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EFFECT OF DECISION PARAMETER EFFICIENCY ON TARGET RELIABILITY

Andrew Way*, Nico de Koker, Celeste Viljoen

Stellenbosch University, Faculty of Civil Engineering, Hammanshand Road, Stellenbosch, 7600, South Africa
* corresponding author: Acway@sun.ac.za

Abstract. Target reliability forms the basis of most modern design standards and is intended to represent an optimal balance of safety and risk of failure. Previous research noted discrepancies between target reliability obtained using generic and case-specific cost optimization for SLS cases. The source of the discrepancies was identified as the efficiency of the decision parameter at increasing reliability. This research extends the investigation to cases of ULS failures and found similar discrepancies. Generic cost optimization assumes a linear dependence between the decision parameter and the structural resistance. Where the dependence of resistance on the decision parameter is superlinear, the generic was found to under-predict target reliability by up to 15%. A factor is proposed to adjust generically-obtained ULS target reliability to be more appropriate to specific ULS cases. The factor accounts for the efficiency of the decision parameter, the case-specific cost of safety and parameter variation. The adjustment factor represents a first step towards mapping generic to case-specific target reliability in the ideal of promoting safer and more cost-effective structures.

Keywords: Cost optimization, structural reliability, target reliability.

1. Introduction

Target reliability in structures is determined through the optimization of costs, considering the consequences of failure [1]. Where human safety is at risk in the case of failure, lower limits on reliability are imposed based on societal risk criterion [2]. The costs of increasing safety and the consequences of failure play an important role in the determination of target (optimum) reliability and have been considered extensively in various research contributions [2–6]. The effect that the decision parameter efficiency has on the reliability limit state, however, has not been given as much consideration.

In the cost optimization process, a function describing the structural cost is developed, in conjunction with the reliability limit state that governs the design of a structure. The parameter with the greatest influence on increasing the reliability in the limit state is identified as the decision parameter. The optimal reliability is found at the decision parameter value that minimizes the structural cost function. Previous research [7–9] has shown that in RC structures governed by SLS design, the efficiency of the decision parameter at increasing safety in the limit state equation also plays a role in the determination of the optimum reliability.

Generic target reliability through cost optimization is typically based on research by Rackwitz [4]. In this generic assessment, the structural resistance is linearly dependent on the decision parameter. However, when the decision parameter in the governing limit state takes on a form that is not linear, the target reliability will differ from that obtained through the generic cost optimization. This research aims to investigate various ULS limit states to determine the extent to which the decision parameter efficiency affects target reliability and how deviations from the generic target reliability may be predicted and accounted for.

2. Target reliability from cost optimization

2.1. Generic cost optimization

A summary of the generic cost-optimization framework to determine target reliability is presented here, from [4]. The total cost of the structure normalized by the initial structural cost, $z(d)$, is shown in Equation 1 as a function of a decision parameter, d.

$$z(d) = \frac{C_p}{C_0} + \frac{U}{C_0} \cdot \frac{p_{fSLS}}{\gamma} + \left(\frac{C_p}{C_0} + \frac{A}{C_0}\right) \cdot \frac{\omega}{\gamma} + \left(\frac{C_p}{C_0} + \frac{H}{C_0}\right) \cdot \frac{p_{fULS}}{\gamma} \tag{1}$$

with

C_p	$C_0 + C_1 \cdot d$
C_0	Initial cost of structure independent of d
C_1	Initial costs of structure dependent on d
γ	Age-averaged discount rate
ω	Obsolescence rate
U, A, H	Costs related to SLS failure, obsolescence and ULS failure
p_f	Probability of failure

Costs attributed to inspection and maintenance are assumed to be included in C_p; costs related to fatigue and ageing failure are not considered. The reliability limit states and probabilities of failure for ULS and SLS that are linked to the cost function in Eq. 1 are given as:

$$g_{ULS} = R_{ULS}(d) - S \tag{2}$$
$$p_{f,ULS} = P(g_{ULS} < 0) \tag{3}$$
$$g_{SLS} = R_{SLS}(d) - S \tag{4}$$
$$p_{f,SLS} = P(g_{SLS} < 0) \tag{5}$$

