Engaging in Critical Dialogue about Mathematics[1]

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Abstract

The goal of this paper is to highlight the fact that the Philosophy for Children Approach (P4C) can be used to stimulate pupil's reflection within the framework of school subjects such as mathematics. First we situate P4C within the field of socio-constructivist epistemology. Then, P4C as adapted to mathematics (P4CM) is introduced. Finally, we describe an experiment linked to five types of exchanges (anecdotal, monological, simple dialogical, semi-critical dialogical, and critical dialogical), manifested between the beginning and the end of a school year while the pupils were learning to philosophize about mathematics. In the discussion, emphasis is placed on the educational interventions of the teachers who facilitate the P4CM workshops.

Key Words: Philosophy for Children; Mathematics; Socio-constructivist Epistemology; Critical dialogue.

In many countries, mathematics seems to be the bane of both pupils and teachers. Many pupils fail in this subject, starting in elementary school. Many pupils have a difficult relationship with mathematics. Many are those who do not understand its meaning or do not see its usefulness or who have negative biases with regard to the subject (i.e. too abstract, too difficult to understand, etc.). Some pupils try to avoid any situation which might confront them with problems with mathematical content because they are convinced they cannot succeed in this discipline because they do not have "mathematical logic" or are not a "math wiz." These attitudes reflect established beliefs that are not entrenched or a priori judgments that constitute barriers to the learning process (Lafortune & Solar, 2003; Lafortune et al., 1999, 2003).

According to Lafortune et al. (1999, 2003), in a majority of classrooms, teaching mathematics consists of "teaching to learn." So educational approaches focused on reflection are hardly used, or not at all. A justification that is often cited by teachers is lack of time, too little time to do activities with the pupils other than those that are directly related to the subject content. Consequently, many pupils come to think that "learning math" means memorizing procedures, applying them and finding exact answers.

At the end of the 1990's, Philosophy for Children adapted to Mathematics (P4CM) was proposed with the objective of guiding second-cycle elementary school pupils in critical reflections regarding prejudices about mathematics and about philosophical-mathematical concepts adapted to teaching programs for these pupils. Experiments conducted with groups of pupils aged 9 to 12 years have indicated that P4CM positively stimulates reflection and dialogical competencies in these pupils.

In the following pages, we introduce three socio-constructivist epistemological principles, which are also inherent in P4C. Then, we present the P4CM material that was used in the classrooms. Finally, we describe an experiment that involved learning to engage in philosophical dialogue about mathematical questions.

Philosophy for Children and Socio-constructivist Epistemology

The Philosophy for Children (P4C) approach, introduced in Quebec in 1982 by Anita Caron, was conceived by Matthew Lipman and his colleagues at Montclair State University (N.J.) early in the seventies. This approach differs from traditional educational approaches in that instead of searching for "the" correct answer (the one in the manuals and expected by the teacher), youngsters learn to ask relevant questions, to think on their own and to discuss problems through philosophical dialogue among peers (Lipman, 2003; Lipman et al., 1980).

P4C is inspired by the pragmatist educational philosophy, which focuses on the child's global development and emphasizes the needs and interests of youngsters as well as their everyday experiences (Daniel, 1997). And P4C is related to a socio-constructivist epistemology (Daniel, 2005). In view of the strong influence of socio-constructivism in Quebec school-teaching programs (MELS, 2002) and programs in other countries, it is important to present three of its principles, which reflect the pragmatist aims of the Lipmanian approach. The first principle concerns the constructed nature of knowledge; the second concerns the viable nature of knowledge; and the third concerns its social nature (see among others: Glasersfeld von, 1994, 2004; Jonnaert & M'Batika, 2004; Jonnaert & Masciotra, 2004; Legendre, 2004; Masciotra, 2007; Masciotra et al., 2008; and Pallascio, 2004).

The first principle, concerning the constructed nature of knowledge, implies that knowledge is not an objective reality predetermined outside the subject, but rather a construction of the subject-in-search-of-knowledge (Dewey, 1967; Ruel, 1994). In other words, reality only exists within the subject and depends on the subject; it must be constructed and not discovered; it thus places the subject in a position of active researcher (actor) rather than passive reception (receptor). In P4C (and P4CM), three consequences of applying this principle are: developing responsibility in the pupil, developing the pupil's involvement in the collective process of knowledge production and, in so doing, developing the pupil's self-esteem.

