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COMPARISON BETWEEN CONCRETE STRESS DIAGRAMS ACCORDING TO THE BRAZILIAN STANDARD ABNT NBR 6118: 2014

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Abstract.

This study aims to compare methods for the determination of concrete properties by means of the stress diagrams present in the Brazilian standard ABNT NBR 6118: 2014. The area under the stress diagram, the internal reactions, and the application point of the resulting reactions for the parabola-rectangle and rectangular block diagrams are present in order to compare them. Deductions and numerical examples were used, and different results were obtained for each formulation. This is due to non-consideration of the relationship between stress and strain in the simplified rectangular block. The rectangular block is applicable only for cases in which the concrete reaches the ultimate strain. These cases are those that concrete crushing determines the section failure in compression with steel yielding in tension (domain 3) or without steel yielding (domain 4).

Keywords: Concrete behavior, parabola-rectangle diagram, rectangular block.

1. Introduction

Exhaustion of the stress capacity of a reinforced concrete element subjected to normal stress can be achieved by crushing the concrete under compression or by an excessive plastic strain of the steel. However, concrete failure is difficult to identify. It is conventional to assume that this failure occurs when the material reaches a maximum compressive strain determined by experimental results [1]. According to American standard ACI 318-14 [2], the maximum compressive strain of concrete is around 0.003 to 0.004 under normal conditions, and it may reach 0.008 under special conditions. The Brazilian standard ABNT NBR 6118: 2014 [3] states a maximum specific strain of 0.0035 for concretes up to class C50 (characteristic compressive strength up to 50 MPa).

This Brazilian standard adopts an idealized stress-strain diagram to represent concrete behavior, in which the stress distribution takes place according to a parabola-rectangular diagram [3]. This standard allows to replace the parabola-rectangle diagram with a rectangle of equivalent depth, as a simplification of calculation [3]. ABNT NBR 6118: 2014 [3] states that the difference in results with both formulations is small, so there is no need to make any adjustment. The rectangular block does not represent the actual stress distribution within the compressed concrete zone, but provides reasonably the same compressive force [4].

However, Mendes Neto [5] noticed that the design of structures with the rectangular block provides a different and unsafe result compared to the parabola-rectangle diagram result. Formulations for stress, internal reaction, and the point of application of the resulting reaction are presented in this paper by means of the parabola-rectangle diagram and the rectangular block. Solutions are compared for some cases to verify if the use of the rectangular block leads to reasonable and safe results.

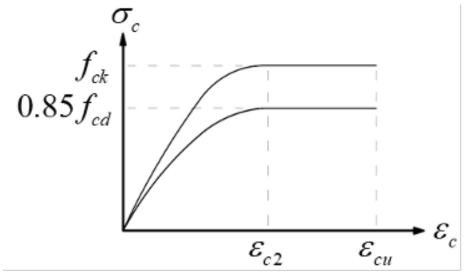

FIGURE 1. Idealized stress-strain diagram. Source: Adapted from the ABNT NBR 6118: 2014 [3].

2. MATERIALS AND NUMERICAL METHODS

This section presents the concepts and formulations considered for the comparison between parabola-rectangle and rectangular block diagrams. ABNT NBR 6118: 2014 [3] uses an idealized stress-strain diagram to represent concrete compression at the ultimate limit state, as shown in Figure 1, in which ε_c is the concrete strain; ε_{c2} is the assumed specific compression strain of concrete at the beginning of the plastic level; ε_{cu} is the assumed maximum useable compression strain in the concrete; f_{ck} is the characteristic strength of the concrete; f_{cd} is the design strength of the concrete; and σ_c is the concrete stress [3, 6]. This diagram is similar to that presented in section 7.2.3.1.5 of the *fib* Model Code 2010 [7] and section 3.1.7 of the Eurocode 2 [8].

