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ESTABLISHING MODEL UNCERTAINTY OF SLS REINFORCED CONCRETE CRACK MODELS APPLIED TO LOAD-INDUCED CRACKING

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Abstract. Although some research has been performed, serviceability limit state (SLS) concrete crack models have yet to be calibrated fully in probabilistic terms in structural design standards. This is partly due to the fact that the SLS is generally not the critical limit state in structural design. However, in some specialist structures such as water retaining structures, the SLS such as cracking is the limiting design criterion, specifically the crack width required to control leakage and thus requires a proper probabilistic analysis. In probabilistic crack models, the reliability of the crack model is determined by the performance function whereby the design limiting crack width is greater than the estimated crack width calculated using the appropriate design crack model. As cracking in concrete is a random mechanism with a high degree of variability and crack models tend to be at least semi-empirical with inherent uncertainty, model uncertainty of the crack model is significant and is applied in the reliability model as a random variable. A database was established of both short and long term cracking experimental data for the tension load case to quantify model uncertainty. However, the data for long term cracking is limited which meant that the model uncertainty in this case was not definitively established. This paper discusses the determination of model uncertainty for tension cracking, which could be extrapolated to other models where data is limited.

Keywords: concrete crack models, model uncertainty, reliability.

1. Introduction

Serviceability limit state (SLS) concrete crack models are treated nominally in probabilistic terms in structural design standards. This is partly due to the fact that SLSs are generally not the critical limit states. However, in some specialist structures such as water retaining structures, SLS cracking is the limiting design criterion, specifically the crack width required to control leakage. In addition, crack models are mostly empirically or semi-empirically derived so have not been assessed to any extent in reliability terms. However, cracking in concrete is a random mechanism with a high degree of variability indicative of a significant level of model uncertainty which, in turn, influences the level of treatment of model uncertainty in probabilistic analyses. Retief (2015) [1] and Holický et al (2016) [2] outlined a framework to assess model uncertainty. The relative influence of model uncertainty can be divided into model uncertainty classes [1] classified by an increasing level of treatment, as having Nominal effect, Significant effect or Dominating effect, with, respectively. When model uncertainty is at least significant, it is deemed to be justified to treat model uncertainty as a random variable in the probabilistic model. This contribution discusses the quantification of model uncertainty for short- and long-term tension cracking, which could be extrapolated to other models where data is limited and where SLS governs. The selection of the theoretical model for the general probabilistic model (GPM) is also important as model uncertainty is specific to the chosen model. Load-induced tension cracking is controlled in structural concrete design standards by limiting the crack width for a given loading, section and reinforcement configuration. The maximum crack width is considered at the tension surface of the cross section. In probabilistic assessments of SLS crack models, the reliability of the crack model is determined by the performance function g, expressed as $g = w_{lim} - \theta \cdot w_{predict}$ where w_{lim} is the design limiting crack width, θ is the model uncertainty as a random variable and $w_{predict}$ is the estimated crack width calculated using the appropriate design crack prediction model.

As model uncertainty is specific to the prediction model concerned, suitable load-induced crack models were investigated, applicable to structures where serviceability cracking governs the design, such as reinforced concrete water retaining structures (WRS). The South African industry utilises the now withdrawn British standard BS 8007 (1987) [3] and is at present updating the standard SANS 10100-1 (2004) [4] for the design of reinforced concrete structures by adopting the corresponding Eurocode [5], concurrently with the development of SANS 10100-3 (Draft) (2015) [6] for the design of WRS. McLeod (2013) [7] in a deterministic analysis of typical WRS wall configurations, showed that the Eurocode crack model was conservative considering the reinforcement ratio compared to BS 8007' especially for the tension loading case (> 38 % increase), for a limiting crack width of 0,2 mm. This has significant economic consequences if EN 1992 adopted in South Africa. The release of the *fib* Model Code MC 2010 [8] included an update of the fundamental crack model of *fib* MC 90 on which the Eurocode crack model is based and allows for long term effects. The load-induced crack models for pure tension of BS 8007 (1987) [3], BS EN 1992-1 (2004) [5], *fib* MC 2010 [8] were thus considered and compared to the results obtained for flexural cracking, as reported in [9] and [10]. The results of this research are also of relevance to researchers developing Eurocode-related crack models and are applicable to structures other than WRS where serviceability cracking has a significant level of importance in the design thereof, such as bridges. The results of this research will also be utilised in the probabilistic assessment of SLS cracking in existing structures such as WRS and bridges.