The decision parameter that which most efficiently increases reliability at a specific limit state. In a generic case, it typically affects only the structural resistance, which is modelled lognormally, with mean and coefficient of variation (V_i) as $R_{ULS} \sim LN(d, 0.3)$ and $R_{SLS} \sim LN(d/1.5, 0.3)$ for ULS and SLS, respectively [4]. In effect, this definition reflects the fact that the loading at ULS is typically ≈ 1.5 times that at SLS and that the decision parameter required at SLS is assumed to be affected in a similar fashion. The load is therefore modelled as $S \sim LN(1, 0.3)$ for both ULS and SLS.

Target reliability from cost optimization is defined by the value of decision parameter which minimizes the total cost of the structure. This corresponds to the point $d : \partial z(d)/\partial d = 0$. From Eq. 6, the optimal value of d is dependent on various cost ratios, as well as γ and ω, all of which have been investigated in previous research [3, 10–13].

$$\frac{\partial z(d)}{\partial d} = 0 = \left(1 + \frac{H}{C_0} + \frac{C_1 d}{C_0}\right)\frac{\partial p_{f,ULS}}{\partial d} + \frac{C_1}{C_0}\left(\frac{\omega+1}{\gamma} + 1\right)p_{f,ULS} + \frac{U}{C_0\gamma}\frac{\partial p_{f,SLS}}{\partial d} \tag{6}$$

Little consideration has been given, however, to the derivative of the probability of failure with respect to the decision parameter at ULS and SLS ($\partial p_{f,ULS}/\partial d$ and $\partial p_{f,SLS}/\partial d$, respectively). This "efficiency parameter" describes how the probability of failure is affected by changes in d and is a measure of how efficient d is at increasing the reliability of the limit state.

2.2. Efficiency parameter

From Eq. 2 to 5, the efficiency of the decision parameter at increasing safety is dependent on the limit state equations for ULS and SLS. Previous research by Van Nierop et al. [7–9] investigated target reliability in reinforced concrete (RC) structures where the design is governed by SLS considerations. This work indicated that discrepancies exist between the target reliability derived using the generic cost optimization and that derived from a case-specific cost optimization. This discrepancy was attributed to the difference in efficiency of the decision parameter in the generic SLS formulation compared to that of the specific case, i.e.

$$\overbrace{\frac{\partial p_{f,SLS}}{\partial d}}^{Generic} \neq \overbrace{\frac{\partial p_{f,SLS}}{\partial d}}^{Specific}$$

and proposed a parameter, ν, that could be used to map the specific case to the generic, of the general form:

$$\overbrace{\frac{\partial p_{f,SLS}}{\partial d}}^{Generic} = \nu \cdot \overbrace{\frac{\partial p_{f,SLS}}{\partial d}}^{Specific}.$$

In the context of target reliability, however, it is more convenient to consider it in terms of the reliability index, as the conversion from probability of failure to reliability index, β, is case-independent:

$$\overbrace{\frac{\partial \beta_{gen}}{\partial d}}^{Generic} = \nu \cdot \overbrace{\frac{\partial \beta_{spec}}{\partial d}}^{Specific}. \tag{7}$$

Van Nierop et al.[8] investigated this discrepancy in target reliability for RC structures governed by SLS design. In the current research, the efficiency parameter is investigated in a broader structural context to determine the extent to which it affects target reliability in general.

