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ORIGINAL RESEARCH ARTICLE

A MATHEMATICAL MODEL FOR PREDICTING THE CUTTING ENERGY OF COCOYAM (COLOCASIA ESCULENTA)

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ABSTRACT
Information on the optimum energy requirements for cutting vegetables is useful in order to estimate the amount of energy needed to cut given products with known physical characteristics. This will aid engineers and product designers in the development of appropriate cost effective cutting systems consuming minimum amount of energy while still providing high quality cut products. In this study, predictive equations were developed for describing the cutting energy of cocoyam (Colocasia esculenta) cormels. Dimensional analysis based on the Buckingham pi theorem was used to obtain the functional relationship between the cutting energy of the selected vegetable and the independent variables such as tool weight (w), height of tool drop (Hd), tool edge thickness (t), cutting speed (v), crop size(s), crop moisture content (φ), crop contact area (A) and crop density (σ). The developed model was validated with experimental data and a high coefficient of determination of R2 = 0.982 between the predicted and measured values was established. The obtained predictive model proved appropriate for determining the cutting energy required for cocoyam cormels up to 98%.

1.0 Introduction

Agricultural products often occur in sizes too large to be used, and therefore they need to be reduced and put into different sizes and shapes like cubes, thin slices or rings to facilitate further processing. Cutting, which is a size reduction activity, is an energy consuming operation. However, the cutting energy requirements of vegetables have been estimated using different cutting systems and approaches with the following objectives: to know the most efficient cutting conditions necessary for optimization of the cutting process; and to determine the optimal energy required to cut and to achieve an overall efficient cutting operation. Blade sharpness, slicing angle, contact area, depth of cut, cutting speed, and the engineering properties of vegetables such as crop variety, maturity stage, moisture content, average diameter, orthogonal dimensions and fibre orientation have been identified as some of the parameters affecting the energy requirements for cutting of vegetables (Ciulica and Rus 2011; Elżbieta and Agnieszka, 2012 and Singh et al., 2016).

Cocoyam (Colocasia esculenta) is the root vegetable of interest in this study. It is a member of Araceae family and in the group of monocotyledons, constituting of one of the six most important root and tuber crops worldwide (Edet and Nsukka, 2000). Cocoyam is an aroid because it is grown mainly for its edible cormels and some of the varieties include; tannia (Xanthosoma sagittifolium), taro (Colocasia esculenta) eddo (Colocasia antiquorum), tarul, arum, Alocasia macrorrhizos and alocasia odora (Ekanem and Osuji, 2006). Nigeria is the largest producer of cocoyam in the world with an annual production of 3.450 million metric tonnes in 2012, representing 72.2 %, 57.7 % and 45.9 % of total production in West Africa, Africa and the World, respectively (FAOSTAT, 2012). Cocoyam is said to be a giant crop, because its cormels, leaves, stalks and inflorescence are all utilized for human consumption (Chukwu et al., 2008). It has been found to have nutritional, medicinal and industrial benefits; reported to be more suitable for several food products, especially as food for potentially allergic infants, and persons with gastro-intestinal disorders (Opara, 2002; Ekwe et al., 2009; Eleazu et al., 2013). The cocoyam cormels could be processed into many products including poi (fresh or fermented paste, canned, and canned-acidified), flour, cereal base, beverage powders, chips, sun-dried slices, grits, drum-dried flakes, a binding agent in tablet manufacture and a source of industrial starches. (Onayemi and Nwigwe, 1987; Iwuoha and Kalu, 2000; Subhadhirasakul et al., 2001; Lawal, 2004; Lewu et al., 2009 and Owuamanam et al., 2010)

However, in spite of its many industrial, medicinal and nutritional benefits, studies reveal that it is an under-exploited and insufficiently studied crop (Nguyen and Nguyen, 1987; Goenaga and Heperly, 1990; Giacometti and León, 1994; Watanabe, 2002) which portends a danger of gradual disappearance from our meal time table and eventual extinction unless urgent attention is given to it. Also, it suffers a lot of postharvest losses due to inadequate post-harvest handling technologies which makes the crop scarce and expensive when it is not in season. Hence in postharvest processes such as drying, frying, grinding, pelleting, packaging, transportation and storage of cocoyam cormels, size reduction by cutting is a necessary operation that equally increases the surface area of the processed cormel.

Furthermore, extensive documentation on the properties of foods and food products exist, however data related to the cutting energy of different vegetables is scarce, even though such data is important in the design of cutters. This observation is affirmed by researchers like Saravacos and Kostaropoulos (2002) who noted that less work has been performed on energy involved in the cutting of different food materials, and Brown et al. (2005) who opined that limited published literatures on specific energy requirement in cutting of fruits and vegetables are available. Also, Mitcham et al. (1996) agreed that literature related to cutting of fruits and vegetables are limited.

