it very attractive and a focus of research efforts especially where computer-based models are used to support the operation and management of canal irrigation. Starting from the work of Rouse (1936), the relationship between depth at the brink "h_e" and critical flow depth upstream "h_c", for a smooth rectangular horizontal channel, the ratio h_e/h_c was obtained as 0.715. Similarly, Markland (1965), as reported by Davis *et al*. (1999), used a relaxation method to integrate the potential flow equations and obtained solutions for the nappe and upstream profiles in a rectangular free overfall. Strelkoff and Moayeri (1970) used numerical integration and potential flow theory to compute free overfall conditions in rectangular, triangular and parabolic channel sections respectively. Hager (1983) used an analytical approach to solve the conservation of energy equation (Bernoulli equation), extended to take account of curvature effects, and applied it to rectangular free overfall conditions. Marchi (1993) also solved the same equations analytically by expanding the stream function in a power series. With the exception of Hager (1983) and Marchi (1993), all the approaches are rigorous, requiring numerical solutions (Davis *et al*., 1999). They all applied to critical and super-critical conditions and do not apply to sub-critical condition. Davis *et al*. (1999) modified the method of Hager (1983) and used parabola method which assumes two-dimensional flow (a prismatic rectangular channel with a confined nappe). It also assumed that a particle in a streamline in the nappe follows a free fall parabolic path, once the overfall structure is passed. That is, it accelerates downward in the vertical direction under the influence of gravity and possesses no acceleration in the horizontal direction. Progressively, Cavailhé (2001) further improved the work of Davis *et al*. (1999) and came out

28with a slightly different equation which predicts the water profile. The three equations of Cavailhé, Hager and Davis were developed to predict water profile over hydraulic structure of weir in this study.

2.1 Presentation of Water Surface Profile Equations

The basic principles of free-overfall equations can be understood from Figure 1 which illustrates a typical situation of water profile and dynamics of forces and other relevant parameters.

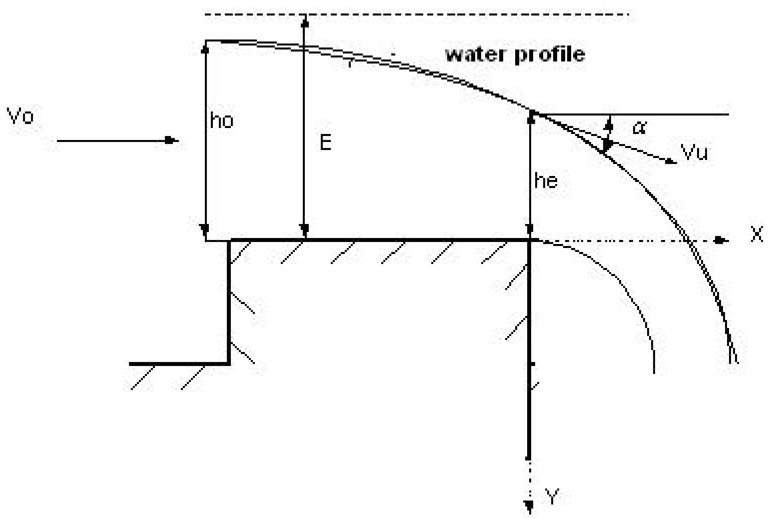

Figure 1: Schematic diagram of free overfall water profile passing over hydraulic structure

The three selected equations developed by Cavailhé (2001), Davis *et al.* (1999) and Hager (1983) under evaluation were as follows:

$$Y = \frac{2}{3}y_c - x\tan\alpha - \frac{1}{2y_c}\left[\frac{x}{\cos\alpha}\right]^2 \quad (1a)$$

$$Y = h_e - x\tan\beta - \frac{g}{2}\left[\frac{x}{V\cos\beta}\right]^2 \quad (2a)$$

$$Y = he - X\tan(\beta 1) - \frac{g}{2}\left[\frac{X}{Vu\cos(\beta 1)}\right]^2 \quad (3a)$$

