Characteristics of a High School Classroom Community of Mathematical Inquiry

Raymond Siegrist

When a community of inquiry is adopted for teaching mathematics, characteristics need to be identified to determine if a community of inquiry forms. One essential characteristic of a community of mathematical inquiry is that members develop their understanding of mathematics through dialogue. Scholars in fields other than mathematics have emphasized the importance of incorporating dialogue in instruction:

- The learning of content through the dialogical process enhances the understanding, retention, and application of the content (Fernandez-Balboa & Marshall, 1994, p. 174).
- Dialogue ... [is] the best available means we have for identifying among ourselves acceptable answers, workable solutions, and reasonable accommodations (Burbules, 1993, p. 16).
- [Meaningful learning] is much more likely to occur when dialogue is present, for the simple reason that direct communication among equals is the only reliable and systematic dynamic available to those who would incorporate the viewpoints of others into their own personal perspectives (Splitter & Sharp, 1995, p. 83).

Dialogue is the medium by which students in a community of mathematical inquiry communally make their own meaning of mathematics. Burbules (1993) views dialogue as “a continuous, developmental communicative interchange through which we stand to gain a fuller appreciation of the world, ourselves, and one another” (p. 8). A definition of dialogue is elusive, but several studies have described its characteristics. Dialogue occurs when there is an open, unforced sharing by two or more participants. Even in the light of disagreements a participant remains interested in, concerned for, and respectful of all other participants by following agreed upon maxims. All participation is guided by discovery on a topic of common interest with value given to all contributions of the participants. A continuous dialogue enables participants to construct knowledge and then reconstruct knowledge as diverse voices synthesize expressed ideas. Transformation of knowledge occurs in a dialogue as participants propose, defend, abandon, and accept ideas causing new situations and possibilities to be discovered. Inherent in dialogue is the need for each participant to take responsibility for his/her ability to influence the outcome (Burbules, 1993; Fernandez-Balboa & Marshall, 1994).

Another characteristic is that members of a community of mathematical inquiry work together in groups to solve more difficult mathematical problems than they could as individuals. Vygotsky’s (1986) “zone of proximal development” supports the use of groups in communities of inquiry because students obtain assistance. If Vygotsky is right, groups permit students to solve general problems that would be out of the reach of individuals. For students to be in their “zone of proximal development” during small groups, they have to be focused on the task at hand, and they have to express, listen to, defend, and evaluate the options created for solving any type of problem (Schoenfeld, 1987).

One strategy for using groups to solve mathematical problems reported by empirical studies is to have groups develop their processes and then present what they have created to the class for critical review (Clarke, 1997; Cobb, Wood, Yackel, Nicholls, Wheatly, Trigatti, & Perlwitz, 1991). In fact, instead of training for mastery of mathematical procedures presented by their teacher, in this process students become apprentices. Students acting as apprentices take on more of the problem-solving role as they gain experience. As they gain experience under their teacher's watchful eye, students are less likely to develop misconceptions about mathematics. Students also benefit from another aspect of apprenticeship, initiation into a mathematical culture (Schoenfeld, 1987). "Having experienced mathematics in this way, students are more likely to develop a more accurate view of what mathematics is and how it is done" (Schoenfeld, 1987, p. 205). One justification for adopting a community of inquiry approach put forth by Splitter and Sharp (1995) is that it encourages students to think like mathematicians or for students to learn to create mathematics the way mathematicians do.

The third characteristic of a community of mathematical inquiry is self-correction; as students check their solutions they often correct mistakes. In mathematics, students typically are not asked to listen to explanations by other students to determine which ideas make the best sense; they are given what the teacher thinks is the best way to solve a mathematical problem. When students begin to negotiate their own mathematical meaning within the community, they must decide whether a given explanation is good enough to change their minds. When a conflict between ideas is resolved through communal dialogue, a self-correction occurs within the community.

On the constructivist model, authentic learning is self-correction – the deliberate, intelligent reconstruction of one's habits of thoughts, feeling and action in order to meliorate some aspect of one's experience. Self-correction further requires self-verification: that a student verify new knowledge to herself (Gregory, 2002, p. 400).

Students begin to internalize standards of mathematical judgment modeled by the teacher, so they have criteria to gauge correctness of arguments and solutions (Schoenfeld, 1987, 1996).

The fourth crucial characteristic of a community of mathematical inquiry is that community members take risks. In this process, criticism and attacks can be made on ideas, but are not acceptable on the personal level. On this point students need practice in not attaching their personal worth to the acceptance of "their" idea. Robertson (1999) points out that all members of the community must entertain the "possibility of being obligated to lose" (p. 5). Members of the community need to abide by the idea that the explanation making the best sense is the one accepted. As Schoenfeld (1996) states, "There is a feeling of trust, in that we must feel free to have our ideas (and not ourselves) compete." (p.16). Students free from personal assessment by the community are more likely to take risks when proposing assertions. Polya (1954) asserts three "moral qualities" a person needs to do mathematics that also pertain to taking risks:

> Intellectual courage: We should be ready to revise any one of our beliefs.
> Intellectual honesty: We should change a belief when there is a good reason to change it ...
> Wise restraint: We should not change a belief wantonly, without some good reason, without serious examination (pp. 7-8).