1.1 Modeling Agricultural Processes

Modeling, simply defined is a representation in mathematical terms of the behavior of real devices and objects, or a system of postulates, data and inferences presented as a mathematical description of an entity or state of affairs (Dym and Ivey, 1980). It could be descriptive, explanatory or predictive. In the prediction part, which can be envisaged as organized thinking of the possible, models are exercised to give information on a yet-to-be-conducted experiment. These predictions are then followed by observations that serve either to validate the model or to suggest reasons that the model is inadequate (Dym, 1994 and Cha et al., 2000). Various mathematical modeling techniques that are in use include: dimensional homogeneity and analysis, abstraction and scaling, conservation and balance principles and consequences of linearity.

However, dimensional analysis has become a widely applicable and very powerful technique that is adopted in mathematical modeling. This is due to its simplicity in planning, presentation, and interpretation of experimental data, providing an organized way to plan and carry out experiments, and enables one to scale up results from model to prototype (Bahrami et al., 2006; Maheedhara and Diwakar, 2014). The dimensional analysis is a mathematical technique used in identifying the factors involved in a physical situation or phenomenon and forming a relationship between them. It offers a method for reducing complex physical problems to the simplest form prior to obtaining a quantitative answer (Andrzej, 2015). Although there are other methods of performing dimensional analysis, notably the indicial method, the method based on the Buckingham pi theorems gives a well-organized procedure for obtaining a solution. The Buckingham pi theorem states that the number of dimensionless and independent quantities required to express a relationship among variables in any phenomenon is equal to the number of quantities involved minus the number of dimensions in which those quantities may be measured (Fox and McDonald, 1992). Mathematically speaking, if there is a physically meaningful equation involving a certain number (n) of physical variables, then the original equation can be rewritten in terms of a set of $p = n - k$ dimensionless parameters ($\pi 1, \pi 2, ..., \pi p$) constructed from the original variables (Hart, 1995), (Here k is the number of physical dimensions involved; which in this situation, (k = 3 namely Mass, Length and Time).

The application of mathematical models in different aspects of agricultural engineering including tillage operations (Fielke, 1999), spraying machines (Teske et al., 1991), crop handling machines (Gorial and O'Callaghan, 1991), harvesting (Baruah and Panesar, 2005a and b) and many versatile topics on post harvesting aspects have been successfully attempted such as; modeling flow rate of egusi-melon (Colocynthis citrullus) through circular horizontal hopper orifice (Asoegwu et al., 2010); modeling the grain cleaning process of a stationary sorghum thresher (Simonyan et al., 2006) and development of a model to describe infrared radiative and convective drying characteristics of onion slices for optimum management of operation parameters (Jain and Pathare, 2004). Although a few research attempts have been made to model cutting processes like mathematical models and laboratory tests of Impact cutting behavior of forage crops, (McRandal and McNulty, 1978); performance modeling of the cutting process in sorghum harvesting (Mohammed, 2002), mathematical modeling of laser based potato cutting and modeling yield efficiency of peeling (Somsen et al., 2004, Ferraz et al., 2007), much is left undone on modeling the energy requirement for cutting vegetables.

Some researchers used the dimensional analysis based on the Buckingham's pi theorem as veritable instrument in establishing a prediction equation of various systems which include the development of screw-conveyor performance models using dimensional analysis, (Degrimencioglu and Srivastava, 1996); a mathematical model for predicting output capacity of selected stationary grain threshers (Ndirika, 2006); a mathematical model for predicting the cracking efficiency of vertical-shaft centrifugal palm nut cracker (Ndukwu and Asoegwu, 2011); and modeling flow rate of Egusi-melon (Colocynthis citrullus) through circular horizontal hopper orifice (Asoegwu et al., 2010).

The size reduction operation by cutting in most postharvest processes is a most laborious, time wasting and energy consuming task which deserves research attention. These studies are necessary in order to discover methods of efficiently improving the cutting process by maximizing the scarce energy, time and resources available and optimally enhancing the postharvest operations concerned. Hence the need to undertake the present study which will establish a mathematical model using dimensional analysis based on the Buckingham's pi theorem to predict the cutting energy requirement for cocoyam (Colocasia esculenta) cormels. The mathematical model will become a design tool for machine designers in the development of energy saving and cost effective cutting systems that will provide good quality cut products.