3. Effect of decision parameter on target reliability

3.1. Form of decision parameter in limit state equations

To investigate the effect of the decision parameter efficiency on target reliability, the governing limit state equation, g (Eq. 2 - 5), is considered in more detail. In the generic cost optimization, a linear dependence is assumed between the decision parameter and the resistance in the generic limit state equation. That is, linear increases in the decision parameter linearly increase the mean of the resistance, and thereby increase the reliability. A typical example of this is the bending resistance of a simply supported RC beam, given by the simplified limit state in Eq. 8, where A_s, f_y, x, w and L are the area of tension reinforcement, reinforcement yield stress, moment lever arm, distributed load and length, respectively. The decision parameter is A_s, which clearly has a linear effect on the resistance and therefore the limit state. In cases like this, the assumption of linearity, and by implication the target reliability derived from it, will also be appropriate.

$$g_1(A_s) = f_y \cdot A_s \cdot x - \frac{w \cdot L^2}{8} \tag{8}$$

In cases where the decision parameter form in the limit state equation is not linear, whether sub-linear or super-linear, the generic assumption of linearity is no longer appropriate. Consider the tension resistance of a structural steel rod with radius r, subjected to a tensile load, T_u, as shown in the limit state in Eq. 9.

Comparing Eq. 8 to Eq. 9, it is clear that increasing r is more efficient at increasing the resistance than increasing A_s, and thus that $\partial\beta_{g_1}/\partial A_s < \partial\beta_{g_2}/\partial r$. In the latter case, a higher target reliability will result.

$$g_2(r) = \pi r^2 f_y - T_u \tag{9}$$

This suggests that it is the form of the dependence of the governing limit state equation on the decision parameter that governs the efficiency of the decision parameter and thereby the target reliability. Decision parameter efficiency will apply to any structure and not just RC structures governed by SLS requirements. Cases of ULS limit states are investigated below.

3.2. ULS Test cases

The target reliability of a number of specific ULS limit states are determined and compared with those obtained from generic cost optimization. Varying degrees of decision parameter efficiency are implied by the chosen limit state equations, to determine the effect that the decision parameter form has on the target reliability.

A more generalized structural cost equation is used to determine the target reliability as shown in Eq. 10, where C_F are all costs associated with failure of the limit states under consideration. The normalized failure cost is varied, with particular points of interest at $C_F/C_0 = 1, 3.5$ and 7.5, which are representative of small, medium and large consequences of failure (CoF), respectively [11]. Similarly, the normalized costs of increasing safety (CoS) are varied, using $C_1/C_0 = 5\times10^{-2},\ 5\times10^{-3},$ and 5×10^{-4} for high, moderate and low costs of increasing safety, respectively [3, 14]. Furthermore, d_n in Eq. 10 represents a normalized decision parameter, as discussed below.

$$\begin{aligned} z(d) =& 1 + \frac{C_1 d_n}{C_0} + \left(1 + \frac{C_1 d_n}{C_0} + \frac{A}{C_0}\right)\frac{\omega}{\gamma} \\ &+ \left(\frac{C_F}{C_0}\right)\frac{p_f}{\gamma} \end{aligned} \tag{10}$$

For the intents of evaluating the effect of decision parameter efficiency on target reliability, the rest of the cost optimization parameters are taken directly from [4], with the exception of the case of lower variation, and are shown in Table 1. The various ULS failure mechanisms in Table 1 are chosen to illustrate the effect of decision parameter efficiency on target reliability, in comparison to that obtained from the generic cost optimization. The following assumptions/simplifications are made, and are discussed below:

- All action effects (S, M, T, N) are considered with both high and low V_S and are modelled as $LN(\mu_S = 1, V_S = 0.3)$ and $LN(1, 0.15)$, in turn;
- All resistance effects are modelled with a mean value stemming from the resistance term from Table 1 and $V_R = 0.3$ and 0.1, in turn. E.g. for Case 2: $R \sim LN(f_y\ A_s\ z,\ 0.3)$ and $LN(f_y\ A_s\ z,\ 0.15)$, in turn;
- All decision parameters are normalized with respect to the value of decision parameter, d_0, that gives $\beta = 0$, i.e. $d_n = d/d_0$;
- Model factors are included as part of V_i.

The above-mentioned assumptions are made so that meaningful comparisons can be drawn between decision parameter efficiencies, in terms of target reliability. In reality, the statistical distribution and variation of each resistance and load parameter in the various limit states will vary; they are kept consistent here, for the sake of comparison with the generic case. Similarly, the model factors assumed to be included in V_i will vary for each case. The normalization of the decision parameter is performed so that the normalized costs of increasing safety (C_1/C_0) can be used for all the cases in Table 1. This avoids the subjectivity of considering costs specifically for each case of decision parameter, and is similar in principle to that performed for target reliability in existing structures from [15]. All reliability analyses are performed using the first order reliability method.