Equation (1) defines this diagram (Figure 1):

$$\sigma_c = 0.85 f_{cd}\left[1-\left(1-\frac{\varepsilon_c}{\varepsilon_{c2}}\right)^n\right] \tag{1}$$

in which $n = 2$ for $f_{ck} \leq 50$ MPa; and n is given by Equation 2 for $f_{ck} > 50$ MPa.

$$n = 1.4 + 23.4\left[(90 - f_{ck})/100\right]^4 \tag{2}$$

For concretes with $f_{ck} \leq 50$ MPa: $\varepsilon_{c2} = 0.002$; and $\varepsilon_{cu} = 0.0035$. Equations 3 and 4 give the values of ε_{c2} and ε_{cu} for concretes with $f_{ck} > 50$ MPa.

$$\varepsilon_{c2} = 0.002 + 0.000085\,(f_{ck} - 50)^{0.53} \tag{3}$$

$$\varepsilon_{cu} = 0.0026 + 0.035\left[(90 - f_{ck})/100\right]^4 \tag{4}$$

The Brazilian standard [3] states that it is possible to replace this diagram with a rectangle of height $y = \lambda x$ (in which x is the effective depth of the neutral axis), and constant stress $\alpha_c\, f_{cd}$, without loss of quality in results. For concretes with $f_{ck} \leq 50$ MPa: $\lambda = 0.8$, and $\alpha_c = 0.85$. Equations 5 and 6 give values of λ and α_c for concretes with $f_{ck} > 50$ MPa. Thus, the strains and stresses in a rectangular cross-section at the ultimate limit state can be represented as in Figure 2.

$$\lambda = 0.8 - (f_{ck} - 50)/400 \tag{5}$$

$$\alpha_c = 0.85\left[1 - (f_{ck} - 50)/200\right] \tag{6}$$

The area under the stress diagram can be obtained by integrating stresses in the domain of the specific strains. Thus, Equation 7 is used when strain values at the most compressed fiber are less than ε_{c2}, and Equation 8 is used if strain values are larger than ε_{c2}.

$$A(\varepsilon) = 0.85\, f_{cd}\int_0^{\varepsilon_c}\left[1-\left(1-\frac{\varepsilon}{\varepsilon_{c2}}\right)^n\right]\mathrm{d}\varepsilon \tag{7}$$

$$\begin{gathered}A(\varepsilon) = 0.85\, f_{cd}\int_0^{\varepsilon_{c2}}\left[1-\left(1-\frac{\varepsilon}{\varepsilon_{c2}}\right)^n\right]\mathrm{d}\varepsilon\ + \\ 0.85\, f_{cd}\int_{\varepsilon_{c2}}^{\varepsilon_c}\mathrm{d}\varepsilon\end{gathered} \tag{8}$$

Considering the Bernoulli hypothesis (plane sections remain plane), the longitudinal strain varies proportionally with the distance to the neutral axis [2]. Therefore, if the specific strain is known, the neutral axis is determined. Thus, as the resultant of internal reactions equals the volume under the stress diagram, its value becomes known after determining the stress and cross-sectional compressed area.

The center of gravity of the compression region is obtained by dividing the stress-strain diagram into infinitesimal areas stress-strain diagram into infinitesimal areas $\sigma\,\mathrm{d}\varepsilon$. The point of application of the resultant of normal concrete stresses can be determined by Equation 9, in which the integral of the stress area in the strain domain is determined by Equation 7 and 8.

$$\overline{\varepsilon} = \frac{\displaystyle\int_A x\mathrm{d}A}{\displaystyle\int_A \mathrm{d}A} = \frac{\displaystyle\int_\varepsilon \varepsilon\sigma\mathrm{d}\varepsilon}{\displaystyle\int_\varepsilon \sigma\mathrm{d}\varepsilon} \tag{9}$$

2.1. STRAIN DOMAINS

Conventionally, failure of a reinforced concrete section occurs when the specific strain of concrete or steel (or both) reaches its ultimate value. The standard ABNT NBR 6118: 2014 [3] defines that the ultimate limit state is characterized when the distribution of strains in the section is within one of the specified strain domains. Figure 3 illustrates these strain domains, in which ε_{yd} is the strain in the tension reinforcement at failure; h is the overall height of a cross-section; d is the distance from the most compressed fiber to the centroid of the longitudinal reinforcement on the tension side of the member; and d' is the distance from the most compressed fiber to the centroid of the longitudinal compression steel [3, 6]. Thus, section failure can occur under excessive plastic strains of the steel characterized by the straight-line