2. Model Uncertainty Treatment

Uncertainty exists in all models and requires quantification as far as possible for proper reliability assessment and may be divided into two main types of uncertainty [1]. Inherent random variability (aleatory) exists in model parameters such as material properties and loads. As these parameters are treated as random variables in the reliability model, this inherent variability is measured by their coefficients of variation. Epistemic uncertainty (that due to incomplete knowledge and statistical uncertainty) may be measured by a model factor treated as a random variable in the reliability model. This model factor is defined as the ratio of measured to predicted values, in this case, crack widths. This ratio, or model factor (MF), is then quantified in terms of its statistical parameters of mean, coefficient of variation (CoV) and probability distribution function (PDF). Model uncertainty is a measure of the performance of the prediction model where the mean of model factors indicates bias in the model. Thus, a MF mean of 1 indicates an unbiased model. A MF mean greater than 1 represents underprediction; conversely, a mean less than 1 indicates an overprediction by the model. The variation of model factors is indicative of the level of uncertainty in the model - a higher variation signifying a higher uncertainty. For a model to be suitable over a reasonable range of design applications, it should behave consistently in terms of model factor bias and uncertainty over the range of application. Interdependence between model uncertainty and parameters results in drift in the MF mean and an inconsistent model.

With a large degree of randomness in concrete crack mechanisms and multiple influencing variables, crack models have high degree of variability (CoV well in excess of 0,1 which is the reference value for general structures). Model uncertainty (θ) is thus a significant variable in the reliability crack model, as shown by McLeod (2019) [9], requiring full probabilistic treatment. Some research (e.g. [11]) has been done in establishing model uncertainty as a model factor but mostly for flexural short term cracking. The mean and CoV of parameters were reported but not the PDF of model uncertainty necessary for reliability assessment.

3. Deterministic analysis of predicted crack width

In determining the predicted crack width for the MF, typical configurations of WRS and thus critical load cases for cracking were investigated. Load-induced pure tension cracking occurs in the wall of circular water retaining structures (WRS) due to hoop stresses in the horizontal plane induced by water load, which is considered as a quasi-permanent load by both Eurocode and SANS design standards. The main reinforcement is thus placed in the horizontal direction with any tension cracks running perpendicular to this reinforcement. As tension cracks in this case tend to be through the full cross section, this is a critical load case for water tightness.

A brief summary of the chosen design standard formulations used to calculate the predicted crack width for the critical pure tension load case in a WRS follows. A more comprehensive discussion may be found in [9]. Crack mechanism philosophies differ in the modelling of the development and transfer of the stresses between the steel and concrete around a crack in the section, and the resulting strain incompatibility. As cracking is a serviceability limit state, linear elastic stress-strain theory applies. The predicted maximum crack width corresponds to a 5 % probability of exceedance by most design code formulations [9],[10]. This may not probabilistically match the measured maximum crack width, and therefore contributes to model uncertainty.

3.1. BS 8007 crack model

The BS 8007 crack model [3] is a no bond-slip empirical model. The maximum surface crack width, w, for tension is calculated using:

$$w = 3\, a_{cr}\, \varepsilon_m \tag{1}$$

Crack spacing is assumed to be a function of a_{cr}, as the distance from the crack considered to the nearest longitudinal reinforcing bar. The mean strain at the surface, ε_m, is calculated from:

$$\varepsilon_m = \varepsilon_1 - \varepsilon_2 \tag{2}$$

where ε_1 is apparent steel strain (ε_s) at the surface and ε_2 is the concrete tension-stiffening effect. The equations to calculate tension stiffening strain for tension were derived empirically, calibrated to the specified maximum crack width limit (w_{lim}) as follows:

$$\varepsilon_2 = \frac{2\,b_t\,h}{3\,E_s A_s} \;\; for\;\; w_{lim} = 0,2\,mm \tag{3}$$

$$\varepsilon_2 = \frac{b_t\,h}{E_s\,A_s} \;\; for\;\; w_{lim} = 0,1\,mm \tag{4}$$

E_s is the steel modulus of elasticity, h is section depth, b_t is the width of the section in tension and As is the area of the tension reinforcement. Interpolation for other crack widths is not possible which limits the application of this crack model.