Where

y_c = critical depth, the upstream depth above the weir

It is given by Cavailhé, (2001) as

$$y_c = \left[\frac{Q^2}{gL^2_a}\right]^{1/3} \quad (1b)$$

Q = discharge passing over the weir, *m^3/s*

g = acceleration due to gravity, *m/s^2*

L_a= width of the canal, *m*

$\alpha = \tan^{-1}\left[\frac{\frac{1}{3} y_c}{L}\right]$ Refer to Figure 1

L = length of the weir, *m*

$h_o = y_c$

$$h_e = \frac{2}{3} y_c = \frac{h_o fr^2}{fr^2 + \frac{2}{5}} \qquad (2b)$$

Where, fr = Froude Number

$$\beta 1 : \text{angle} = \sqrt{\frac{3(fr^2 - \frac{he}{h_o})\left(\frac{he}{h_o} - 1\right)^2}{fr^2}} \frac{180}{\Pi} \qquad (3b)$$

$$\beta 2 : \text{angle} = (20.555 - 5.686\text{fr}) \frac{\Pi}{180} \qquad (2c)$$

Vu : flow velocity = $\sqrt{2g(E - he)}$

E : energy at upstream = $h_o + \frac{Vo^2}{2g}$

Vo: average flow velocity of the discharge upstream

$Vudav$: flow velocity on the crest = $\sqrt{2g(Edav - he)}$

$Edav$: upstream energy as modified by David $= h_o + C\frac{Vo^2}{2g}$

$C = \frac{2.264}{\sqrt{fr}}$ (Davis *et al.*, 1999).

Detailed procedures of determining these equations could be obtained from Davis *et al.* (1999), Hager (1983) and Cavailhe (2001) respectively. However, Figure 1 could assist in placing all the parameters that appear in the equations.

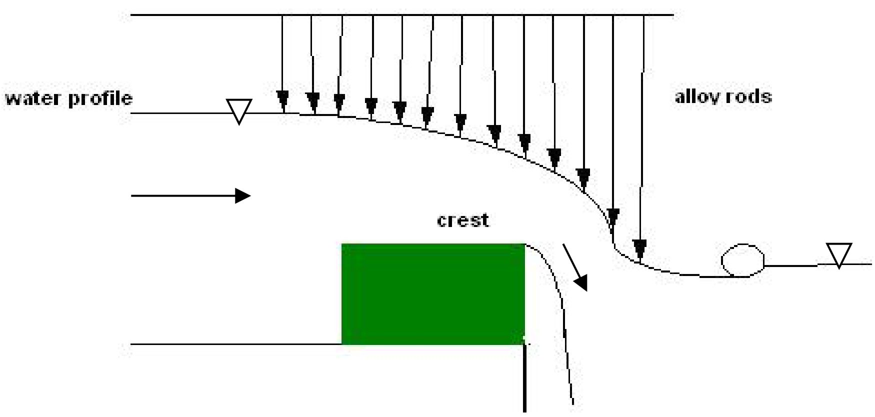

Figure 2: The experimental set-up

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3. Materials and methods

A Laboratory experiment was conducted to evaluate the three equations of water profile passing over crest of hydraulic structure as an overfall. The experiment took place at the Irrigation Laboratory of *Ecole Nationale Supérieur d'Agronomie de Montpellier (*ENSAM,) Montpellier, France in 2002. A regulator was installed at the upstream of the system, which supplies the canal with water for a desired discharge from 10 l/s up to maximum of 60l/s at interval of 5 l/s. The reference was taken at the upstream, 30 cm from the terminal end of the crest and the distance coincided with the thickness of the broad-crested weir used for the experiment. The broad-crested weir used for the experiment was constructed as a reduced model of the component of a gated-structure (Begemann gate) for upstream water control in the North Main Canal (NMC) of Hadejia Valley Irrigation Project (HVIP), Nigeria. Consequently, the dimensions of the weir were determined using the principle of dynamic similarity which implies the identity of all the dimensionless laws governing the model and prototype phenomena. Thus, a comparison of froude numbers of the two structures (the one at HVIP and the prototype model) at a critical depth with maximum canal capacity of NMC at HVIP and the prototype model in the Irrigation Laboratory canal of ENSAM was done using the following relations (Yalin, 1971):