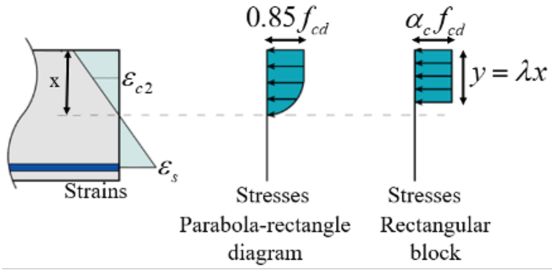

Figure 2. Strain and stress diagrams of concrete.

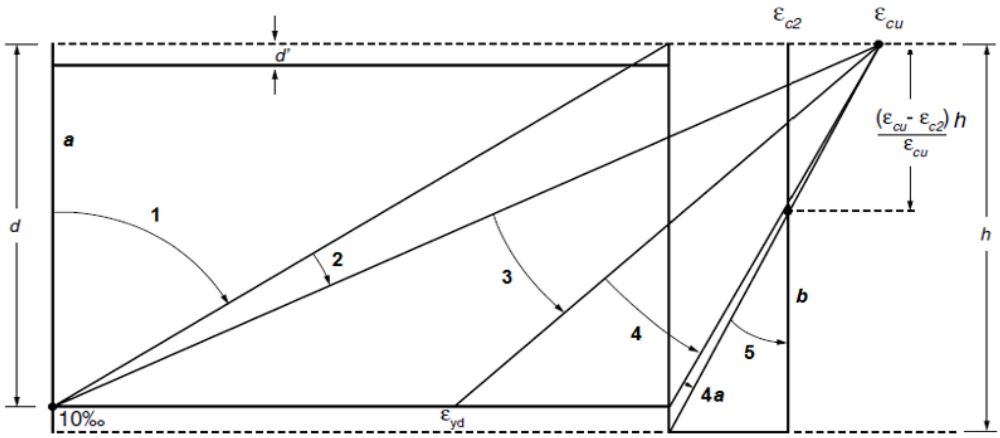

Figure 3. Strain domains in the ultimate limit state. Source: Adapted from the ABNT NBR 6118: 2014 [3].

a, within domains 1 and 2, or by crushing of the compressed concrete defined by domains 3, 4, 4a, 5, and the straight-line *b*.

The ultimate limit state of domain 2 is characterized by the strain $\varepsilon_{yd} = 1\%$, and the failure of the cross-section occurs without the concrete reaching its maximum strength. Fusco (1981) [1] divides domain 2 into two subdomains: 2a (ε_c from zero up to ε_{c2}) and 2b (from ε_{c2} to ε_{cu}). This subdivision aims to identify the value at which the use of compression reinforcement becomes efficient [1]. Such consideration becomes important here because it marks the point at which pseudo-plastification of concrete begins. While the section is in subdomain 2a, concrete stresses are less than 0.85 f_{cd}. When concrete strain exceeds ε_{c2}, the concrete pseudo-plastification and the constant compression region arise. When the section deformation approaches domain 3, the specific strains become higher, and the plasticized zone expands. Therefore, two different equations are required to represent the behavior of compressed concrete, Equations 7 and 8.

The strain ε_{cu} characterizes the ultimate limit state of domains 3, 4 and 4a. In these domains, the failure of the section occurs by crushing the concrete in compression. Although the neutral axis goes down in the section and the compressed area increases, the stress diagram proportions remain constant. In these domains, it is possible to find the stresses and reactions in concrete only with Equation 8.

The strain ε_{cu} also characterizes the ultimate limit state of domain 5, with concrete failure in compression. In domain 5, the entire section is under nonuniform compression. The strains converge to a constant value ε_{c2} as the section approaches the line b. Thus, the concrete properties in domain 5 are defined by Equation 8, changing the lower limit of the first integral.