The second principle emphasizes the viable nature of knowledge; it thus confronts the notion of "truth." Indeed, according to pragmatists and constructivists, truth is never final; it is an open process that is never reached since it is relative to norms that are accepted at the time and to criteria temporarily established by a group, a society or a culture. For example, we know that each scientific community, each school of thought accepts as "true" what suits it, and yet at times the interpretations of some of these communities happen to be contradictory, each one justifying its perspective according to specific and different theoretical frameworks. Constructivists speak of the "sustainability" or viability of knowledge, terms that refer to a search for explanations and sustainable interpretations of how the world works; pragmatists use the term "contextualization," which means that these explanations and interpretations result from an adaptation linked to the context and that they stem from experience. Thus, sustainability or contextualization of knowledge implies a diversity of supported and justified explanations and it rests on sharing within a community of peers. Indeed, it is to the extent that it is experientially or theoretically justified, then reaffirmed by various subjects, that constructed knowledge is validated and considered "true" (Bayles, 1966; Glasersfeld, von, 1994, 2004). In the context of P4C, it is the sustainability of the pupils' constructed knowledge that determines its validity – more than its correspondence to established scientific knowledge. Consequently, pupils are called upon to experiment, justify and confront their points of view.

The third principle emphasizes the social character of the knowledge production process. It is based on, among others, Vygotsky's (1985) theoretical principles, which postulate that people construct their knowledge through language and social interaction. As Mead (1972) explains, socio-cognitive processes are deployed based on stimulation of complex thinking skills, which develop through a person's relationships with her environment. In other words, social interactions are as fundamental to cognitive development as language itself, and increasing sophistication in the knowledge-construction process cannot occur without the subjects' interactions, in particular language interactions (Auriac, 2007; Larochelle & Désautels, 2001; Le Cunff, 2009; Vygotsky, 1985).

The third principle leads pupils to transcend simple memorization and actively engage in a conscious and voluntary inquiry process with their peers. It is through cognitive conflicts that emerge from divergent points of view among peers that pupils are brought to question and to doubt reality, suggest alternative solutions, criticize suggested alternatives, and identify criteria to bring to light the most coherent solution (Dewey, 1925, 1983; Fourez, 1998; Larochelle, 1998; Larochelle & Désautels, 2001). Thus, the third socio-constructivist principle refers to the complexity of pupils' thinking. This complexity is made possible through philosophical dialogue among peers, without which it would be difficult to construct, transform and enrich meanings regarding mathematical problems and concepts.

Philosophizing about Mathematics

Inspired by the P4C approach, a team comprised of a philosopher of education and two mathematicians conceived material to help youngsters at the end of elementary school to dialogue about and reflect on mathematics in a critical manner (Daniel et al., 1999, 2004) . This material does not pretend to fall within the scope of the philosophy of mathematics. It simply aims to stimulate pupils toward an autonomous and critical comprehension of mathematical problems and concepts included in their study programs , as well as biases and stereotypes that are often attached to this subject.

The Philosophy for Children Adapted to Mathematics (P4CM) material includes a philosophical novel and a manual for teachers. The novel, titled *The Mathematical Adventures of Matilda and Damian*, presents the questions and reflections of end-of-elementary school pupils regarding mathematics. It tells the story of Matilda who succeeds very well in math but is worried that her reputation as a "math wiz" may make her unattractive to Matthew. Damian, who excels at Art, hates math and has all kinds of negative prejudices toward the subject. The manual includes approximately 300 philosophical-mathematical discussion plans and activities to explore questions such as: Is there such a thing as a perfect cube? Is zero equal to nothing? If you were able to count every grain of sand on the planet, would you say the number is infinite or indefinite? Is math useful in your life? Does the teacher know everything?, etc. Following is an excerpt from the novel:

> Matilda goes into her bedroom and slams the door shut. As usual, she takes off her shoes, drops her backpack in a corner and throws herself on her bed. Ohhhh she feels good!
> Matilda likes her bedroom. It is a little green room with a square floor.
> -Oh! One could say it's almost a cube! Isabel spoke to us about cubes this morning, in geometry class. What exactly did she say? Matilda frowns, trying to recall.
> The words of Isabel, her teacher, slowly come back to her. Things always happen like that in Matilda's head. First, her thoughts form a type of large, thick cloud. Then, one by one, her ideas emerge from the thickness of the cloud. Only then can she can grasp and inspect them.
> While continuing to think reflectively, Matilda vaguely looks around her. She wonders:
> - Can a room really be a cube or does it just look like a cube? Isabel said, I remember now, that it was impossible to have an absolutely perfect cube on Earth. That's astonishing!
> Matilda tries to think some more about this question, but she is tired. She gets bogged down in her ideas, gets impatient and, finally, gives up.
> "Tomorrow, I will ask Isabel to clarify this. After all she is the teacher! She must surely know all about geometry."
> Matilda's thoughts take wing, freed from their mathematical problem. She starts to dream about Matthew. She would so like him to be her boyfriend. (need pp. #)

To fully exploit the first philosophical-mathematical concept in this chapter *(Can a perfect cube exist?)* – which pupils do not fail to question – the teachers' manual suggests a concrete activity for pupils, that is, the drawing of a cube. Then, to guide the teacher in her Socratic maieutics regarding the perfection (or imperfection) of a cube, the manual proposes the following discussion plan:

- Are the drawings you just made cubes or do they just look like cubes? Why?
- Is a cube a square? Explain the similarities and the differences.
- How many square faces, summits and edges can you find on a cube's surface?
- Does the number of faces, summits and edges vary according to the size of the cube? Why?
- Establish a parallel between a cube and some other thing. For example, take the word "tree." Is the word itself a tree or just a concept that encompasses every type of tree that exists on the planet?
- What is the difference between the word "tree" and a real tree?
- Can you answer the question Matilda asks herself: What is the difference between "being a cube" and "looking like a cube"? (need pp. #)

The P4CM material is said to be philosophical in that it contains open-ended questions that enable pupils to reflect upon a diversity of viable and reliable answers. To answer these philosophical-mathematical questions, pupils must actively engage in a critical reflection process. And since the process is too complex for individuals to achieve results, it is carried out with peers. Reflection within a community of inquiry, because of the diversity of points of view it presupposes, stimulates pupils at the cognitive level. Also, as verbal interactions are required to answer the philosophical-mathematical questions, pupils are stimulated on the discursive level (among others: Daniel et al., 2002, 2005). The following section focuses on learning philosophical dialogue.

Description of an Experiment

The P4CM material, because it resides within the scope of the philosophical process suggested by Lipman and his colleagues, is likely to foster critical exchanges among pupils. Indeed, first, pupils use the story of Matilda and Damian as a pretext to question philosophical-mathematical concepts or biases regarding mathematics. Then, with reference to the story, pupils formulate questions they would like to discuss with their peers. And, finally, they exchange among each other, with teacher guidance, so that as a group they can construct elements of answers that are reliable and viable. The objective of the third step is not to let pupils "talk" (Daniel, 2000, 2009), nor to encourage argumentation, as in a competition (see Dolz and Schneuwly, 1998; Le Cunff, 2009), but to encourage pupils to dialogue philosophically in a perspective of cooperation, where each individual intervention contributes to enriching the group's perspective. The essence of P4CM is found in learning to dialogue about mathematics.

Philosophical dialogue requires regular and continuous practice (approximately one hour per week for at least one school year). A study conducted in Australia, Mexico and Quebec , in eight classrooms of pupils aged 10 to 12 years who used P4CM, revealed that the exchanges can remain anecdotal or monological during the entire school year if they are not adequately guided by the teacher (Daniel et al., 2002; Daniel & Delsol, 2005). We considered an exchange to be anecdotal when youngsters speak in a non-structured manner about personal and particular experiences. When this is the case, pupils are not engaged in a process of inquiry, they do not share a common goal and they are little or not at all influenced by peer interventions. Furthermore, they do not justify their points of view and their opinions are presented as conclusions. Following is an example of an anecdotal exchange centred on personal experiences:

Teacher: In the story, why doesn't Ramon like math exams?
Pupil 1: I become nervous during exams.
Pupil 2: Because sometimes, I, because I worry.