4. Results and discussion

The results of the optimization for higher variation are shown in 1 and 2. Red vertical lines indicate small, medium and large CoF. The magnitudes of the target reliability in Figures 1 and 2 is lower than typical due to the high variation, which is intended to cater to a wide array of applications. From Figures 1 and 2, the case of concrete column buckling (decision parameter of a 4^{th} order) has the highest target reliability.

ULS Case	**Limit state equation**	**Decision parameter**	Form (order)
1 - Generic	$R(d) - S$	Generic decision parameter - d	Linear (1^{st})
2 - RC beam bending	$f_y \cdot A_s \cdot z - M$	Area of tension reinforcing - A_s	Linear (1^{st})
3 - Steel tension rod	$\pi r^2 f_y - T$	Steel rod radius - r	Quadratic (2^{nd})
4 - RC column buckling	$E_c\pi^3 D^4/64L^2 - N$	Concrete column diameter - D	Quartic (4^{th})
Cost optimization parameters			
$\gamma = 0.035$ $\omega = 0.02$	$A/C_0 = 0.2$	C_F/C_0: $0.1 - 7.5$	C_1/C_0: 5×10^{-2}; 5×10^{-3}; 5×10^{-4}

Table 1. ULS Test case limit state equations and cost optimization parameters.

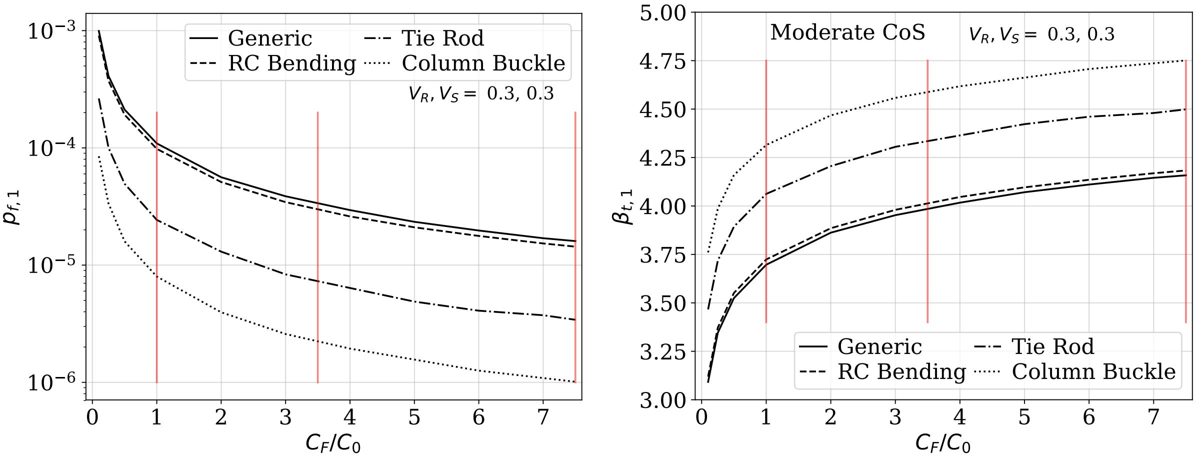

Figure 1. Annual probability of failure (left) and annual target reliability (right) for the considered ULS limit states for moderate CoS ($C_1/C_0 = 5 \times 10^{-3}$) with high variation.

The difference in target reliability with respect to the generic case is $\Delta_{\beta_{t,1}} \approx 0.6$ for all three cases of CoS. The generic case and that of bending in an RC beam (both linear decision parameters) have the lowest target reliability and are practically identical. The case of a steel tension rod (quadratic decision parameter) falls in between the generic and concrete buckling cases, with $\Delta_{\beta_{t,1}} \approx 0.3$. These results confirm that target reliability increases with increasing decision parameter efficiency.