2.0 Materials and Methods

2.1 Theoretical Development

Factors affecting the cutting energy of vegetables obtained from literature include tool parameters like (sharpness, rigidity of cutting tools and knife speeds) and physical properties of the plant material like (crop variety, size, maturity stage, crop moisture content, crop density, fiber orientation) (Szot et al., 1987; Nadulski, 2001; McGorry et al., 2003; Blahovec, 2007; McCarthy et al., 2007 and Atkins, 2009).

In the development of the model, dimensional analysis was employed which is a technique used in identifying factors involved in a physical situation and forming a relationship among them. However dimensional analysis based on the Buckingham pi theorem which states that the number of dimensionless and independent quantities required to express a relationship among variables in any phenomenon is equal to the number of quantities involved minus the number of dimensions in which those quantities may be measured (Fox and McDonald, 1992) was adopted.

If N = number of variables involved in a physical situation

x = number of dimensions in which quantities may be expressed = (3), M, L, T

n = number of dimensionless groups as given in Eqn (1).

Hence,

$$n = N - x \qquad (1)$$

However, due to the large number of variables influencing the energy of cut of the selected root vegetable, assumptions will be made in order to bring these large numbers to a reasonable and manageable number. (Simonyan et al., 2006).

2.2 Assumptions made in Model Development

Variables that are design parameters which are measurable were considered.

Variables that are functions of other variables were not considered e.g. volume which is a function of crop size. However, cutting speed which is a function of time was considered instead of time.

The fiber orientation, textural properties and crop variety are considered negligible in the model development.

2.3 Development of the Functional Energy Equation

These assumptions helped in reducing the number of variables involved to the under listed as these were considered to have greater influence on the energy of cut of the selected vegetable and are measurable. The chosen variables were: Tool weight (w), Height of tool drop (Hd), Tool edge thickness (t), Cutting speed (v), Crop size(s), Crop moisture content (φ), Crop contact area (A) and Crop density (σ).

Having identified the core variables influencing the energy required for cutting the selected vegetable, Eqn 2 represents the functional equation of the predictive model.

$$E = f(w, Hd, t, v, s, \varphi, A, \sigma) \qquad (2)$$

where

E = cutting energy (J), w = Tool weight (kg), Hd = Height of tool drop (m), t = Tool edge thickness (m), v = Cutting speed (m/s), s = Crop size (m), φ = Crop moisture content (%), A = Crop contact area (m2) and σ = Crop density (kg/m^3).

Three primary dimensions M= Mass, L= Length, T= Time were chosen in the description of the variables. From the Buckingham pi theorem (Fox and McDonald, 1992) the total number of dimensionless groups to be formed is as given in Eqn 1 above.

for $n = N - x$

and N = number of variables involve in the situation studied = 9

X = number of dimensions for describing these variables = 3

then, $n = 9 - 3 = 6$

Six dimensionless groups were formed namely; π_1, π_2, π_3, π_4, π_5 and π_6. In determining the dimensionless groups, the following procedure was adopted (Fox and McDonald, 1992).

The variables utilized in the establishment of the model equation for the cutting energy of the selected vegetable were expressed in terms of their dimensions as shown in Table 1.

Table 1: List of variables and their dimensions

S/N	VARIABLE	SYMBOL	UNIT	DIMENSION
1	Energy	E	kgm^{2}s^{-2}	ML^{2}T^{-2}
2	Tool weight	w	kgms^{-2}	MLT^{-2}
3	Height of tool drop	Hd	m	L
4	Tool edge thickness	t	m	L
5	Cutting speed	v	ms^{-1}	LT^{-1}
6	Crop size	s	m	L
7	Crop moisture content	φ	φ	M^{0}L^{0}T^0
8	Crop contact area	A	m^2	L^2
9	Crop density	σ	kgm^{-3}	ML^{-3}

The dimensional matrix of the variables is shown in Table 2. This is needed to develop the indices of the involved variables.

Table 2: Dimensional matrix of variables

S/N	VARIABLE	SYMBOL	M	L	T
1	Energy	E	1	2	-2
2	Tool weight	w	1	1	-2
3	Height of tool drop	Hd	0	1	0
4	Tool edge thickness	t	0	1	0
5	Cutting speed	v	0	1	-1
6	Crop size	s	0	1	0
7	Crop moisture content	φ	0	0	0
8	Crop contact area	A	0	2	0
9	Crop density	σ	1	-3	0

From Table 2, it was seen that φ is dimensionless and therefore excluded from the dimensionless terms determination and is to be added later (Simonyan et al., 2006) while the other variables were combined to form the Π groups. Cutting speed (v), tool weight (w) and crop size (s) were selected as the major parameters (i.e. recurring set) because they contained all the primary dimensions involved in this problem and their combination does not form a dimensionless group. Having selected w, v and s as the recurring set, the exponents a, b and c are attached to them respectively so that when their product $w^a v^b s^c$ divide the remaining variables E, Hd, t, A and σ, the dimensionless groups Π1, Π2, Π3, Π4 and Π5 are obtained as given in Eqns 3 to 7 (Ndirika, 2006; Simoyan et al., 2006; Asoegwu et al., 2010; Ndukwu and Asoegwu, 2011) .This is the basis of the Buckingham pi theorem of dimensionless groups.