3.2. EN 1992 crack model

The EN 1992 crack width design equation [5] for pure tension cracking is a semi-analytical bond-slip model, developed from the compatibility relationship for cracking and assuming the stabilised cracking phase has been reached, namely:

$$w_m = S_{rm} \cdot \varepsilon_m \tag{5}$$

where w_m is the mean crack width, S_{rm} is average crack spacing and ε_m the mean strain. This crack width prediction model is based on the more fundamental model of *fib* MC 90. The mean strain is:

$$\varepsilon_m = \varepsilon_{sm} - \varepsilon_{cm} \tag{6}$$

where ε_{sm} is the mean strain in the reinforcement under loading calculated using linear elastic theory. The mean concrete strain, ε_{cm}, also known as tension stiffening, is calculated using:

$$\varepsilon_{cm} = k_t\,\frac{f_{ct,eff}}{\rho_{ct,eff}}\,(1 - \alpha_e\,\rho_{peff})\,/E_s \tag{7}$$

where α_e is the modular ratio E_s/E_c, $\rho_{p,eff}$ is the effective reinforcement ratio between the reinforcement area and the effective area of concrete in tension, $f_{ct,eff}$ is the mean tensile strength of the concrete at the time of cracking, E_c is the concrete modulus at the time of cracking, and k_t is a factor dependent on the duration of load. A minimum limit of $0,6\,\sigma_s/E_s$ (where σ_s is the steel stress) is placed on the mean strain.

The design (or "maximum") crack width is required rather than the mean width. This maximum crack width is related to the mean crack spacing by the equation:

$$w_k = (\beta_w\,S_{rm})\;\varepsilon_m \tag{8}$$

where $(\beta_w\,S_{rm})$ is the maximum crack spacing ($S_{r,max}$) and the factor β_w (the ratio of $S_{r,max}$ to Srm) has a value of 1,7 corresponding to $1,64\times$ standard deviations from the mean (normal distribution of crack widths). The EN 1992 maximum crack spacing is:

$$S_{r,max} = k_3\,c + k_1\,k_2\,k_4\,\varphi/\rho_{p,eff} \tag{9}$$

where φ is the bar diameter, c is the concrete cover to the longitudinal reinforcement and k_1 is a coefficient taking into account of the reinforcement bond properties, having a value of 0,8. The values of the factors k_3 and k_4 are determined by European Union individual member countries' National Annexes, with recommended values of 3,4 and 0,425 given for k_3 and k_4, respectively. The factor k_2 allows for stress distribution and has a value of 1 for pure tension load-induced cracking. All k-values are empirical factors, therefore would contribute to model uncertainty.

3.3. MC 2010 crack model

The *fib* MC 2010 crack model [8] is an update of the *fib* MC 1990 model. The design crack width, w_d, as a maximum (assumed to be the 95$^{\text{th}}$ percentile, as EN 1992) is determined from:

$$w_d = 2\,l_{s,max}(\varepsilon_{sm} - \varepsilon_{cm} - \eta\,\varepsilon_{sh}) \tag{10}$$

where $l_{s,max}$ is the transfer length over which slip occurs (equal to half the crack spacing) and ε_{sh} is the free shrinkage strain over time. The factor η is zero for short-term cracking, and 1,0 for long-term cracking. The minimum limit on mean strain of EN 1992 is also specified by MC 2010. The transfer length is determined using:

$$l_{s,max} = k \cdot c + 0.25\,\frac{f_{ctm}}{\tau_{cms}} \cdot \frac{\varphi_s}{\rho_{p,eff}} \tag{11}$$

where k is an empirical parameter to account for the influence of the concrete cover ($k = 1,0$ can be assumed), τ_{cms} is the mean bond strength between steel and concrete (considered to be evenly distributed between two cracks) and φ_s is the nominal diameter of reinforcing bars. The ratio between the concrete tensile strength and mean bond strength (f_{ctm}/τ_{cms}) is 1/1,8 for stabilised cracking for both short and long-term loads. MC 2010 allows for crack width determination over both the crack formation stage and the stabilised cracking phase. The MC 2010 crack model is valid for $c \leq 75$ mm.