$$fr^2 = \frac{Q^2 b}{gS^3} = \frac{Q^2}{gb^2h^3} \qquad (4)$$

$$Q^2 = gfr^2b^2h^3 \qquad (4a)$$

if $\quad h' = \lambda h \qquad (5)$

and $\quad b' = \lambda b \qquad (6)$

$$Q'^2 = \lambda^5 gfr^2b^2h^3 \qquad (6a)$$

Where *Q'* discharge of the model, *Q* discharge of the gated-structure in the field (HVIP), *b'* and *h'* width and breath of the model while *b* and *h* are width and breath of gated-structure in the field. Combining (4a) and (6a) yields (7a):

$$\lambda = \left(\frac{Q'}{Q}\right)^{\frac{2}{5}} \qquad (7a)$$

λ can be found by using the maximum discharge of the laboratory canal of 0.06 m^3/s and that of the gate in the field (HVIP) which was found from the design value to be 0.91 m^3/s

$$\lambda = \left(\frac{0.06}{0.91}\right)^{\frac{2}{5}} = 0.337 \qquad (7b)$$

h and *b* measured in the field were 93.5 cm and 112.8 cm, respectively and the height and width of 31.5 and 38 cm respectively were computed and used for the construction of the prototype model . Small alloy rods installed into a plastic frame following the pattern of the water trajectory was used to measure the water profile passing over the structure for each of the desired discharge supplied from upstream (see Figure 2). The water depth was taken from one-end of the crest length up to distance of 65 cm at interval of 5 cm for flow rates of 10 l/s to 60 l/s at interval of 5 l/s. The measurements were done with meter rule following the length of the alloy rod from its end to where it touches the water profile. The measurements were repeated four times at each point and the average was determined. Thereafter, the three

	Discharge (l/s) in the upstream supplied from the source with sub critical flow										
Distance (cm)	**60**	**55**	**50**	**45**	**40**	**35**	**30**	**25**	**20**	**15**	**10**
	Water depths from the bottom of the down stream bed (cm)										
0	69	68,3	67,3	66,4	65,2	64,2	62,9	61,4	60,2	58,9	56,7
5	68,3	67,1	66,8	65,2	64,4	63,3	61,8	60,6	59,1	57,5	55,6
10	66,8	66,4	65,2	64,3	63,2	61,9	60,5	59,3	57,9	56	54,2
15	65,5	64,7	63,7	62,7	61,7	60,3	59,1	57,9	56,4	55,2	53,9
20	63,7	62,6	62	61,2	60	59	57,8	56,8	55,7	54,7	53,6
25	61,8	61,2	60,4	59,7	58,7	58	57	56,2	55,5	54,4	53,5
30	60,2	59,5	59	58,5	57,6	56,7	56,2	55,4	54,7	53,8	52,8
35	58,5	58	57,4	56,8	56,3	55,5	55	54,4	53,4	52,7	51,4
40	56,1	55,8	55	54,3	55,7	53,2	52,5	51,6	50,7	49	47,2
45	53,2	52,5	52	51,3	50,9	49,6	48,5	47,7	46,2	43,4	40,2
50	49,3	48,6	47,9	47,4	46,6	45,4	44,7	42,7	39,8	38,7	33
55	44,5	44,2	43	42,1	40,7	39,6	35,7	35,1	30,5	27,3	21,9
60	38,5	37,9	37,3	35,7	34,5	28,6	26,5	22,7			
65	31,5	30	29,6	26,5							

equations were used for the prediction of the profiles using the different discharges as used during the experiment. This was done using spreadsheet while substituting all the known parameters and then determining the profile depth at each point for the different discharges used. The profile predicted by each of the three equations was compared with the measured data using quantitative analysis. Similarly, regression analysis and percentage error were conducted between the predicted and measured data for three selected flows in which R^2 values were determined to measure the level of accuracy for each of the equations. The percentage error was computed using the equation $(|D_t-D_p|)/D_t)$, where D_t is measured depth and D_p predicted depth.