$$\Pi_1 = \frac{E}{w^a v^b s^c} \quad (3)$$

$$\Pi_2 = \frac{H_d}{w^a v^b s^c} \quad (4)$$

$$\Pi_3 = \frac{t}{w^a v^b s^c} \quad (5)$$

$$\Pi_4 = \frac{A}{w^a v^b s^c} \quad (6)$$

$$\Pi_5 = \frac{\sigma}{w^a v^b s^c} \quad (7)$$

where a, b, and c are exponents needed to make the groups non-dimensional. The variables are substituted with their dimensions and the non-dimensional Πs are replaced with M0L0T0 which is a dimensionless group. In order to obtain values for the exponents, the principle of dimensional homogeneity is used to equate the dimensions on each side of the equations of the Π groups.

Eqn 3 being expressed in terms of the dimensions on both sides becomes Eqn 8

$$M^0L^0T^0 = \frac{ML^2T^{-2}}{(MLT^{-2})^a(LT^{-1})^bL^c} \quad (8)$$

Cross- multiplying Eqn 8 gives Eqn 9.

$$M^0L^0T^0(\left(MLT^{-2})^a\left(LT^{-1}\right)^bL^c\right) = ML^2T^{-2} \quad (9)$$

Using dimensional homogeneity for M, L and T, the exponents a, b and c are got in Eqns 10, 11, 12 and 13.

For M; $M^{0+a} = M^1$

0 + a = 1

a = 1 (10)

For L; $L^{0+a+b+c} = L^2$

0+a+b+c = 2; from Eqn 10, a = 1

1+b+c =2

b+c = 2-1 =1

b+c = 1 (11)

for T; $T^{0-2a-b} = T^{-2}$

-2a – b =-2

Since a =1

-2(1) – b = -2

-b = -2+2 =0, b = 0 (12)

From Eqn 11, b + c = 1, put b=0

0+c=1; c=1 (13)

Hence, a = 1, b = 0, c = 1; replacing the exponents with their values,

Π_1 becomes Eqn 14

$$\Pi_1 = \frac{E}{w^1 v^0 s^1} = \frac{E}{ws} \tag{14}$$

Similarly, solving for the remaining Π groups, we obtain Eqns 15 to 19.

$$\Pi_2 = \frac{H_d}{s} \tag{15}$$

$$\Pi_3 = \frac{t}{s} \tag{16}$$

$$\Pi_4 = \frac{A}{s^2} \tag{17}$$

$$\Pi_5 = \frac{\sigma v^2 s^2}{w} \tag{18}$$

$$\Pi_6 = \varphi \tag{19}$$

Combining these equations gives Eqn 20, whose components are dimensionless.

$$\text{i.e.} \quad \frac{E}{ws} = f\left(\frac{H_d}{s}, \frac{t}{s}, \frac{A}{s^2}, \frac{\sigma v^2 s^2}{w}, \varphi\right) \tag{20}$$

Combining the dimensionless terms to reduce it to a manageable level (Shefii et al. 1996) by multiplication and/or division, we obtain Eqns 21 to 23 which are dimensionless.

$$\Pi_{12} = \frac{\Pi_1}{\Pi_2} = \frac{E}{ws} \times \frac{s}{H_d} = \frac{E}{wH_d} \tag{21}$$

$$\Pi_{34} = \frac{\Pi_3}{\Pi_4} = \frac{t}{s} \times \frac{s^2}{A} = \frac{ts}{A} \tag{22}$$

$$\Pi_{56} = \frac{\Pi_5}{\Pi_6} = \frac{\sigma v^2 s^2}{w} \times \frac{1}{\varphi} = \frac{\sigma v^2 s^2}{w\varphi} \tag{23}$$

The new dimensionless functional relationship becomes Eqns 24 and 25.

$$\Pi_{12} = f(\Pi_{34}, \Pi_{56}) \tag{24}$$

$$\frac{E}{wH_d} = f\left(\frac{ts}{A}, \frac{\sigma v^2 s^2}{w\varphi}\right) \tag{25}$$

From Eqn 25, the Equations for E are obtained as Eqns 26 and 27.

$$E = wH_d\left(\frac{ts}{A}, \frac{\sigma v^2 s^2}{w\varphi}\right) \tag{26}$$

$$E = f\left(\frac{tswH_d}{A}, \frac{\sigma v^2 s^2 H_d}{\varphi}\right) \tag{27}$$

