Acta Polytechnica CTU Proceedings **3**:1–6, 2016
doi:10.14311/APP.2016.3.0001

available online at http://ojs.cvut.cz/ojs/index.php/app

STATISTICAL EVALUATION OF FATIGUE DATA OF COMPONENTS

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Abstract. A variety of steels, cast iron grades and other metals have long been used for the production of machine components. In recent years, however, new materials such as sintered materials and plastics become increasingly important. Because of the large number of different fibers, matrices, stacking sequences, processing conditions and processes and the variety of resulting material configurations it is not possible to rely on proven fatigue models for conventional materials. Moreover, the development of models, which are valid for all composites are generally extremely difficult.

In this work, a possible application of high-performance composites as materials for machine elements are investigated. This study attempts to predict the fatigue behavior and the consequent durability, based on laboratory measurements. Using the statistics program JMP, the aquired data was subjected to a reliability analysis in order to ensure the plausibility, validity and accuracy of the measured values.

Keywords: fatigue models, statistical evaluation, reliability, techniques of parameter fitting, JMP.

1. Introduction

There was a tremendous advance in the field of plastics took in the past few decades. Plastics have become an integral part of our daily life. Polymers are flexible materials which can cover a great range of applications. They replace more and more metal, glass, wood and other materials. The development of novel artificial materials often opens a door to new technologies. Electrical and automotive industry are hard to imagine without plastics, the use of polymer materials has revolutionized medicine.

Cyclically stressed components have a limited durability, therefore it is important to perform fatigue tests or simulations on critical components to predict their lifetime. Figure 1 shows representative loading patterns with constant amplitudes for the whole loading level [1].

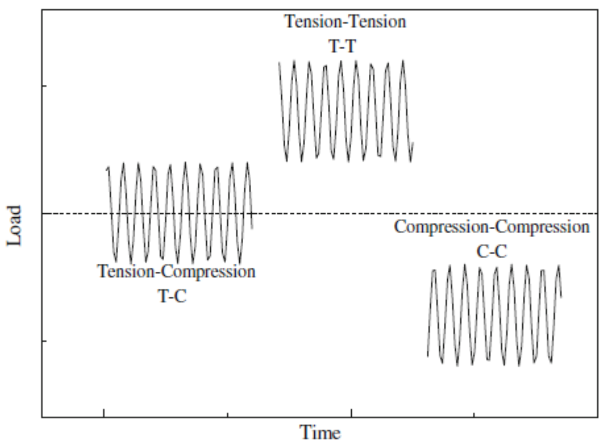


Figure 1. Constant amplitude loading patterns.

Fatigue tests are performed to study the relationship between the fatigue resistance of a material, component or structural element and cyclic loading [2]. It is a slowly progressing damage process. The strengths lie far below the static strength and yield strength.

The objective of this work was to create high cycle fatigue curves from experimental datasets, using suitable material models, similar to the Wöhler curve. The curve should as closely as possible reflect the experimental values. Since specimens and test conditions are never 100 % identical, in two discrete measurements there is always scattering in the results, which can span over a decade in fiber reinforced polymers [1]. Therefore to handle and correctly interpret experimental results, statistical methods must be used.

2. Fatigue Models

In general, Fatigue models are quantifications of physical material properties. They are independent of the shape of the component and are usually based on experimentally acquired data. The aim of all models is to predict how a component behaves in certain conditions. The relationships are reproduced mathematically afterwards, therefore they are mathematical models.

In this work, the focus will be on presenting fatigue models [3]. In order to design a component correctly in terms of fatigue, a complete set of experimentally acquired data is usually required. Since this is not possible for reasons of time and cost, engineers have to rely on predictive models. This models predict the durability N (in cycles) under a given cyclic loading.

2.1. The Basquin Model

The Basquin fatigue model is a linear regression model. In a logarithmic scale, the durability (logN) is plotted versus the stress amplitude (log$\Delta\sigma$).

$$\log N = A - B \cdot log\Delta\sigma,\ \Delta\sigma \geq \Delta\sigma_0,\ B \geq 0 \qquad (1)$$

A and B are material parameters that need to be approximated with appropriate fitting methods. In the model, $\Delta\sigma$ is limited by $\Delta\sigma_0$, the endurance limit. In this work, the endurance limit is at 40^6 cycles. $\Delta\sigma_0$ itself has no direct influence on the model, since it is not considered in the formula.