The fifth characteristic of a community of mathematical inquiry is that students consider, propose, and build on alternate approaches to problem solving. A common, general problem-solving strategy is to list several methods that might result in a solution. When a process is successful, students should not decide inquiry is finished. Other ideas might lead to better insights, additional knowledge, or other areas of study (Lampert, 1990). Self-correction eliminates some alternate approaches to problem solving as unfeasible, but open inquiry requires investigating all ideas within the constraints of the initial problem. Cobb, Wood, and Yackel (1992) offer some guidance: "the teacher might frame conflicting interpretations or solutions as a topic for discussion, thus encouraging students to explicitly negotiate mathematical meanings by engaging in mathematical argumentation" (p. 11).

Members of a community of mathematical inquiry should be able to provide justification for each step taken in the solution process. Reasons need to be given by a student to the community for accepting or rejecting a claim. Mason presents an outline for justifying mathematical arguments: First convince yourself. Then convince a friend. Finally convince an enemy (Mason, Burton, & Stacey, 1984). "If the teacher's guide is the source of right answers, for example, this suggests that the basis for epistemic authority in mathematics does not rest within the knower" (Ball, 1991, p. 7). But if the student has defended a conjecture or answer with reason, not only does the "epistemic authority" reside within the knower, the student has created her/his own meaning.

Students too often see the polished product of a teacher's struggle with a topic or hours of investigation that did not lead directly to a viable process. "Naïve conjecture and counterexamples do not appear in the fully fledged deductive structure: The zig-zag of discovery cannot be discerned in the end product" (Lakatos, 1976, p.42). Students need to engage in the polishing process, but for this to happen teachers must get students to change their focus. Instead of a race to the correct answers, students need to focus on the investigation process, the connections among concepts, and the structure of mathematics (Lakatos, 1976; Peressini & Knuth, 2000; Tymoczko, 1985). In a community of inquiry, "Knowledge is never presented as complete and sacred; rather, it is always open to further question and criticism" (Harpaz & Lefstein, 2000). Schoenfeld argues that, "entry into that culture...may be necessary to understand and appreciate mathematics" (1987, p. 214).

A sixth characteristic of a community of mathematical inquiry is that students inquire into the procedures of inquiry. In mathematics, when students think about what they are doing during the problem-solving process, they inquire into the procedures of inquiry (problem solving). When a teacher adopts a cooperative problem-solving approach, an underlying objective is for students to abandon rehearsals of rules and to plan how to solve complex problems. This objective requires students to adopt and practice a way to manage their problem solving.

Aspects of management include (a) making sure that you understand what a problem is all about before you hastily attempt a solution; (b) planning; (c) monitoring, or keeping track of how well things are going during a solution; and (d) allocating resources, or deciding what to do, and for how long, as you work on a problem (Schoenfeld, 1987, p. 109-110).

Schoenfeld (1987) contends that if teachers fail to provide students a general way to "self-regulate" their problem solving attempts, students become frustrated when they cannot start a problem, they continue a strategy that never converges to a solution, or they run into a dead end with their selected strategy. Therefore, in addition to learning a collection of processes and concepts, students must learn and practice a management system in order to become successful problem solvers. In other words, managing or thinking about problem solving differs from solving problems, but it is a necessity for problem solvers. Schoenfeld (1987) suggests students reflect on the problem-solving process by asking, "How well do you keep track of what you're doing when (for example) you're solving problems, and how well (if at all) do you use the input from those observations to guide your problem solving actions?" (p.190).

The seventh characteristic of a community of mathematical inquiry is that it supports students doing mathematics like mathematicians. One essential ingredient in developing a community of inquiry environment in high school mathematics is the creation of a culture of mathematicians (Clarke, 1997; Cobb, Wood, & Yackel, 1991; Lampert, 1990; Shoenfeld, 1987, 1996). In a community of mathematical inquiry, axioms and definitions remain open to reexamination by the community. When members of the community engaged in constructing a proof discover a deficiency, the need for revision is obvious. Revisions of assertions by community members in the light of recently discovered inadequacies allow mathematics to advance (Lampert, 1990). Helped by a mathematician's educated guessing about relationships, mathematics develops in a back-and-forth process between adjusting assumptions and revising conclusions. A conjecture is proposed to the mathematical community and stands until a counterexample is discovered or the conjecture is proven. Students need practice in taking the risk to make conjectures; they must also understand that, because of limited insight, their conclusions might be incorrect (Lampert, 1990).

Schoenfeld (1987) points out three important types of metacognition that mathematicians need to develop: a) A problem solver's knowledge about his/her own thought process, b) a problem solver's self-regulation, and c) a problem solver's beliefs and intuitions (p. 190). Participation in a community of mathematical inquiry allows each of these qualities to be developed. Students become better general problem solvers when these qualities are introduced and their application is practiced. Many attempts at solving mathematical problems depend on the prior mathematical knowledge of the students. When students have a good sense of what they know, their efficiency as general problem solvers increases. In addition, students learn to explain how they solved general problems and constantly reflect upon their thought processes in a community of inquiry (Carpenter & Fennema, 1992). Simply put, a student who self-regulates during a general problem solution is a better problem solver than one who does not keep track of what is happening. Student beliefs, such as the belief that students must memorize rules to be successful in mathematics, should begin to change as their participation in the community matures (Schoenfeld, 1987, 1996).