Eqn 27 gives the cutting energy E, with all the parameters in Eqn 2, as a function of two energy components $\frac{tswH_d}{A}$ and $\frac{\sigma v^2 s^2 H_d}{\varphi}$ which are represented as P and Q, respectively in Eqn 28. Eqns 29 and 30 give the energy components represented by P and Q.

$$E = f(P,Q) \tag{28}$$

Where

$$P = \frac{tswH_d}{A} \tag{29}$$

$$Q = \frac{\sigma v^2 s^2 H_d}{\varphi} \tag{30}$$

2.4 Experimental procedure

Fifty cocoyam cormels (Colocasia esculenta spp) bought from the Eke-Onunwa Market at Owerri, Imo state, Nigeria were thoroughly cleaned of all impurities and unwholesome cormels, thereafter sorted into four size ranges of 10 corms each and some physical properties obtained at an initial moisture content of 71.8±0.16% (wb)(AOAC, 2000). The measured physical properties of the selected cocoyam cormels were; mass (kg) determined with an electronic weighing balance (Model GF-200, A & D company Ltd, Japan of an accuracy of ±0.01); the circumference at the point of cut (obtained by winding round with a tape and measuring out on the meter rule); the area (mm2) was determined by tracing out on a graph sheet and summing up the number of squares; the orthogonal dimensions (major, intermediate and minor diameters) were determined with a digital venier caliper (Model 500-196, Mitutoyo Products, America), these were used to calculate the equivalent diameter of the cocoyam samples. With an automated vegetable cutter, at a preset speed of 40mm/min, the cutter drops from a height of 30cm, cutting the sample as it travels through it. The slope of the force-distance relationship displayed on the monitor was used to calculate the cutting energy. Figure 1.0 shows the experimental set-up of the cutting mechanism.

Figure 1.0: Experimental set-up of the cutting mechanism

The automated vegetable cutter consists of the hardware and software components. Arduino controlled processors automatically and effectively measures, records and stores cutting variables and other basic parameters with minimum human supervision, thus making the entire cutting process automated. Connecting to an electric power source switches on the cutter and a predetermined speed value is selected on the keypad. As the knife presses against the sample, the reactive force exerted on the load cell is amplified and measured. Connecting to a computer using USB port the measured values are relayed and MATLAB intercepts the values and plots the resulting graph of force of cut against distance travelled which is used in calculating the energy of cut.

2.5 Model input parameters and Validation

In the determination of validation parameters, every other parameter was held constant while the crop size varied. The selected tool edge thickness (t) measured with a digital venier caliper (Model 500-196, Mitutoyo Products, America) was 0.2cm. The weight (w) of the cutting tool determined with an electronic weighing balance (Model GF-200, A & D company Ltd, Japan of an accuracy of ±0.01) was 300g. The cutting tool was dropped from a pre-determined height (h) of 30cm measured with a meter rule. The crop density (σ) determined by dividing the weight of the crop by the volume of water displaced when sample is placed inside a measuring cylinder was taken as 1.35g/cm^3. The orthogonal dimensions of the sample were determined with the digital venier caliper (Model 500-196, Mitutoyo Products, America). The crop contact area (A) taken as 28.75cm^2 was determined by multiplying the circumference at the point of cut (obtained by winding round with a tape and measuring out on the meter rule) with the major diameter of the crop sample. The moisture content of the crop sample determined with the method described in AOAC (2000) was taken as 71.8 %(wb).

2.6 Prediction energy equation

The predicted cutting energy equation was established by allowing one of the energy components $P(\frac{tswH_d}{A})$ or $Q(\frac{\sigma v^2 s^2 H_d}{\varphi})$ to vary at a time while keeping the other one constant and observing the resulting changes in the function (Shefii et al., 1996). This was achieved by plotting the experimental values of E against $P = (\frac{tswH_d}{A})$ while keeping Q constant. $P = (\frac{tswH_d}{A})$ was evaluated by substituting the measured values (as listed out in section 2.5) for tool edge thickness (t), crop size(s), tool weight (w), height of tool drop (Hd) and crop contact area (A) into P. Also, E against $Q = (\frac{\sigma v^2 s^2 H_d}{\varphi})$ was plotted while keeping P constant. Values for $Q = (\frac{\sigma v^2 s^2 H_d}{\varphi})$ were obtained by substituting the measured values (as listed out in section 2.5) for crop density (σ), cutting speed (v), crop size(s), height of tool drop (Hd) and crop moisture content (φ) into the equation. In evaluating the energy values for P and Q, the crop size was varied while the other factors remained constant (Simonyan et al., 2006; Shefii et al., 1996). The obtained values for the predicted cutting energy were plotted against the measured (experimental) values on regression curves using the statistical package, Microsoft excel 2007 and the coefficient of determination (R^2) and Root Mean Square Error (RMSE) values obtained. The validity or suitability (goodness of fit) of the models developed for the cutting energy requirement of the selected crop was tested by comparison with experimental data. Values obtained for the Root mean Square Error (RMSE) and the coefficient of determination (R^2) were indicators of the suitability of the developed models.