2.2. The Strohmeyer Model

In contrast to the Basquin model the models of Strohmeyer and Weibull are nonlinear. They are shaped by smoothing a piecewise linear function. Again A and B are the material parameters.

$$\log N = A - B \cdot log(\Delta\sigma - \Delta\sigma_0),\ B \geq 0 \qquad (2)$$

2.3. The Weibull Model

The Weibull model is more complex, since there are more parameters to fit.

$$\log(N+D) = A - B \cdot log\left(\frac{\Delta\sigma - \Delta\sigma_0}{\Delta\sigma_{st} - \Delta\sigma_0}\right),\ B \geq 0 \quad (3)$$

A, B and D describe the material parameters and $\Delta\sigma_{st}$ denotes the ultimate strength.

2.4. The Bastenaire Model

$$\frac{logN - B}{\Delta\sigma - \Delta\sigma_0} = A \cdot exp[-C \cdot (\Delta\sigma - \Delta\sigma_0)] \qquad (4)$$

A, B and C express the material parameters. This model will be henceforth denoted as Bastenaire1, because one can find another model of Bastenaire in the literature, which is called Bastenaire2 here. It is quite similar to equation 4 and it is also discussed for the purpose of comparison.

$$N = A \cdot exp[-C \cdot \frac{\Delta\sigma - \Delta\sigma_0}{B}]/(\Delta\sigma - \Delta\sigma_0) \qquad (5)$$

3. Techniques of Parameter Fitting

In this section, two exemplar fitting methods are discussed in greater detail [4–6]. Both methods are current and proven estimation methods in statistics.

3.1. The Method of Least Squares (L2-Norm)

The method of Least Squares is a mathematical standard procedure for compensation calculation. Here a function is determined, which fits a point cloud as close as possible. A point cloud is a scatter plot, a graphical representation of statistical measurements in a coordinate system.

To illustrate the method, the Basquin model is used as an example. Consider a dependent variable y (in this case, logN), which depends on one or several variables (in this case A, B and log$\Delta\sigma$). The relationship between the dependent variable and the arguments are described in a model function f [7]. The model function f can be linear, as in this case, but it can also have any other shape (parabolic, exponential, ...). The general form is:

$$y(x) = f(x; a_1.........a_m) \qquad (6)$$

In this case:

$$\log N = A - B \cdot log\Delta\sigma,\ \Delta\sigma \geq \Delta\sigma_0,\ B \geq 0 \qquad (7)$$

The parameters A and B should be adapted such that bad data points (outliers) have only little effects on the fitting. If no unique solution that perfectly fits the point cloud can be found, then a compromise solution with the smallest overall deviation from the point cloud is the valid criterion. For this purpose, the sum of the squares of the differences between the model function f and the measured values y_i is formed.

$$\sum_{i=1}^{n}(f(x_i; a_1.........a_m) - y_i)^2 \qquad (8)$$

The parameters A and B are adapted until the sum of squares becomes minimal. In this work the fitting procedure carried out with the help of a solver implemented in Excel.

3.2. The Method of Least Absolute Deviations (L1-Norm)

The L1 Norm is a more robust fitting method. Outliers are not so strongly weighted. In principle, this method works similarly to the previously explained Method of Least Squares. Instead of the sum of the squares, the absolute sum of the differences between the model function f and the measured values y_i is calculated.

$$Abs \parallel \sum_{i=1}^{n}(f(x_i; a_1.........a_m) - y_i) \parallel \qquad (9)$$

Subsequently, the parameters A and B are also be adjusted until the absolute sum is minimized.

4. Reliability

The reliability is a measure of the accuracy of the measurement, as well as for the reliability of data. Measurement series with very high reliability are therefore almost free of random errors, which means that they are repeatable at any time under the same measurement conditions and thereby provide approximately the same results [8, 9]. Therefore, one gets a high reliability, when performing controlled and standardized measurements. To examine the reliability, different techniques can be used [10]. Some known techniques are:

- Test - Retest Reliability
- Parallel - Forms Reliability
- Split - Half Reliability
- Internal - Consistency Reliability.

Reliability analysis can be easily carried out with various computer programs. Known programs are SPSS or, as used in this work, JMP.

5. Evaluation of Fatigue Data

Two different materials were tested. Material#1 is a glass fiber reinforced, semi-crystalline thermoplastic. Material#2 is a carbon fiber reinforced, semi-crystalline thermoplastic, where the bearing was simulated as a compliant bearing. Material#3 is the same material as Material#2, however it has been simulated as a rigid bearing.