4. Quantification of crack model uncertainty

The model factor for the chosen concrete crack models was defined as the ratio between measured crack width and predicted crack width, w_{exp}/w_{pred}. A database of experimental values for load-induced tension cracking was thus compiled to establish the stochastic parameters of model uncertainty for the crack models of EN 1992-1-1, MC 2010 and BS 8007. The database was that from McLeod (2019) [9] updated to include more recent testing by Gribniak et al (2020) [13].

The statistical parameters and probability distribution necessary for probabilistic analyses were determined for the model factors of each crack model using standard statistical test methods to a 95 % confidence level. The same procedure of analysis was followed

Researcher	Element Type	Test Duration	No. of samples**
Farra & Jaccoud [12]	Ties with single reinforcing bar, square cross section.	Short-term	71
Gribniak [13]	Ties with 4 no. reinforcing bars, cover varied. 10 samples total, repeat samples.	Short-term	4
Hartl (1977), UPM data [14], [15]	Ties with single or 2 No. reinforcing bars, square cross section.	Short-term	48
Hwang [16]	Slab elements reinforced in both directions, axial tension in one direction. Variation of cover and reinforcement	Short-term	34
Wu [17]	Ties with single reinforcing bar, square cross section.	Short (7) & long-term (4)	7 + 4
Eckfeldt [15]	Ties with 1or 2 No. reinforcing bars, square cross section. Repeats 2×4 No. ties & 3×1 ties	Long-term	11

*Final load steps considered only, EN 1992 minimum strain complied with.

Table 1. Sources of experimental data - direct tension load-induced cracking.

as detailed in [9] and [10]. Non-parametric normality tests such as Kolmogorov-Smirnov (using Lilliefors significance correction) and Shapiro-Wilks (corrected), as appropriate, were performed to a significance p of 0,05 to establish the estimated probability distribution of the model factor. Graphical methods such as probability plots and box plots were used to validate the estimated probability distribution. Through the statistical analysis of model factors, a first assessment of the crack models' performances was done. Pearson's correlations were used to investigate the relationship between significant parameters and model uncertainty, and thus assess the consistency of each crack model.

4.1. Experimental database

With data for existing WRS lacking and challenging to obtain, experimental data was used to establish model uncertainty of the crack models of EN 1992, BS8007 and MC2010. Both short- and long-term data was considered, given the quasi-permanent nature of the water load in WRS. However, the data for long term tension cracking [9] is limited. Sources for the database are summarised in Table 1.

Given that the first filling of a WRS only occurs once concrete has reached at least its 28-day strength and considering the quasi-permanent nature of the water load, only data from the stabilised cracking stage was considered. Where steel stress is small, crack widths may be underpredicted resulting in an overestimation of model uncertainty, unduly increasing the upper tail of the model uncertainty distribution. In addition, the applied loads and section geometry of water retaining structures typically result in a mean strain well above the minimum strain limit. Therefore, data was selected where the calculated mean strain was at least the specified minimum limit of $0,6\sigma_s$ of EN 1992 and MC 2010 where σ_s is the steel stress determined using linear elastic theory. Crack widths measured on the final load step only were considered to ensure independence of samples. Results for any repeat samples were averaged to prevent undue sample bias. Analyses were performed for EN 1992 and MC 2010 using the updated database, whilst the results for BS 8007 were taken from [9].