2.2. Calculations with a computer program

The MATLAB software was used for the preparation of scripts in order to make a comparison between the diagrams. The stresses, internal reactions, and center of gravity of the parabola-rectangle diagram were obtained through these scripts, that solve equations 7, 8, and 9. The results were compared with those obtained by the rectangular block. This analysis is called "Approach 1".

Another comparison between the diagrams, with a different approach (Approach 2), was performed. Values were assigned to the position of the neutral axis, and the internal reactions were obtained with Equation 10 for the parabola-rectangle diagram, and with Equation 11 for the rectangular block.

$$R_c = \int_A \sigma \,\mathrm{d}A \tag{10}$$

$$R_c = \alpha_c\, f_{cd}\, b_w\, \lambda\, x \tag{11}$$

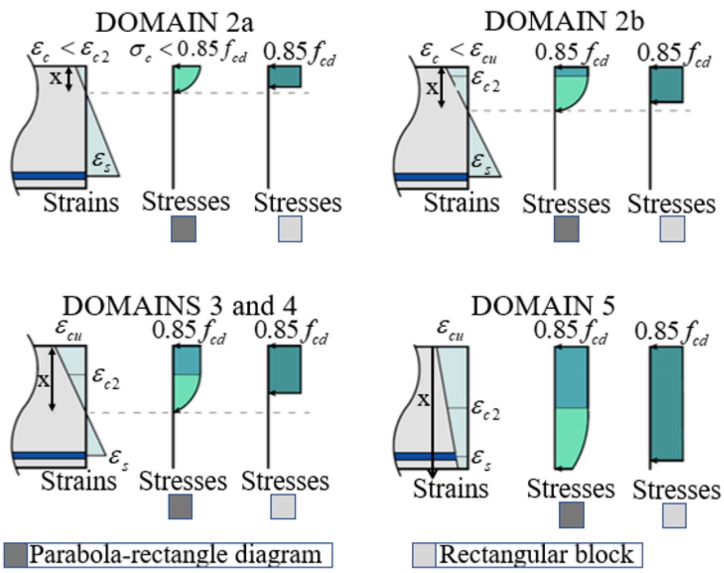

Figure 4. Concrete strains and stresses in the ultimate limit state.

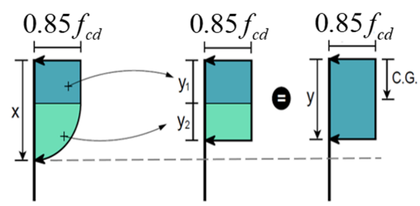

Figure 5. Representation of calculation solution.

3. Results and discussions

Initially, it should be emphasized that the use of stress diagrams does not apply to line a and domain 1, since there are no compressive stresses in concrete, and its tensile stresses are neglected at the ultimate limit state.

3.1. Approach 1

Based on the parameters analyzed, the rectangular diagram presents only one way to describe the behavior of the concrete. However, the parabola-rectangle diagram has five different behaviors:

1. While the structure is in domain 2a, there is only a parabolic excerpt ($\sigma_c < 0.85\, f_{cd}$);
2. When the pseudo-plastification begins in domain 2b, the excerpt of constant stress 0.85 f_{cd} appears, and there is an expansion of this plastic zone until the section reaches domain 3;
3. For domains 3, 4 and 4a, there is an expansion of the compressed region, in which the stress distribution increases proportionally in the parabolic excerpt and in the constant stress excerpt;
4. For sections in domain 5, there is an overlap of the plastic zone, where the specific strains across the section converge to ε_{c2} and the stresses converge to 0.85 f_{cd};
5. For line b, the entire section is under uniform compression ($\sigma_c = 0.85\, f_{cd}$).

Figure 4 illustrates the stress distribution and behaviors in the strain domains.

Results of the parabola-rectangle diagram can be obtained by equations 7 and 8, calculating the area as a function of the neutral axis position (x). For comparison, the solution is represented as proportions of equivalent rectangles of constant stress of 0.85 f_{cd} and nominal height y_1 and y_2, referring to the portions of constant and parabolic stresses distribution, respectively. Figure 5 illustrates the solution, and the results are listed in Table 1.