5.1. Fatigue Models

Figure 2 exhibits the applied fatigue models for material#1. Since for Material#2 the ultimate strength and the load at the endurance limit could not be investigated, only the Basquin model could be applied 3. In the case of Material#3 the fitting with the Strohmeyer and the Weibull model resulted in the same curve 4.

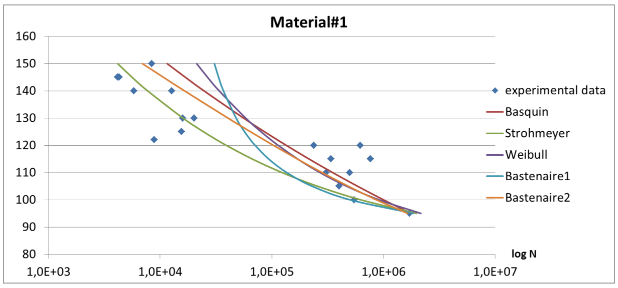


Figure 2. Applied fatigue models for Material#1.

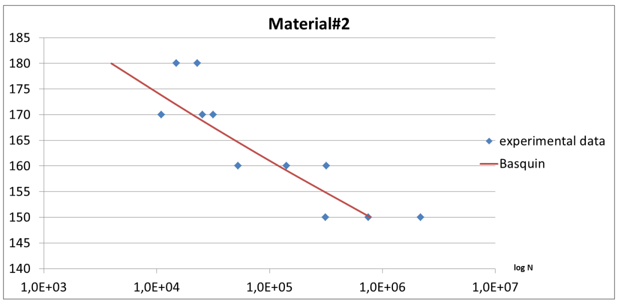


Figure 3. Applied fatigue models for Material#2.

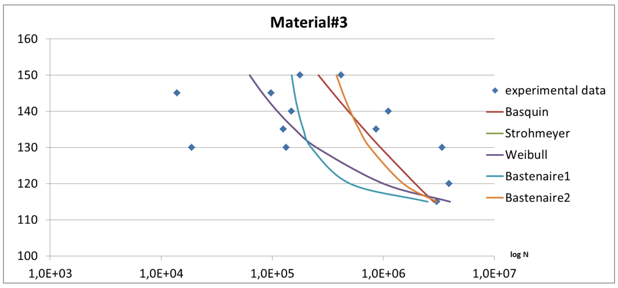


Figure 4. Applied fatigue models for Material#3.

The software minimized the deviation between the models and the measured values. By comparing the obtained model parameters the optimal model can be easily determined, not only qualitatively but also quantitatively. Figure 5 and 6 show the optimal fatigue models for the materials. Since for Material#2 only the Basquin model was applied, it was omitted below.

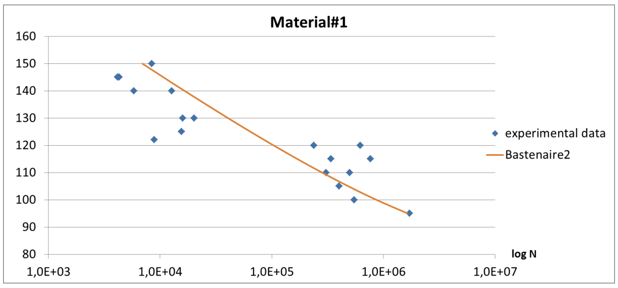


Figure 5. Optimal fatigue model for Material#1.

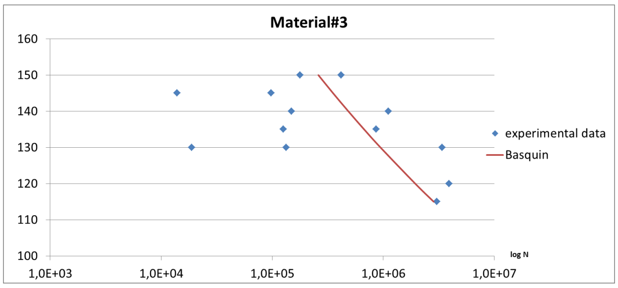


Figure 6. Optimal fatigue model for Material#3.

5.2. Reliability Analysis in JMP

In JMP custom tables can be created, or files of different formats (Excel, SAS, text files, ...) can be processed. For a correct data input, the appropriate settings in the import wizard must be applied.