4. Result and discussion

4.1 Measured depths of water profile

Table 1 shows the result of the surface water profile measurements for the different discharges supplied from 0 to 60 l/s respectively. The values in Table 1 represent the profile of the upper nappe of the flowing water passing over crest with a shape of a trajectory of free overfall. This means that the flow accelerates downward in the vertical direction under the influence of gravity and no acceleration in the horizontal direction. As the acceleration in the horizontal direction is small and hence negligible, the thickness of the nappe could be determined using relationship given by Rousse (1943). From the table and the graphs plotted, the results show that the influence of the upstream curvature started before the thickness of the crest, where there exist hydrostatic pressure condition which agrees with the reasoning of Davis *et al.* (1999) that the curvature or downstream effects never reach upstream more than a distance equal to four times the upstream depth.

Table 1: Average water profile measured depths (cm) at different points and flows passing over broad-crested weir

32***4.2 Models Comparison with Experimental Data***

Figure3 shows the comparison of the data between the predicted values by the three models and measured values at channel flow of 60 l/s. From the Figure, the Cavailhé and Hager Models tend to obey the law where the acceleration in the horizontal direction is the lowest. On the other hand, Davis model seem to obey the law with lesser precision indicative of lower slope. Similarly, Figures 4 and 5 show the comparison at 30 and 10 l/s flows respectively. The models predictions commence from where the nappe start. At this point, there is a complete aeration below and above the nappe, allowing the pressure to be atmospheric. Under this condition hydrostatic pressure could be assumed to be zero. However, there is still exist considerable internal pressure due to convergence of the stream filaments between upper and lower surfaces of the flow. This internal pressure is dependent on the stream size, the higher the stream size, the higher the internal pressure and the higher the velocity of the jump. Consequently, the shape of each channel flow is different as shown by the three figures. The slope for the curves at 60 l/s is lower as also shown in the figures, covering more than 35 cm horizontal distance from crest end. The slope increases for the curves at 30 l/s covering less than 30 cm horizontal distance from crest (Figure 4) while the distance covered by 10 l/s was less than 25 cm from crest with the highest slope. Again, from the models, the profile depth is dependent on the relationship between variable channel flow (flow velocity) and a constant geometrical form of the flow changes. Therefore, the complete geometrical and dynamic similarities of any two different flows are hardly attainable as pointed out by Rouse, (1936, 1943) and Davis *et al.* (1999). Thus, the points on the curves from the experimental data and those obtained from the models predictions could attest to this point (Figures 3, 4 and 5).

Table 2: Statistical comparisons of measured values with the three models at the maximum, minimum and medium flows (l/s)
M = measured values, P = predicted values, %E = percentage error