Domain 4a represents a small region of the domains, and it ends when the section is fully compressed, so its relationships are similar to those of domains 3 and 4. For domain 5 and line b, the neutral axis relation is no longer represented as a function of x/d and becomes a function of x/h, as shown in Table 2.

Thus, the nominal compression of the parabola-rectangle stress diagram is similar to the rectangular diagram in domains 3, 4 and 4a, where the height $y = 0.8095x$ is equivalent to the height adopted by simplification ($y = 0.80x$). However, the point of application of the resulting reaction is different in each type of diagram. In addition, a considerable differ-

(x/d)	y/x			y_1/x	y_2/x	C.G.$/x$		
Domain 2a								
0.01	1460 / 29403	≅	0.050	-	1460 / 29403	391 / 1168	≅	0.335
0.02	710 / 7203	≅	0.099	-	710 / 7203	191 / 568	≅	0.336
0.03	1380 / 9409	≅	0.147	-	1380 / 9409	373 / 1104	≅	0.338
0.04	335 / 1728	≅	0.194	-	335 / 1728	91 / 268	≅	0.340
0.05	260 / 1083	≅	0.240	-	260 / 1083	71 / 208	≅	0.341
0.06	630 / 2209	≅	0.285	-	630 / 2209	173 / 504	≅	0.343
0.07	8540 / 25947	≅	0.329	-	8540 / 25947	337 / 976	≅	0.345
0.08	590 / 1587	≅	0.372	-	590 / 1587	41 / 118	≅	0.347
0.09	3420 / 8281	≅	0.413	-	3420 / 8281	319 / 912	≅	0.350
0.10	110 / 243	≅	0.453	-	110 / 243	31 / 88	≅	0.352
0.11	11660 / 23763	≅	0.491	-	11660 / 23763	301 / 848	≅	0.355
0.12	255 / 484	≅	0.527	-	255 / 484	73 / 204	≅	0.358
0.13	12740 / 22707	≅	0.561	-	12740 / 22707	283 / 784	≅	0.361
0.14	3290 / 5547	≅	0.593	-	3290 / 5547	137 / 376	≅	0.364
0.15	180 / 289	≅	0.623	-	180 / 289	53 / 144	≅	0.368
0.16	860 / 1323	≅	0.650	-	860 / 1323	16 / 43	≅	0.372
0.166667	2 / 3	≅	0.667	-	2 / 3	3 / 8	≅	0.375
Domain 2b								
0.17	172 / 255	≅	0.675	2 / 85	166 / 255	22019 / 58480	≅	0.377
0.18	94 / 135	≅	0.696	4 / 45	82 / 135	6451 / 16920	≅	0.381
0.19	68 / 95	≅	0.716	14 / 95	54 / 95	9977 / 25840	≅	0.386
0.20	11 / 15	≅	0.733	1 / 5	8 / 15	43 / 110	≅	0.391
0.21	236 / 315	≅	0.749	26 / 105	158 / 315	39211 / 99120	≅	0.396
0.22	42 / 55	≅	0.764	16 / 55	26 / 55	3697 / 9240	≅	0.400
0.23	268 / 345	≅	0.777	38 / 115	154 / 345	49859 / 123280	≅	0.404
0.24	71 / 90	≅	0.789	11 / 30	19 / 45	3481 / 8520	≅	0.409
0.25	4 / 5	≅	0.800	2 / 5	2 / 5	33 / 80	≅	0.412
Domains 3 and 4								
0.259259 ∼ 1	17 / 21	≅	0.810	3 / 7	8 / 21	99 / 238	≅	0.416

TABLE 1. Section properties in the ultimate limit state (Domains 2, 3 and 4).

ence is noted for the nominal compression and the point of application of the resultant reaction within sections in domains 2 and 5, and in line b.

3.2. APPROACH 2

A rectangular section with $b_w = 1$, $d = 1$, $h = 1.2$ and $f_{ck} = 20$ MPa was analyzed, in which b_w is the section width. Regardless of the values adopted for the cross-section, the difference obtained by the diagrams remains proportional. For this reason, dimensionless values were used for b_w, h and d. The results for the ultimate limit state are shown in Figure 6. As a reference, the limits of the strain domains were marked, where CA-50 steel (steel with 500 MPa yield strength) was adopted.