In order to determine the best distribution for the measured values, a life distribution is performed [11]. The program returns a table with the appropriate distributions for the respective materials. JMP sorts them in descending order, the best model being on the top. The distribution of the measured data is weighted by 3 criteria.The Akaike Information Criterion (AIC), the Bayesian Information Criterion (BIC) and Log-likelihood (maximum probability) used as estimation methods for the selection of models in statistics.

Tables 1, 2 and 3 show the "Model Comparisons" for each materials. According to the distribution analysis in JMP, the measured values from Material#1 follows a natural logarithmic distribution, from Material#2 a Frechet distribution and from Material#3 a Weibull distribution.

Distribution	AICc	Loglike	BIC
Lognormal	503.65435	498.90435	504.79323
Weibull	503.66928	498.91928	504.80816
Frechet	505.47261	500.72261	506.61149
Loglogistic	505.74393	500.99393	506.88280
Exponential	518.31862	516.08332	519.02776
LEV	538.93104	534.18104	540.06992
Logistic	546.20378	541.45378	547.34266
Normal	550.30607	545.55607	551.44495
SEV	563.21318	558.46318	564.35205

Table 1. Comparisons of the distribution of the measured data for Material#1.

Distribution	AICc	Loglike	BIC
Frechet	297.91571	292.41571	297.21150
Lognormal	299.16181	293.66181	298.45760
Loglogistic	300.06766	294.56766	299.36345
Weibull	301.43592	295.93592	300.73171
Exponential	305.28286	302.83841	305.23631
LEV	320.31613	314.81613	319.61192
Logistic	326.13477	320.63477	325.43056
Normal	329.93763	324.43763	329.23342
SEV	337.33938	331.83938	336.63517

Table 2. Comparisons of the distribution of the measured data for Material#2.

Distribution	AICc	Loglike	BIC
Weibull	407.25467	402.16376	407.44188
Loglogistic	409.33460	404.24369	409.52181
Lognormal	409.74094	404.65003	409.92814
Frechet	415.33191	410.24100	415.51912
Exponential	416.03305	413.69971	416.33877
LEV	432.98688	427.89597	433.17409
Logistic	439.48273	434.39182	439.66994
Normal	439.98404	434.89313	440.17125
SEV	445.68650	440.59560	445.87371

Table 3. Comparisons of the distribution of the measured data for Material#3.

Based on this findings, the durability evaluation was performed. Tables 4, 5 and 6 display the mean estimates for each material. σ is the standard deviation from the measured data, β_0 and β_1 denote positional and shape parameters. StdError stands for standard error and describes the standard deviation, but from the estimate function. Additionally the tables show for each estimation the 95 % confidence interval.

μ is the estimated average value of cycles, which is dependent on the loading. Strictly speaking, this designation should be logμ, because the relations are from logarithmic nature.

Par.	Estimate	StdError	Low95 %	Up95 %
β_0	79.25810	6.98804	64.83945	93.67675
β_1	-14.17640	1.44835	-17.16484	-11.18796
σ	0.92287	0.14971	0.69226	1.31688

Table 4. Mean estimations for Material#1.

$$\mu = 79.2581 - 14.1764 \cdot log(loading) \qquad (10)$$

Par.	Estimate	StdError	Low95 %	Up95 %
β_0	126.9802	20.30212	82.06733	168.4553
β_1	-22.7520	3.98476	-30.90587	-13.9531
σ	0.6862	0.15914	0.45366	1.1457

Table 5. Mean estimations for Material#2.

$$\mu = 126.9802 - 22.75204 \cdot log(loading) \qquad (11)$$

Par.	Estimate	StdError	Low95 %	Up95 %
β_0	86.41222	14.55656	53.28540	112.4174
β_1	-14.89503	2.94585	-20.13681	-8.1682
σ	1.18943	0.25506	0.80843	1.8933

Table 6. Mean estimations for Material#3.

$$\mu = 86.41222 - 14.89503 \cdot log(loading) \qquad (12)$$

Figures 7, 8 and 9 illustrate the quantile analysis. The quantile is a measure of location in statistics. It represents a threshold. A certain amount of the value is below, the residual amount above this threshold. Using the example of Material#1 the threshold is 21675.91 cycles. That means here that for a loading of 132.5 there is a 50 % probability of failure. The blue dotted lines represent the 95 % confidence interval.

JMP can also perform a custom estimate. For example, the failure probability at the endurance limit is calculated for each material. The results are given in figures 10, 11 and 12. In the case of Material#3 the material can sustain 4.7*10^6 cycles at a loading of 93 and a failure probability of 5 %. At a loading of 93 and cycles of 40^6 the failure probability becomes 4.4 %.