	Water profile depths (cm) in Y-direction																				
	Discharge of 60 l/s							Discharge of 30 l/s							Discharge of 10 l/s						
Distance (x-direction)	Models							Models							Models						
		Cavailhe		Davis		Hager			Cavailhe		Davis		Hager			Cavailhe		Davis		Hager	
	M	**P**	**%E**	**P**	**%E**	**P**	**%E**	**M**	**P**	**%E**	**P**	**%E**	**P**	**%E**	**M**	**P**	**%E**	**P**	**%E**	**P**	**%E**
0	60.2	64.1	6.5	59.7	0.8	59.7	0.8	56.2	58.9	4.8	56.1	0.1	56.1	0.1	52.8	54.3	0.0	53.0	0.0	53.0	0.0
5	58.5	63.1	7.9	59.4	1.5	59.1	1.1	55.0	57.6	4.6	55.6	1.1	55.1	0.3	51.4	51.9	0.0	51.9	0.0	51.0	0.0
10	56.1	60.9	8.6	58.4	4.1	57.4	2.3	52.5	54.3	3.5	54.1	3.0	52.4	0.2	47.2	45.6	0.1	48.7	0.1	45.2	0.0
15	53.2	57.6	8.2	56.8	6.8	54.4	2.3	48.5	49.2	1.5	51.5	6.2	47.7	1.6	40.2	35.4	0.4	43.4	0.4	35.6	0.1
20	49.3	53.0	7.6	54.5	10.6	50.3	2.1	44.7	43.2	5.5	47.9	7.2	41.2	7.7	33.0	21.3	0.5	36.0	0.5	22.1	10.3
25	44.5	47.3	6.3	51.6	16.0	45.1	1.3	35.7	33.3	6.6	43.3	21.4	32.9	7.9	21.9						
30	38.5	40.4	4.8	48.1	24.9	38.6	0.3	26.5	22.6	14.4	37.7	42.4	22.7	14.4							
35	31.5	32.2	2.3	43.9	39.4	31.0	1.5														
R^2 Values	**0.9983**			**0.9962**		**0.9961**		**0.9981**			**0.9956**		**0.9957**		**0.9958**			**0.9948**		**0.9945**	
Average % Error			**6.5**		**12.8**		**1.5**			**15.9**		**11.6**		**14.6**			**0.1**		**0.0**		**0.1**

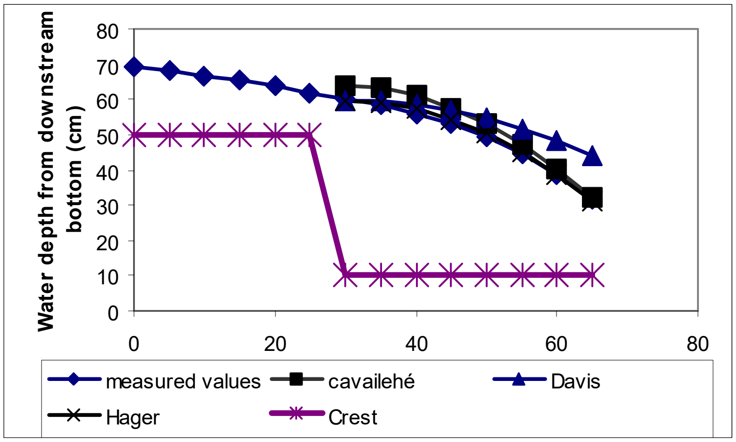

Figure 3: Graph of comparison between the three equations and measurement at 60 l/s

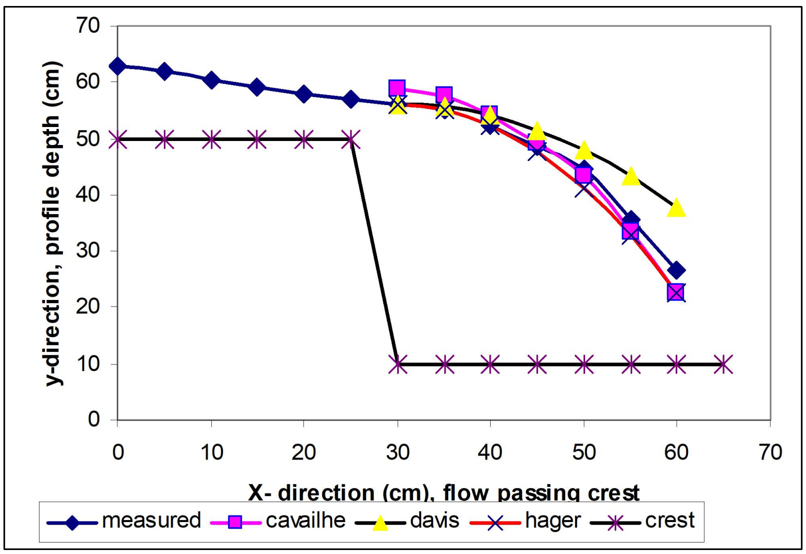

Figure 4: Graph of comparison between the three equations and measurement at 30 l/s