The same fact can be verified for concrete strength group II (C55 to C90): there is a considerable difference between results with the parabola-rectangle diagram and the rectangular block simplification. A rectangular section with $b_w = 1$, $d = 1$, $h = 1.2$ and $f_{ck} = 80$ MPa was analyzed. The relationship between internal reaction and neutral axis position is shown in Figure 7.

4. CONCLUSIONS

Although the rectangular block simply represents the behavior of the concrete at the imminence of failure, it does not reflect the relation between stresses and strains where concrete is under limit strain of failure. This simplification considers that the material works at its full capacity of strength all over the compressive zone.

The nominal compression and point of application of the concrete reaction are almost equal with both diagrams (parabola-rectangle or rectangular block) in the design of linear elements sections submitted predominantly to bending load (domains 3 and 4). However, it can result in unsafe design in sections in domains 2 or 5, typically slabs or columns, respectively.

The difference between the diagrams is amplified for high strength concrete elements. Since high resistance concretes have lower ductility and an explosive and brittle failure, they have a smaller plastic region and, consequently, values of ε_{c2} close to ε_{cu}, with a smaller excerpt of constant stresses. Thus, the calculated difference between the stress diagrams becomes larger.

(x/h)	y/h			y_1/h	y_2/h	C.G.$/h$		
Domain 5								
1.00	17 / 21	≅	0.810	3 / 7	8 / 21	99 / 238	≅	0.416
1.05	133349 / 158949	≅	0.839	3 / 7	65228 / 158949	805443 / 1866886	≅	0.431
1.10	39989 / 46389	≅	0.862	3 / 7	20108 / 46389	247923 / 559846	≅	0.443
1.15	188621 / 214221	≅	0.881	3 / 7	96812 / 214221	1192347 / 2640694	≅	0.452
1.20	13709 / 15309	≅	0.895	3 / 7	7148 / 15309	87963 / 191926	≅	0.458
1.25	10085 / 11109	≅	0.908	3 / 7	5324 / 11109	13095 / 28238	≅	0.464
1.30	71741 / 78141	≅	0.918	3 / 7	38252 / 78141	470187 / 1004374	≅	0.468
1.35	323861 / 349461	≅	0.927	3 / 7	174092 / 349461	2139027 / 4534054	≅	0.472
1.40	5669 / 6069	≅	0.934	3 / 7	3068 / 6069	37683 / 79366	≅	0.475
1.45	403829 / 429429	≅	0.940	3 / 7	219788 / 429429	2698803 / 5653606	≅	0.477
1.50	4469 / 4725	≅	0.946	3 / 7	2444 / 4725	30003 / 62566	≅	0.480
1.75	27725 / 28749	≅	0.964	3 / 7	15404 / 28749	37791 / 77630	≅	0.487
2.00	2477 / 2541	≅	0.975	3 / 7	1388 / 2541	17019 / 34678	≅	0.491
3.00	1685 / 1701	≅	0.991	3 / 7	956 / 1701	2343 / 4718	≅	0.497
4.00	13061 / 13125	≅	0.995	3 / 7	7436 / 13125	91107 / 182854	≅	0.498
5.00	335 / 336	≅	0.997	3 / 7	191 / 336	234 / 469	≅	0.499
10.0	94205 / 94269	≅	0.999	3 / 7	53804 / 94269	131823 / 263774	≅	0.500
50.0	≅ 1.000			3 / 7	0.571	≅ 0.500		
Line b								
∞		1		3 / 7	4 / 7	0.5		

TABLE 2. Section properties in the ultimate limit state (Domain 5 and line b).