Pupil 3: Because I am nervous.

We considered an exchange to be monological to the extent that pupils begin to engage in a personal process of inquiry that is essentially oriented toward a quest for "the" correct answer. Each pupil intervention is independent from the others, and answers are juxtaposed to one another. The analysis illustrates that in this type of exchange, pupils have difficulty in justifying their opinions.

It is only when philosophical *praxis* is established and when pupils have become aware of their individual rights (e.g. to think in an autonomous matter) and responsibilities (e.g. to share their thoughts) that they start to de-center from their own points of view and start to listen to those of their peers. Pupils are then able to dialogue; they begin to form a "community of inquiry" and they actively participate in the reflection as they are motivated by a common goal (Daniel *et al.*, 2000). But to dialogue does not inevitably mean to dialogue in a philosophical or critical manner, as pupils may co-construct their point of view without evaluating the validity, usefulness, or viability of the statements or criteria in question. In a non-critical dialogue, pupils respect differences of opinion; they construct their points of view based on those of their peers; they begin to justify their positions. However, they do not yet "philosophize" concerning mathematics even though they are situated within a philosophizing learning process. Following is an example of a non-critical dialogical exchange:

Teacher: Why do you say geometry is an interesting subject?
Pupil 1: Because it is part of our daily lives.
Pupil 2: That's true because at school, for example, we learn to measure figures and when we're older and we will want to buy some land, we will know how much land we have.
Pupil 3: I agree with pupil 2. And also because with geometry, for example, architects can build schools, buildings and all, stores and all that we need in our lives like Pupil 1 said.

Non-critical dialogue, or simple dialogue, is the type of exchange that seems the most valued by the current school systems. Indeed, intrinsically, it includes values that are highly regarded socially, such as pluralism, accepting differences, and tolerance of differences of opinion. However, despite its dialogical nature, this is not enough to help young generations meet the challenges of the 21st century. Indeed, our study revealed that simple dialogue fell within the scope of an epistemology we refer to as relativism (Daniel *et al.*, 2005; Daniel, accepted). Relativism carries meanings that are both positive (openness, respect, tolerance, etc.) and negative (tolerance can lead to intellectual complacency to the extent that any premise is acceptable and accepted). Complacency, which is a pitfall in any non-critical exchange, anchors youngsters in a mentality of non-questioning, acceptance, and even passivity.

In fact, our analysis of exchanges among pupils brought to light three types of dialogical exchanges: non-critical, semi-critical and critical. The semi-critical dialogical exchange is manifested when, in a context of interdependence, some pupils are sufficiently critical to question the statements of their peers, but the latter are not critical enough to be cognitively influenced by this criticism, so that the criticism does not lead to a transformation (nuance, clarification, etc.) of the perspective. Following is an excerpt of a semi-critical dialogical exchange:

Teacher: Can we speak of a perfect cube?
Pupil 2: I say maybe it's possible, to have a perfect cube, because if you take 4 (sic) squares and if you look at them, then with a blade, you take a little bit away... You keep on taking away little bits until they become equal...
Pupil 3: At the end, you're going to need instruments that are too small to do

	something.
Pupil 4:	You'd have to be lucky [...].
Pupil 3:	No. I don't think that if you measure the centimeters... After, you have to comeround to millimeters, then you come to hundredths of mm, then to thousandths of mm, you keep going like that. You'll never be able to make a perfect cube if you measure [...]
Pupil 2:	You could take geometry blocks.
Pupil 5:	Yes, but geometry blocks aren't all equal. [... the makers] make them as equal as possible, the most perfect possible, but that doesn't mean they're perfect, perfect, perfect. They may seem perfect to us, but...
Pupil 2:	I say it might be possible to have a perfect cube.