3.0 Results and Discussion

Average values and standard deviations of the measured physical parameters of the cocoyam cormels (as described under the experimental procedure) are shown in Table 3.

Table 3: Average values for physical properties of cocoyam cormels at 71.8± 0.16% (w.b)

S/N	Corms	Mass (g)	Diameter (mm)	Circumference (mm)	Area (mm^2)
1	C_1	300±6.29	60.50±0.22	205.2±1.44	2874.75±32.89
2	C_2	140±1.30	48.31± 0.62	165.7±1.02	1833.01±40.11
3	C_3	100±4.53	36.20±0.60	120.5±0.84	1029.22±25.16
4	C_4	70±1.24	22.15± 0.48	79.8±0.35	385.33±10.28

From the results displayed in Table 3, it can be observed that the measured physical properties (in terms of mass, equivalent diameter, circumference and area) of the cocoyam samples sorted from the largest (C1) to the smallest (C4) based on measurement decreased in the measured values from C1 to C4. This is indicative of the significant effect of size on the physical characteristics of agricultural products, which is expected to influence the energy required for cutting the products.

Table 4 shows the experimental values of the cutting energy obtained with the automated vegetable cutter and the predictive values of the cutting energy obtained by substituting values of the cutting variables into the energy equations $P = (\frac{tswH_d}{A})$ and $Q = (\frac{\sigma v^2 s^2 H_d}{\varphi})$.

Table 4: Experimental values (Emeas) and calculated values (P, Q) of cutting energy for cocoyam cormels.

S/N	Corms	Crop size (mm)	(E_{meas})	$P = \left(\frac{tswH_d}{A}\right)$	$Q = \left(\frac{\sigma v^2 s^2 H_d}{\varphi}\right)$
1	C_1	60.50±0.22	192.21	378.8	110.2
2	C_2	48.31± 0.62	170.73	302.4	70.2
3	C_3	36.20±0.60	129.25	226.65	39.4
4	C_4	22.15± 0.48	75.22	138.99	14.8

Cutting energy results as observed from Table 4 show the reduction in both the experimental values (Emeas) and the predictive values (P, Q) obtained as the cormel sizes reduced. The results obtained showed the significant effect of crop size on the cutting energy requirement for cocoyam cormels as the highest energy observed was for C_1 which is the largest range of crop sizes tested and the lowest energy obtained was for the least range of crop sizes tested i.e C4 (over 60% reduction in the cutting energy values from highest size range to lowest size range).

Furthermore, while C1 is 63.4% bigger in size than C4, the cutting energy components, E_{meas}, P and Q for C1 were found to be 60.9%, 63.3% and 86.6% higher than for C4. This goes to authenticate the effect of crop size on the cutting energy requirement of the crop being studied.

The plots of the cutting energy (E_{meas}) against P and Q are shown in Figures 1 and 2 with their linear equations and R_2 values expressed in Eqns 31 and 32.

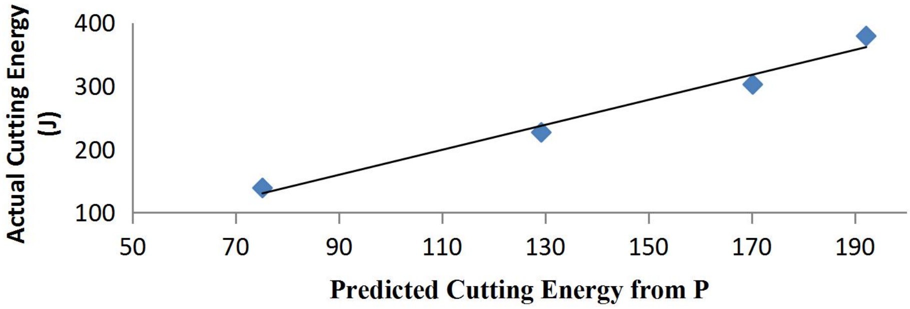

Figure 1: Variation of cutting energy against $P = \left(\frac{tswH_d}{A}\right)$, keeping $Q = \left(\frac{\sigma v^2 s^2 H_d}{\varphi}\right)$ constant.