Studies Showing Outcomes of Classroom Communities of Inquiry

There is very little empirical literature on mathematical communities of inquiry. The most relevant literature incorporates community of inquiry characteristics as part of socio-constructivist approaches to teaching and learning mathematics. This literature is appropriate as a frame for a study on mathematical communities of inquiry because socio-constructivist approaches to teaching share many theoretical underpinnings as well as pedagogical approaches with community of inquiry: 1) "Students have frequent opportunities to discuss, critique, explain, and, when necessary, justify their interpretations and solutions" (Cobb, Wood, Yackel, & Perlwitz, 1992, p.485). 2) "Mathematics learning is a process in which students reorganize their activity to resolve situations that they find problematic" (Cobb, Wood, Yackel, Nichols, et al., 1991, p. 4). 3) Learning occurs when students attempt to make meaning by actively negotiating with their peers (Cobb et al., 1991; Fawcett, 1938; Lampert, 1990; Schoenfeld, 1987, 1996). 4) "Social norms are not static prescriptions or rules to be followed but are instead continually reconstructed in the course of classroom interactions" (Cobb, Wood, Yackel, Nichols, et al., 1991, p. 7). 5) Students create mathematics as mathematicians (Cobb et al., 1991; Fawcett, 1938; Lampert, 1990; Schoenfeld, 1987, 1996).

In a 1991 study, Cobb, et al. designed instruments to assess mathematics achievement, computational proficiency, beliefs about success in mathematics, and motivation to study mathematics. Ten second-grade classes (project students) were taught mathematics using an inquiry approach while eight classes (non-project students) were taught in a traditional manner. The instruments were administered at the end of a year. This study indicated that the project students retained more content, constructed more advanced concepts, cooperated with their peers to a greater extent, and did better on challenging tasks than the non-project students. The amount of time to teach the concepts of the curriculum using the inquiry approach was not a concern mentioned in this study. Students in the project classes believed success in mathematics comes from attempting to understand mathematics while communicating with their peers about their thinking.

In a follow-up study Cobb, Wood, Yackel, and Perlwitz (1992) tested former second grade students from five inquiry classes and six traditional classes. All students were in traditional third grade classes. Scores on standard achievement tests together with results from instruments designed to assess computational development, personal goals in mathematics, and beliefs about mathematical success were used to compare students at the end of the year. The inquiry students maintained their edge in conceptual understanding and mathematical problem solving. Beliefs of the inquiry students, that success in mathematics comes from attempting to understand concepts and from talking with their classmates about their thinking, persisted even after a year of traditional instruction.

An early pretest-posttest, quasi-experimental study conducted by Fawcett (1938) showed the viability of the inquiry approach. Several geometry classrooms totaling seventy-five students were taught by the traditional method in two schools; these students became the control group. In the experimental group, a class of 25 students determined the pace and sequence of their learning, constructed their own definitions, and questioned assumptions. Pretest scores were the same for all groups, but the experimental group scored significantly better on the posttest despite the fact that the experimental group only covered half of the curriculum. Two of Fawcett's (1938) conclusions were that "following these procedures improves reflective thinking" and that "the usual formal course in demonstrative geometry does not improve the reflective thinking of pupils" (p. 119).

Lampert (1990) conducted a classic qualitative, first-person study indicating how a middle school classroom could be set-up in a community of inquiry. She found many examples of how students tried to use irrational strategies to have their assertions accepted by other students. Lampert used the work of Polya (1954) and Lakatos (1976) to identify how to establish a culture of mathematicians in the classroom and bring students to the realization of what it means to do mathematics. Her students studied exponents like mathematicians by making conjectures and dialoguing about their thoughts. In her role as the teacher, Lampert would at different times tell students whether their behavior was appropriate or inappropriate, model appropriate ways of behaving, or do mathematics with her students.

The list of seven characteristics of a community of mathematical inquiry based on current literature establishes a starting point for determining if a community of inquiry exists. Since additional characteristics could certainly be extracted from the literature, there is no claim that this list is exhaustive. Each of the characteristics can be observed and measured by the frequency of its occurrence on the individual or class level. Literature related to high school community of mathematical inquiry was limited to Fawcett's 1938 classic quantitative study about geometry students. Lampert's 1990 classic, first-person, qualitative study generalized student attempts to thwart dialogue in a middle school community of inquiry that would be applicable to the high school level. The Cobb et al. (1991, 1992) studies demonstrated the effectiveness of a community of inquiry approach for mathematics at the second grade level.

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Address correspondences to:

Raymond Siegrist
Suny College at Oneonta
Box 146 Fitzelle
Oneonta, NY 13820
siegrir@oneonta.edu