As to the dialogical critical type of exchange, it is manifested when the pupils' points of view contribute not only to improving the group's perspective, but to transforming it (Rorty, 1990a, 1990b, 1991). This type of exchange implies the following criteria: explicit interdependence among pupils; the inquiry is focused on construction of meaning (*vs*. search for truth); pupils are aware of the complexity of their peers' points of view; they search for divergence and consider uncertainty to be a positive cognitive state; criticism is sought for its own sake, as a tool to further comprehension; pupils spontaneously justify their points of view coherently and completely; a social/ethical preoccupation can be observed in their interventions; statements are articulated in the form of hypotheses to be verified rather than as closed conclusions. Following is an extract of a dialogical critical exchange:

Teacher:	Last week, we looked at the order of animals and the order of maths, which ones you thought were higher. Can one of you (...) take up the discussion where we left off?
Pupil 2:	I would place humans in fourth or third place or maybe second because I don't think we deserve to go at the top for what we've done to all those animals and how we've had wars. And like animals don't care, I mean they have wars sometimes but it's when they need to be in the higher group to be respected more. (...) So I think that animals are a higher level than humans but they respect other people and we tend to be selfish.
Pupil 4:	I think that humans are the only ones that can do math, because it's like
English:	Humans invented English. And math is just like another language that we invented. We use it to understand things, to do the things we have to do well, to understand the reasons behind things. Like why the sky is blue and why we can't float or fly. So we invented maths to explain these things. (...) But the animals they just think sky and they don't really think about it, because they've got one main instinct which is eat and reproduce.
Teacher:	And how does that affect the order of things?
Pupil 4:	Oh, well if it's the order of how smart they are, I think humans would have to be at the top.
Teacher:	Humans would have to be. Why? What criteria are you using?
Pupil 4:	On how complex they are. And that we've got other intelligences, like I said yesterday, empathy and sympathy and stuff like that.
Pupil 5:	I agree because if I had to rank any of the animals in a higher order or whatever, I think I'd put humans on the top as well because (...) we do things for our own pleasure and usually we do them of our own accord. We usually do whatever we want because we've got better resources for it and we've created more things. It's just our brain power is larger. I don't know if it is but I think that our brain power is larger.
Pupil 6:	I disagree with Pupil 5 when he said they don't build things. They build nests,

they build burrows, they have got to work out how to build them, that's not really easy. And they only kill what they need.

Pupil 5: [...] I think I sort of changed my mind. I sort of agree with Pupil 6 (...). Then there are like two different paradigms.

Pupil 6: Yes, there is the intelligence to think how to make things and the intelligence of how to use these things. We are both the most stupid and the most intelligent.

In short, the experiment we conducted in three countries led us to understand that pupils from elementary school have the competencies to philosophize about mathematical concepts and problems. The P4C method, inherent in the P4CM material, helped them to dialogue in a critical manner with their peers.

Discussion

Observations in Quebec, Australia, France, México and elsewhere have shown that most pupils who participate in P4C workshops have trouble philosophizing or engaging in critical dialogue, no matter what the framework in which these sessions take place (Ethics, Violence Prevention, Language Arts, or Mathematics). After analysis, we have come to understand that this difficulty is neither associated with the pupils' cultural background, nor with the school subject in which these philosophical workshops take place. We have also come to understand that difficulties in philosophizing were not due only to a cognitive or epistemological limit in younger pupils – even though age is a factor (see Daniel & Gagnon, 2012). We observed that it stemmed from, among other things, the stimulation-reflection relationship between teacher and pupils. Following are four typical sessions that brought to light some intrinsic relationships between teacher questions and pupils learning to philosophize:

a) When the teacher did not closely monitor the pupils' reflection, in other words when she let them "talk" about questions they had formulated when they were collecting questions (e.g.: *Why is math boring? Why is math stressful? Why do we do math?*), most of the time pupils discussed idiosyncratically on more than one idea at a time – instead of expanding on one by providing definitions, by identifying relationships, by finding causes and consequences, etc. They did not focus on a common objective (e.g.: *the origin of negative prejudices toward mathematics*), instead focusing on a diversity of specific objectives relating to their interests (their fears or frustrations) regarding the subject; their exchange was linear since it was only slightly or not at all argumentative; finally, the classroom remained an aggregation of individuals who generated ideas juxtaposed to one another, instead of actively participating in a dialogue within a community of inquiry.