6. Conclusion

Due to the very large scatter of the measurement results, the determination of the most suitable model

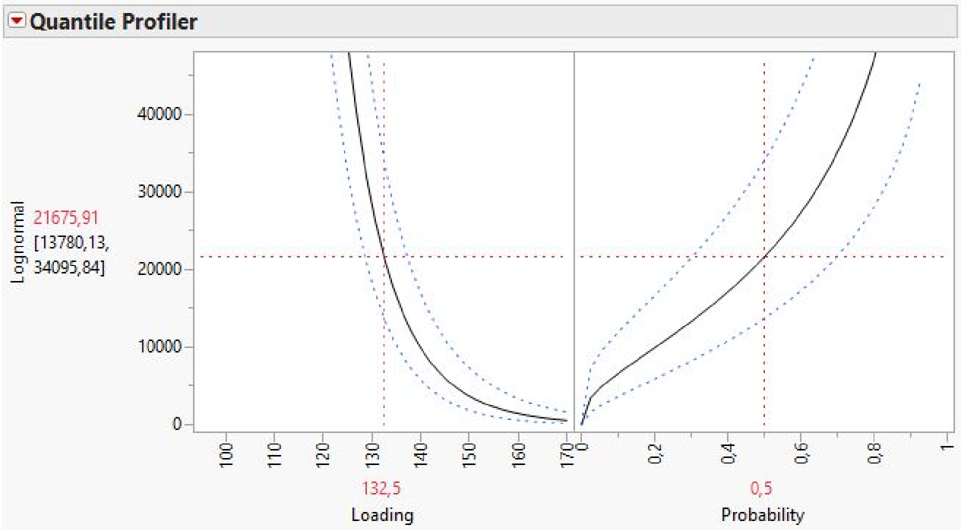


Figure 7. Quantile analysis for Material#1.

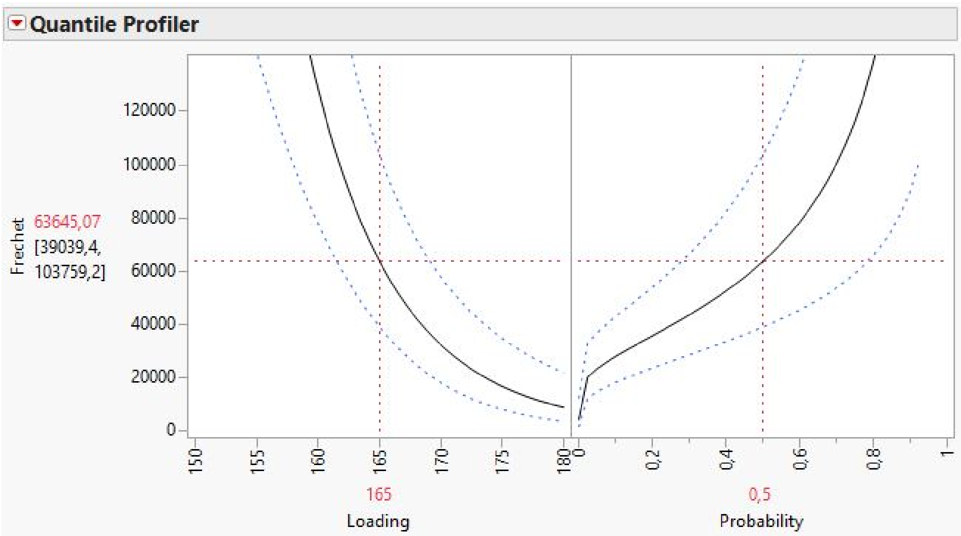


Figure 8. Quantile analysis for Material#2.