5. Discussion on Model Uncertainty Quantification

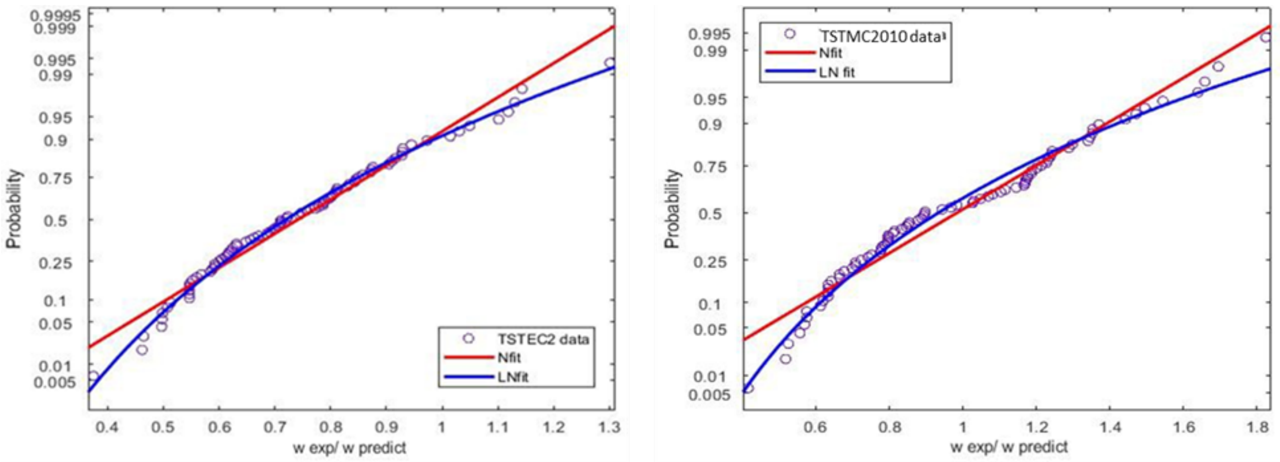
5.1. Short term Tension

The final sample size for short-term tension was 86 after repeats were averaged, which is sufficient to obtain a reasonable estimate of model uncertainty. The statistical parameters for model uncertainty are summarized in Table 2. The values obtained for the statistical parameters are similar to those reported on in [9]. There were some small differences on including the research of [13] which consisted of ties reinforced with multiple bars rather than the mostly single bars or uniform samples of the database of [9].

The statistical analysis showed that the EN 1992 crack model tends to be conservative, overestimating the predicted short-term tension crack widths, with a MF mean of 0,75. The MF mean of around 1 obtained for the MC 2010 crack model shows little bias in this model. The MC 2010 MF CoV of 0,32, however, is higher than that of the EN 1992 model (CoV of 0,25). The BS 8007 crack model underpredicts crack widths with MF means over 1,27. This bias increases significantly when tension stiffening is determined using the smaller limiting crack width, which is problematic when developing a GPM as the model is not consistent over crack widths away from the given limiting crack width (either 0,1 or 0,2 mm). There is also some uncertainty in the determination of a_{cr} in ties reinforced with a single bar as the formulation for a_{cr} results in underprediction of the crack spacing and width.

Statistical Parameter	EN 1992	MC 2010	BS 8007 w_{lim} 0.2 mm [9]	BS 8007 w_{lim} 0.1 mm [9]
Mean	0.747	0.996	1.271	1.430
Standard Error	0.020	0.034	0.032	0.041
Median	0.722	0.956	1.225	1.398
Standard Deviation	0.183	0.319	0.290	0.369
COV	0.245	0.321	0.228	0.258
Sample Variance	0.033	0.102	0.084	0.136
Kurtosis	0.051	-0.581	-0.056	1.234
Skewness	0.540	0.427	0.441	0.777
Range	0.927	1.408	1.516	2.139
Minimum	0.374	0.416	0.582	0.657
Maximum	1.301	1.824	2.097	2.796
PDF	LN	LN	N	N
Count	86	86	82	82

Table 2. Model uncertainty statistical parameters - short-term tension cracking.



EN1992 MC 2010

Figure 1. Probability Plots for EN 1992 and MC 2010 - short-term tension cracking.

Referring to Table 2, both the EN 1992 and MC 2010 crack models exhibit a positive skewness which suggests that the distribution is not normal. For the EN 1992 model uncertainty, the Shapiro-Wilks non-parametric test rejected the null hypothesis, whilst for the MC 2010 crack model, the Shapiro-Wilks significance factor was just greater than 0,05. Thus, it is estimated that both models have a non-normal distribution for model uncertainty. Curve-fitting was done for both crack models, considering normal and lognormal distributions. This indicated that both EN 1992 and MC 2010 tends towards a lognormal distribution, as shown in Figure 1. Considering that a lognormal distribution produces lower reliability estimates than a normal distribution (so more conservative), and that the distribution of the MC 2010 crack model has a positive skewness and is not clearly normal, a lognormal distribution is assumed for both models. BS 8007 model uncertainty appears to have a normal distribution.

5.2. Long term tension

Statistical analyses for long-term tension cracking are summarised in Table 3, and as reported in [9]. The sample size given is the final one after all repeats were averaged.