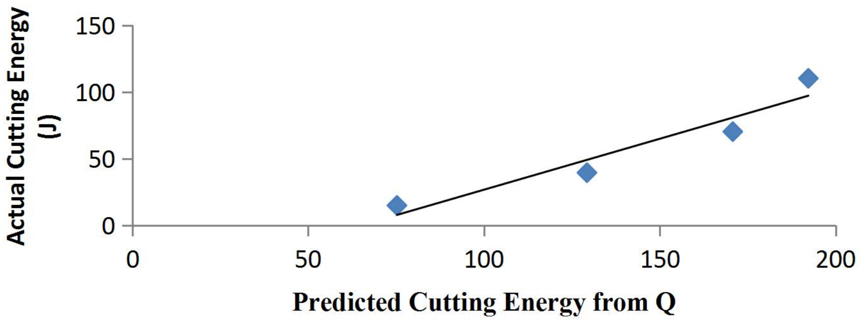

Figure 2: Variation of cutting energy with $Q = \left(\frac{\sigma v^2 s^2 H_d}{\varphi}\right)$, keeping $P = \left(\frac{tswH_d}{A}\right)$ constant.

$$E = 1.973P - 17.96; \qquad R^2 = 0.977 \qquad (31)$$

$$E = 0.764Q - 49.83; \qquad R^2 = 0.916 \qquad (32)$$

The plot of the P and Q terms in Figures 1 and 2 forms a plane surface in linear space and according to Mohammed (2002), it implies that their combination favors summation or subtraction. Therefore, the component equations formed by the subtraction and summation of Eqns 31 and 32 give Eqns 33 and 34 respectively.

$$E = f_1\,(P,Q) - f_2\,(P,Q) + K \qquad (33)$$

$$E = f_1\,(P,Q) + f_2\,(P,Q) + K \qquad (34)$$

It must be noted that;

at f_1, Q was kept constant while P varied

at f_2, P was kept constant while Q varied

Substituting Eqns 31 and 32 into Eqn 33 and performing some algebraic manipulations yields Eqn 35.

$$E = 1.973P - 0.764Q + 31.87 \qquad (35)$$

Also, Substituting Eqns 31 and 32 into Eqn 34 and performing some algebraic manipulations yields Eqn 36.

$$E = 1.973P + 0.764Q - 67.79 \qquad (36)$$

A further manipulation as permitted under the rules of the Burkingham pi theorem (Shefii et al., 1996) is manipulating with a constant factor. Hence, Eqns 35 and 36 are divided with a constant factor of 3 which yields the predicted model Equations expressed in Eqns 37 and 38 respectively. Dividing eqns 35 and 36 with the constant factor of 3 yielded predicted values close to the actual ones.

$$E = 0.658P - 0.255Q + 10.62 \qquad (37)$$

$$E = 0.658P + 0.255Q - 22.60 \qquad (38)$$

Substituting the variables for P and Q into Equations 37 and 38 yield Equations 39 and 40 respectively.

$$E = 0.658\frac{tswH_d}{A} - 0.255\frac{\sigma v^2 s^2 H_d}{\varphi} + 10.62 \qquad (39)$$

$$E = 0.658\frac{tswH_d}{A} + 0.255\frac{\sigma v^2 s^2 H_d}{\varphi} - 22.60 \qquad (40)$$

However, the final predicted model equation will be either of the above two equations that gives the better statistical inference.

3.1 Model validation

The mathematical model was validated using the data generated from the vegetable cutter. The model validation was done at four ranges of crop sizes (20-23mm, 35-38mm, 45-49mm and 60-63mm) and a constant cutting speed of 40mm/min. The method of regression analysis as computed using Microsoft Excel environment was used to describe the relationships, plot the graphs and compute the coefficients of determination (R2). Measured values of parameters were substituted into Eqns 39 and 40 to yield the predicted cutting energy values which were plotted against the experimental energy values on a regression curve in order to obtain the coefficients of determination (R^2) and the Root Mean Square Error (RMSE). Eqns 41 and 42 express the relationship between the predicted cutting and experimental cutting energy with R^2 values of 0.982 and 0.970, respectively. However, Eqn 41 which gives the higher R^2 value of 0.982 with a

RMSE of 0.97 between the experimented and predicted cutting energy values which is less than 1% of the average value of the experimental cutting energy is taken as the predicted model equation for the cutting energy of cocoyam cormels. Figures 3 and 4 presents the regression curves between the predicted and experimental cutting energy respectively with their linear equations and R2 values given in Equations 41 and 42 respectively.