The process is repeated for lower values of $V_R = 0.1$ and $V_S = 0.15$ to determine whether or not similar trends apply to limit states with variation more representative of typical applications [2, 16], where parameters and/or model factors exhibit lower variation. For the sake of brevity, only the results for moderate CoS are shown. From Figure 3, similar trends can be observed for lower variation, with higher values of target reliability. Similar trends were observed for cases of low and high CoS (not shown here). Differences in magnitude of target reliability between the generic and column buckling cases are slightly lower for lower variation, at $\Delta_{\beta_{t,1}} \approx 0.5$.

In the consideration of target reliability, higher values are justified when the CoF are large, when parameter and model uncertainty is low or when the costs of increasing safety are low. In this case, the latter is true for ULS cases 3 and 4. As a result of the heightened decision parameter efficiency, the cost of increasing safety is effectively lowered. The generic cost optimization under-predicts the target reliability for these cases due to the assumed linear form of the relationship between the decision parameter and the resistance term in the limit state equation. Figure 4 illustrates this visually by comparing the target reliability obtained from the generic with that from the specific, for moderate CoS, high variation and a range of CoF of $0.1 \leq C_F/C_0 \leq 7.5$. Similar results were found for other CoS and values of variation. Thus, for any specific limit state with a decision parameter form other than linear, the target reliability obtained from the generic cost optimization will need to be adjusted using a factor that considers the decision parameter efficiency.

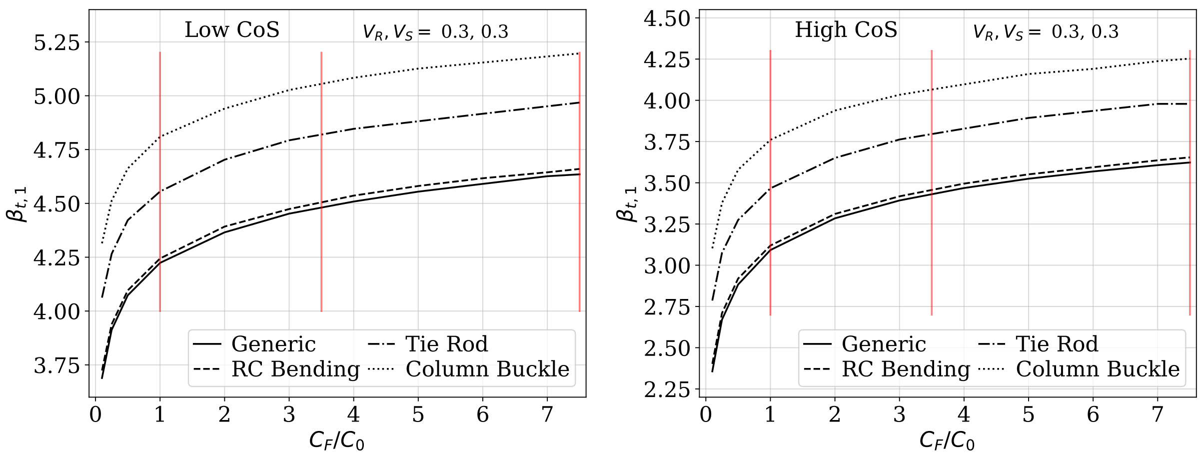

Figure 2. Annual target reliability for the considered ULS limit states for low and high CoS ($C_1/C_0 = 5 \times 10^{-4}$ and 5×10^{-2}, from left to right, respectively) with high variation.

V_i	Cost of safety: Low			Moderate			High		
	Form of decision parameter in governing limit state								
	Linear	Quadratic	Quartic	Linear	Quadratic	Quartic	Linear	Quadratic	Quartic
High	0.0	0.3	0.6	0.0	0.3	0.6	0.0	0.4	0.7
Low	0.0	0.2	0.4	0.0	0.2	0.4	0.0	0.4	0.7

TABLE 2. Target reliability adjustment factor (ν) values for various forms of decision parameter, with reference to Eq. 11 for all considered consequence of failure classes ($0.1 \leq C_F/C_0 \leq 7.5$).