Comparing the long-term model uncertainty statistical parameters [9] to the short-term case, crack models that do not take long term shrinkage into account (BS 8007 and EN 1992) do not have a consistent model uncertainty as load duration increases. The EN 1992 model uncertainty mean increases from 0,75 to 0,90 for short to long-term tension loading. However, when considering the MC 2010 crack model, the MF means for short and long-term loading are very similar. The CoV for all models for long-term loading is lower than that of short-term tension, but still demonstrate the significant influence that model uncertainty has on the crack models. However, some restraint is required on interpreting the data given the small sample size. The sample size is too small to give a good indication of skewness, nevertheless,

Statistical Parameter	EN 1992	MC 2010	BS 8007 w 0,2 mm	BS 8007 w 0,1 mm
Mean	0.895	0.988	1.318	1.603
Standard Error	0.078	0.076	0.211	0.280
Median	0.860	0.946	1.089	1.353
Standard Deviation	0.220	0.214	0.597	0.793
COV	0.246	0.216	0.453	0.495
Sample Variance	0.048	0.046	0.356	0.629
Kurtosis	-1.539	-0.059	3.779	2.151
Skewness	0.442	0.806	1.882	1.447
Range	0.571	0.618	1.807	2.328
Minimum	0.656	0.764	0.836	0.935
Maximum	1.227	1.382	2.643	3.262
PDF (estimated)	LN	LN	N	N
Count	8	8	8	8

Table 3. Model uncertainty statistical parameters for long-term tension cracking [9].

a lognormal rather than normal distribution is suggested from the positive skewness values obtained for both the EN 1992 and MC 2010 crack models. As found for short-term tension, the BS 8007 model uncertainty statistical parameters are not consistent for the different tension stiffening models.

5.3. Evaluation of crack models - Pearson's correlations

The interdependence between selected parameters and model uncertainty for each model were evaluated, together with further assessment of the performance of the EN 1992, MC 2010 and BS 8007 crack models by means of scatter plots, regression analyses and Pearson's correlations. The selected parameters were reinforcing ratio (as % As), steel stress, concrete tensile strength (f_{ctm}), section depth (h), concrete cover (c) and section width (b). The Pearson's correlation factors between the model factor and selected parameters are summarised in Table 4 for each load case and crack model, respectively. The small sample size means that the long-term tension cracking correlations of [9] need to be viewed with caution. Little variation in the sample configurations for long-term tension loading [9] also results in correlations relating to section geometry being overestimated for all models. This applies to some extent to short-term tension loading, as although the sample size is larger, many samples were limited to ties with a single reinforcing bar or have little variation. Adding the experimental programme of [13] (which consisted of repeat samples with multiple bars as opposed to much of the more uniform database of [9] with ties reinforced with single bars) resulted in differences to the short-term tension values obtained for the Pearson's analyses reported in [9] for MC 2010 and EN 1992 even though the sample size increase was small. Further research using different configurations/real structures is required for tension cracking for a more accurate evaluation of interdependence between these parameters and the model factor.

Referring to Table 4 and short-term tension, with the exception of the section depth to width ratio (h/b) for MC 2010, model uncertainty for the EN 1992 crack model has low to moderate correlations with all parameters. As would be expected, model uncertainty has a negative correlation with concrete tensile strength (f_{ctm}) for both EN 1992 and MC 2010. The inconsistency in the BS8007 crack model is clearly demonstrated as the Pearson's correlations vary substantially between the tension stiffening formulations. A larger sample size is required for a true reflection of the Pearson correlations for the long-term case [9], as can be evidenced by the extremely varied values between short- and long-term loading.

5.4. Comparison to Flexure Model Uncertainty

As the database for long-term tension cracking was very limited, comparisons were made to analyses to quantify model uncertainty as reported in [9] and [10] for short- and long-term flexure loading applied to the EN 1992, MC 2010 and BS 8007 crack models. The statistical parameters for the MF for flexural cracking are summarised in Table **??**. Further detail on the flexural crack models analyses can be found in [9, 10].

Comparing the short-term model uncertainty for tension to that of flexure, the means for the MC 2010 crack model are similar at approximately 1, although the variation for the tension case is less than that of flexure. This is probably partly due to the greater uniformity of sample configurations of the tension database compared to flexure. Lognormal distributions are indicated for the EN 1992 and MC 2010 crack models for both flexural and tension cracking. However, model uncertainty parameters are similar enough to conclude that it is not necessary to distinguish between the flexural and tension load cases if the MC 2010 crack model is considered for the GPM. Although the dataset for long term flexural cracking