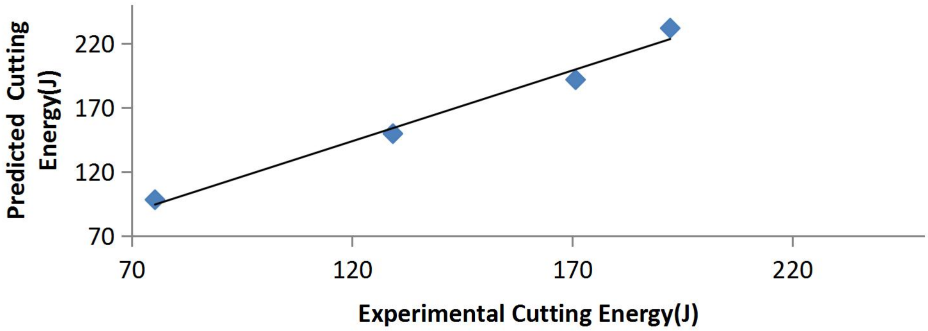

Figure 3: Relationship between experimental and predicted cutting energy (for subtraction of component energy equations)

$$E_{pred} = 1.100\, E_{exp} + 11.74; \qquad R2 = 0.982 \qquad (41)$$

Where E_{pred} = Predicted Cutting Energy, E_{exp} = Experimental Cutting Energy

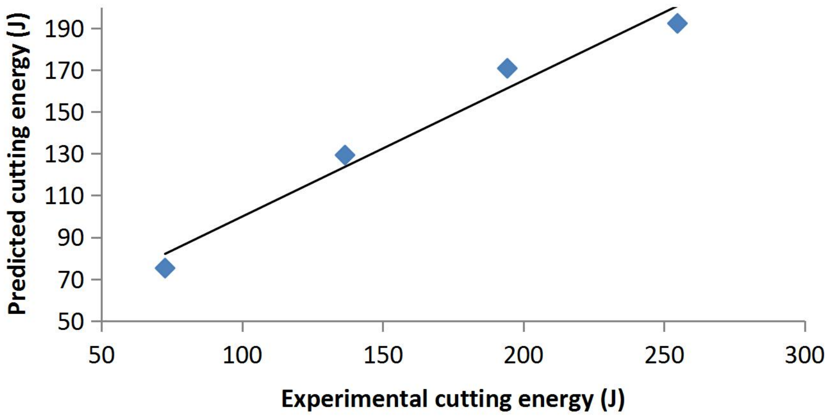

Figure 4: Relationship between experimental and predicted cutting energy. (For summation of component energy equations)

$$E_{pred} = 0.650\, E_{exp} + 34.75; \qquad R2 = 0.970 \qquad (42)$$

Where E_{pred} = Predicted Cutting Energy, E_{exp} = Experimental Cutting Energy

From the results of the correlation graphs, the coefficient of determination obtained when component equations were subtracted was R^2 = 0.982 with a RMSE of 0.97 between the experimental and predicted cutting energy which is less than 1% of the average value of the experimental cutting energy. Also the coefficient of determination obtained when component equations were summed was R^2 = 0.970 with a RMSE of 1.06 between the experimental and predicted cutting energy which is more than 1% of the average value of the experimental cutting energy. However, a higher coefficient of determination of R^2 = 0.982 and a lower RMSE of 0.97 were obtained with subtraction of the component energy equations as compared to coefficient of determination (R^2 = 0.970) and RMSE of 1.06 obtained by the summation of the component energy equations. Hence, the prediction model equation for the cutting energy requirement for cocoyam (Colocasia esculenta) cormels will be the equation which gives a higher value of coefficient of determination of R2 = 0.982 and a lower value of RMSE = 0.97 expressed as Eqn 43.

$$0.658\frac{tswH_d}{A} - 0.255\frac{\sigma v^2 s^2 H_d}{\varphi} + 10.62 \quad (43)$$

Where: w = Tool weight (kg), Hd = Height of tool drop (m), t = Tool edge thickness (m), v = Cutting speed (m/s), s = Crop size (m), φ = Crop moisture content (%), A = Crop contact area (m^2) and σ = Crop density (kg/m3).

4.0 Conclusion

A mathematical model for predicting the cutting energy of cocoyam (Colocasia esculenta) cormels was presented using dimensional analysis based on the Buckingham's Π theorem. The model equation expressed as $E = 0.658\frac{tswH_d}{A} - 0.255\frac{\sigma v^2 s^2 H_d}{\varphi} + 10.62$ was validated with data from an automated vegetable cutter. Results obtained showed a high coefficient of determination (R^2 = 0.982) and a low Root Mean Square Error (RMSE) of 0.97 which implies that the model is good. Therefore, the developed model could be used to predict the cutting energy of cocoyam cormels up to 98%.

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