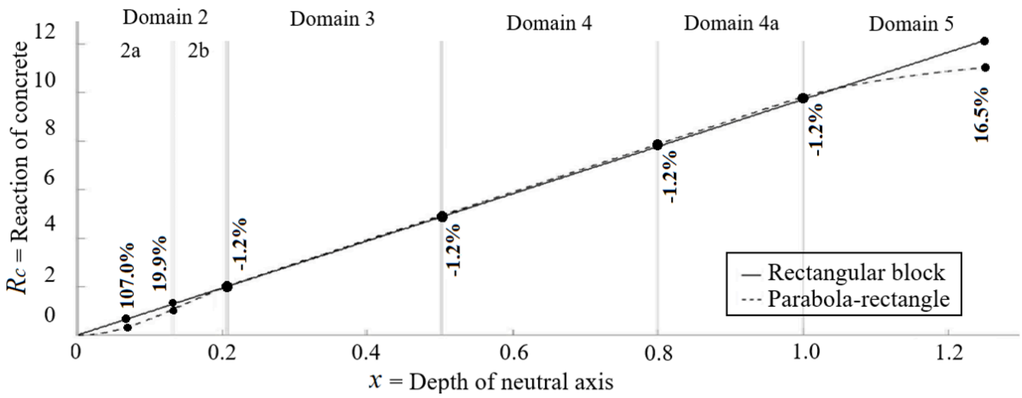

FIGURE 6. Internal reactions for concrete class C20 (f_{ck} = 20 MPa). The percentage difference between the two approaches is marked at some representative points, referred to results obtained with the parabola-rectangle diagram.

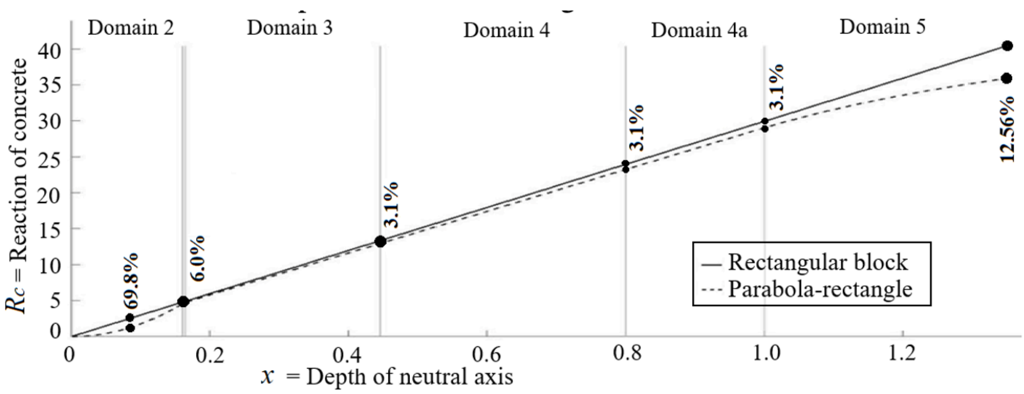

FIGURE 7. Internal reaction for concrete class C80 (f_{ck} = 80 MPa). The percentage difference between the two approaches is marked at some representative points, referred to results obtained with the parabola-rectangle diagram.

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References

[1] P. B. Fusco. Estruturas de concreto: Solicitações normais, Rio de Janeiro: Guanabara Dois, 1981.

[2] American Concrete Institute ACI 318-14. Building code requirements for structural concrete (Farmington Hills: ACI), 2014.

[3] Associação Brasileira de Normas Técnicas. ABNT NBR 6118: 2014: Projeto de estruturas de concreto - procedimento, Rio de Janeiro: ABNT, 2014.

[4] A. H. Mattock, L. B. Kriz, E. Hognestad. Rectangular concrete stress distribution in ultimate strength design. *Journal Proceedings*, 1961.

[5] F. N. Mendes. Bloco retangular versus parábola-retângulo: Comparação na flexão composta Anais do 4° Simpósio EPUSP sobre estruturas de concreto, São Paulo: EPUSP, 2006.

[6] J. K. Wight. Reinforced concrete: mechanics and design, Hoboken: Pearson, 7th edition, 2016.

[7] Fib. Fédération Internationale du Béton. Model Code for concrete structures 2010, Lausanne: fib, 2013. https://doi.org/10.1002/9783433604090.

[8] European Committee for Standardization 2004 EN 1992-1-1. Eurocode2: Design of concrete structures - Part 1-1: General rules and rules for buildings, Brussels: CEN, 2004.