b) When the teacher (contrary to the previous situation) kept to her traditional role, despite the use of philosophical material, and focused on questions related to the discipline (e.g.: *How many square faces are in a cube? How many edges are there?*) without following through on this theoretical learning with a philosophical reflection about, among other things, "similarities and differences" (e.g.: *Does the number of faces and edges vary according to the size of the cube? Why?*), then pupils set as their objective to answer the teacher's questions correctly, rather than aiming at the construction of meanings together with their peers. In addition, they waited for the teacher to question them to state their points of view instead of embarking upon an autonomous inquiry.

c) When the teacher de-emphasized her role as a "transmitter" to favour her role as a "guide" and supported the pupils in their reflections (e.g.: *Who can provide an example to illustrate what X just said? Who can offer another example? Who agrees with what X just said?*) without, however, stimulating their argumentative and critical skills (see examples in d), then pupils learned to think autonomously but did not learn to argue; they sometimes opposed their peer's ideas, but more in a perspective of confrontation than negotiation; they were hardly ever able to completely justify their points of view; instead, they used personal examples to demonstrate their points of view.

d) When the teacher encouraged pupils to reflect on mathematics (e.g.: *Can you define what a cube is? Does what you just said imply that …?*), when she fostered pupil interaction (e.g.: *Regarding what X just said, do you think that…?, Who can answer Y's question? Look at Z when speaking to him*), when she asked for justifications (e.g.: *Why do you say that geometry is an interesting subject? What criteria do you use to say that humans are more intelligent than animals?*), when she stimulated criticism (e.g.: *Who disagrees with X's idea? Who can refine what Y just said? Who has a counter-example? Who can reformulate and clarify what was just said? Among the criteria we have just named, which seems most reliable or appropriate?*), etc., then it was observed that pupils learned to respect divergent points of view, to justify their opinions, to evaluate the statements of their peers and their own in a constructive manner, to realize that criticism, when formulated positively, contributes to enhancing perspectives; in short they learned to philosophize about mathematics.

In sum, as shown from certain analyses, the implementation of philosophical dialogue among youngsters is not spontaneous, and the mobilization of critical thinking is not innate. Also, it is fundamental that, in a philosophical workshop, the role of the teacher be anchored in rigorous and critical questioning.

One of the challenges facing teachers who use P4CM with elementary school pupils consists in becoming aware of the differences between, on one hand, conversation (monological exchange) and dialogue and, on the other hand, between non-critical dialogue and critical dialogue. Without awareness of these differences, youngsters' exchanges may stagnate in simple conversation and, consequently, not help them to think critically about the mathematical problems they must solve, and not help them develop reliable and valid judgments regarding the meaning of concepts related to their learning in this subject (see Daniel, accepted).

Conclusion

It is both appropriate and necessary to help pupils to philosophize in all school subjects, particularly in mathematics, because many pupils have trouble succeeding in the subject.

Philosophizing implicitly refers to learning to dialogue and, more specifically, to learning to engage in *critical* dialogue among peers. This is because, essentially, criticism (presented in the form of questions, doubts, counter-examples, etc.) is likely to create a cognitive imbalance that is sufficient to trigger in pupils a reflective process that can lead to comprehension of complex concepts – for example, in mathematics, concepts such as the perfection of geometric forms, infinite vs. indefinite, the role zero plays, etc. Only critical comprehension is likely to lead to the transformation of perspectives, of negative biases or of unfounded beliefs regarding mathematics.

In the critical dialogue learning process, it is the teacher's responsibility to question pupils. Model questions are suggested at the end of the paper.

Endnotes

1. This paper was originally published in French in 2011. Daniel, M.-F. Philosopher sur les mathématiques par le biais du dialogue critique. In M. Gagnon et M. Sasseville (dir.) *La communauté de recherche philosophique. Applications et enjeux* (pp. 41-57). Québec : Les Presses de l'Université Laval.

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