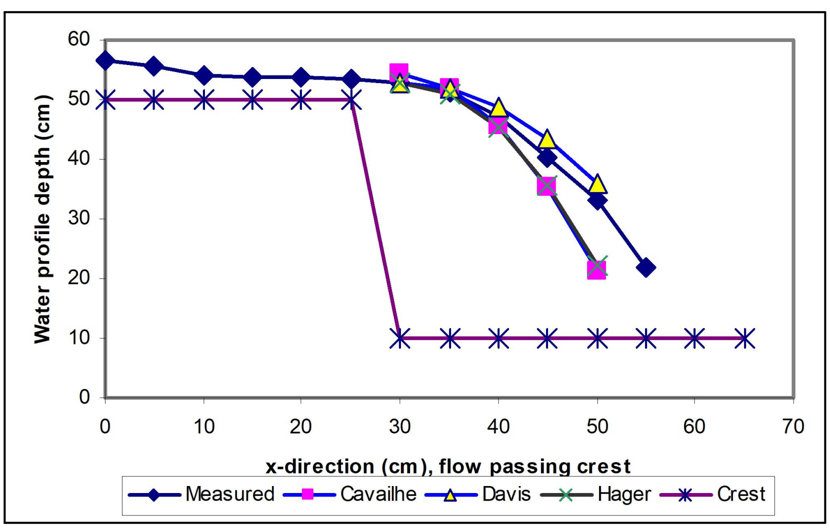

Figure 5: Graph of comparison between the three equations and measurement at 10 l/s

Table 2 shows the statistical comparisons of the models predictions with the experimental data at the channel flows of 60 l/s, 30 l/s and 10 l/s respectively.

From the table, the following can be deduced:

- Cavailhé model predict better than the other two models irrespective of the channel flows where R^2 values of 0.9983, 0.9981 and 0.9958 were obtained at 60 l/s, 30 l/s and 10 l/s channel flows respectively. Davis model follows in performance at 60 l/s and 10 l/s and performed least at 30 l/s while Hager Model shows the least prediction accuracy at 60 l/s and 10 l/s and performed better at 30 l/s.
- The trend of percentage error recorded by the three models to predict water profile over weir has no consistent pattern. (Table 2). The highest percentage errors were recorded by all the three models at 30 l/s channel flow while the least errors occurred at 10 l/s for all the three models. At highest channel flow (60 l/s), Hager model had the least percentage of error (1.5) followed by Cavailhé model (6.5) where Davis model recorded 12.8 % error.
- Comparing the R^2 values and percentage error for all the three models at the three selected channel flows, Cavailhé model predict water profile with highest degree of accuracy followed by that of Davis while Hager model predict with least degree of accuracy. It is logical that the progressive improvement incorporated by Cavailhé in Davis work, who in turn, improved Hager work are empirically justified in this study.

4. Conclusion and recommendations

The laboratory evaluation of the three free overfall models for predicting water profile over weir bring out the differentiation among the most recent models developed. The experimental set-up represent a real life situation where managers of irrigation schemes are confronted with decision on whether to use weirs as solely diversion structures or use them for dual functions of diversion and flow measurements as well as upstream control hydraulic structures. All the three models were found to predict fairly accurately the water profile depths flowing over broad crested-weir in an open channel. The choice among the three depends on the level of precision required for the type function the structure is intended to serve. The study would also guide in the choice of which model to use by irrigation engineers for design of water control and flow measurement hydraulic structures in canal network. Similarly, the choice of which model can predict most efficiently is also useful in selection and in-corporation into a canal irrigation simulation packages. Finally, the ease of manipulations of these models is also instructive of their accuracy in prediction and user friendliness.

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