Load Case	Parameter	Correlation with Model Factor			
		EN 1992	MC 2010	BS8007 $w = 0,2\,\text{mm}$ [9]	BS8007 $w = 0,1\,\text{mm}$ [9]
Short-term tension	Steel stress	0.052	0.204	-0.284	-0.364
	h/b	0.579	0.722	0.049	0.206
	c	0.286	0.420	0.332	0.230
	Bar dia, φ	0.518	0.570	-0.068	-0.241
	f_{ctm}	-0.294	-0.503	-0.030	0.080
	% As	0.352	0.390	-0.036	-0.019
Long-term tension [9]	Steel stress	0.825	0.641	0.840	0.777
	h/b	0.490	0.335	0.602	0.479
	c	-0.825	-0.641	-0.840	-0.777
	Bar dia, φ	-0.414	-0.209	-0.544	-0.407
	f_{ctm}	-0.826	-0.724	-0.792	-0.773
	% As	-0.349	-0.156	-0.426	-0.334

Table 4. Pearson's correlation matrix between model uncertainty & model parameters.

Load Case	Statistical parameter	EN 1992	MC 2010	BS 8007 $w = 0.2$ mm	BS 8007 $w = 0.1$ mm
Short term	Mean	1.107	1.052	1.185	1.112
	CoV	0.397	0.376	0.380	0.459
	PDF	LN	LN	LN	LN
	Count	164	164	164	164
Long term	Mean	1.443	1.127	1.502	1.514
	CoV	0.331	0.380	0.336	0.357
	PDF	LN	LN	LN	LN
	Count	30	30	30	30

Table 5. Summary of statistical parameters of MF for flexural cracking [10].

[9, 10] is larger than that of the tension load case, further research for long-term loading in general is recommended, including expanding the database and application to real conditions, for a more accurate estimation of the statistical parameters of model uncertainty.

Pearson's correlations were performed for flexural cracking, as reported by [10]. Low to moderate correlations were found for all parameters except for % As for long-term flexure (r of 0,58). It is noted that the dataset for flexural cracking had more variation in terms of section and reinforcement configurations than that of tension cracking, which supports the suggestion that the uniformity of the dataset for tension cracking results in apparent dependence between these parameters and the MF.

6. Summary and concluding remarks

In quantifying model uncertainty for pure tension crack models, the significant degree of uncertainty was confirmed, which justifies a full probabilistic treatment of model uncertainty and SLS crack models. The model uncertainty statistical parameters were determined for short and long-term tension cracking, although the latter case does require further investigation to confirm values obtained. However, considering the MF statistical parameters determined for short and long-term flexural cracking ([9, 10]), the short-term MF values for tension cracking may be used for long-term MF's when considering crack models such as MC 2010 which take long-term shrinkage into account. Indications are also that for the MF, it is not necessary to distinguish between flexural and tension load-induced cracking. This simplifies the reliability assessment of load-induced cracking and the development of the GPM.

In terms of crack model performance, the BS 8007 crack model is shown to be inconsistent as its tension stiffening model is dependent on the specified limiting crack width. Any variation in crack width away from this limit results in a high variation in the MF, and thus an inconsistent model. This is a point to note for any empirically-developed model which may be considered for a GPM. It was also noted during the analyses that the point on the cross section where the crack was measured was generally not given in the experimental records, introducing some further uncertainty into the predicted crack widths due to potential mismatches between measured and estimated crack positions. Using a_{cr} based on the geometry of the cross section, rather than a crack spacing formulation, for ties with single bars resulted in underprediction of the measured crack widths. This effect would be mitigated in structures under real conditions. Crack models that omit shrinkage strain (EN 1992 and BS 8007) underpredict crack widths in the long-term loading case. MC 2010 was the most consistent crack prediction model and will thus be the basis for the GPM. Further to this research, data will be compiled for existing structures such as bridges and water retaining structures for use in the probabilistic analysis of the GPM.

The experimental database was compiled and refined with WRS in mind, where SLS cracking is dominant in design, so focused on small crack widths. For long-term tension cracking in particular, it must be noted that as many of the elements tested were mostly ties reinforced with single reinforcing bars, the crack widths recorded may not necessarily be representative of real conditions in structures such as WRS where there are obviously multiple reinforcing bars in both the main and transverse directions. This was noted in [9] and [10], borne out in the further analyses. This requires further research to properly assess the influence of section and reinforcement configurations on tension cracking, and which will include data from existing structures.

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