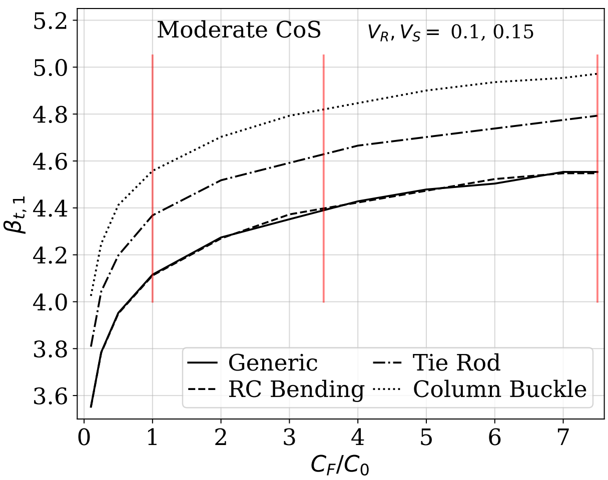

FIGURE 3. Annual target reliability for the considered ULS limit states for moderate CoS ($C_1/C_0 = 5 \times 10^{-3}$) with low variation.

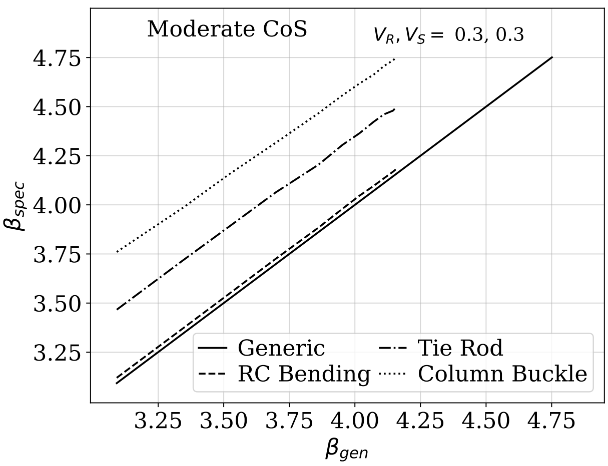

FIGURE 4. Comparison of annual target reliability for generic vs specific cases for moderate CoS, high variation and $0.1 \leq C_F/C_0 \leq 7.5$

From the results, an additive factor of the form in Eq. 11 captures the deviation from the generic target reliability more effectively than the multiplicative form proposed in [7].

$$\overbrace{\beta_{t,1,gen}}^{Generic} + \nu = \overbrace{\beta_{1,t,spec}}^{Specific} \tag{11}$$

Values for the target reliability adjustment factor, ν, are shown in Table 2 for first, second and fourth order decision parameter forms. These values are applicable for low, medium and high CoF and are given for high ($V_R, V_S = 0.3, 0.3$) and low ($V_R, V_S = 0.1, 0.15$) variation. It should be noted however, that these are indicative first estimates and that further research is required, especially for cases where the CoS of a specific decision parameter varies substantially from the considered ratios. Additionally, the determination of similar values for SLS cases should be considered in future research.

5. Conclusions

Target reliability determined by generic cost optimization forms the basis of design of most modern design codes. To maintain acceptable levels of reliability and simultaneously design cost effective structures, target reliability should be as reflective of the specific structure under consideration as possible. Previous research reported discrepancies between SLS target reliability obtained using generic cost optimization and that obtained using structure-specific cost optimization. The efficiency of the decision parameter at increasing safety was identified as the cause of the discrepancies. This research investigated cases of ULS failures and also identified discrepancies in target reliability of up to $\Delta_\beta = 0.7$ between generic and specific optimization. For forms of decision parameter in the limit state equation other than linear, the target reliability differs from that obtained through generic cost optimization and needs to be adjusted to be representative of the specific case. A factor is sproposed to appropriately adjust the target reliability, based on decision parameter efficiency, parameter variation and cost of safety. Future research aims to determine similar values for SLS, as well as a more comprehensive consideration of decision parameter forms in governing limit states.

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