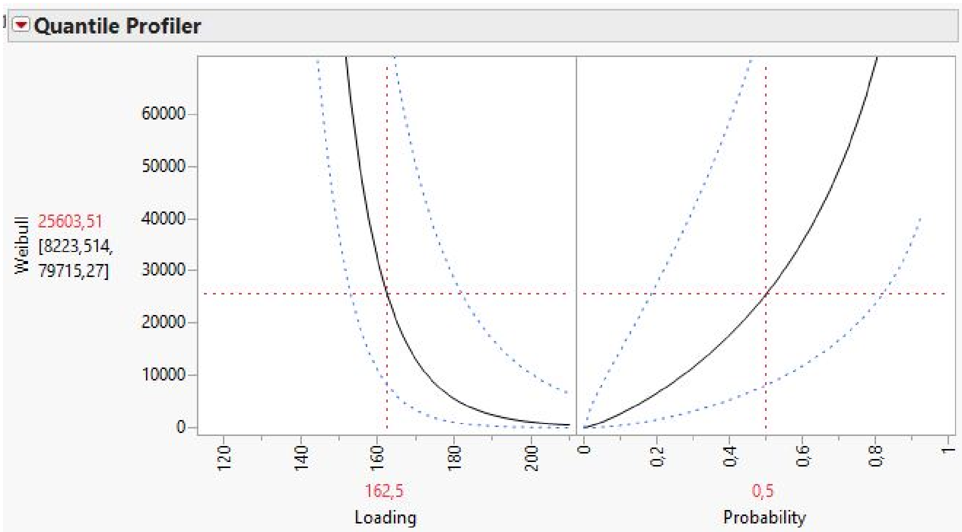
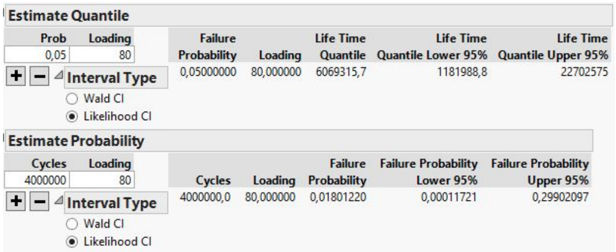


Figure 9. Quantile analysis for Material#3.



Estimate Quantile

Prob	Loading	Failure Probability	Loading	Life Time Quantile	Life Time Quantile Lower 95%	Life Time Quantile Upper 95%
0,05	80	0,05000000	80,000000	6069315,7	1181988,8	22702575

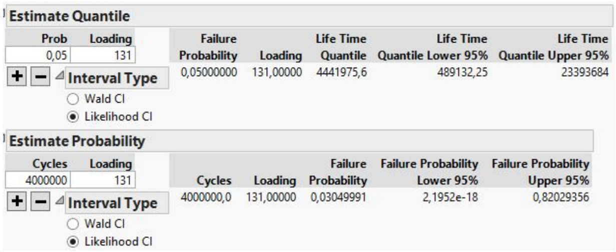
Interval Type: Wald CI / Likelihood CI

Estimate Probability

Cycles	Loading	Cycles	Loading	Failure Probability	Failure Probability Lower 95%	Failure Probability Upper 95%
4000000	80	4000000,0	80,000000	0,01801220	0,00011721	0,29902097

Interval Type: Wald CI / Likelihood CI

Figure 10. Custom estimate for Material#1.



Estimate Quantile

Prob	Loading	Failure Probability	Loading	Life Time Quantile	Life Time Quantile Lower 95%	Life Time Quantile Upper 95%
0,05	131	0,05000000	131,00000	4441975,6	489132,25	23393684

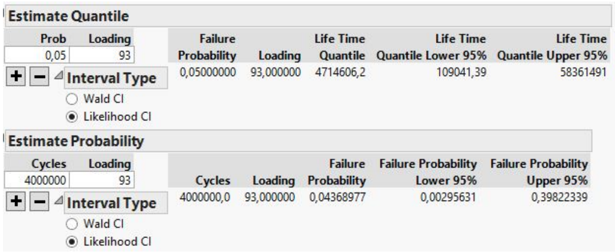
Interval Type: Wald CI / Likelihood CI

Estimate Probability

Cycles	Loading	Cycles	Loading	Failure Probability	Failure Probability Lower 95%	Failure Probability Upper 95%
4000000	131	4000000,0	131,00000	0,03049991	2,1952e-18	0,82029356

Interval Type: Wald CI / Likelihood CI

Figure 11. Custom estimate for Material#2.



Estimate Quantile

Prob	Loading	Failure Probability	Loading	Life Time Quantile	Life Time Quantile Lower 95%	Life Time Quantile Upper 95%
0,05	93	0,05000000	93,000000	4714606,2	109041,39	58361491

Interval Type: Wald CI / Likelihood CI

Estimate Probability

Cycles	Loading	Cycles	Loading	Failure Probability	Failure Probability Lower 95%	Failure Probability Upper 95%
4000000	93	4000000,0	93,000000	0,04368977	0,00295631	0,39822339

Interval Type: Wald CI / Likelihood CI

Figure 12. Custom estimate for Material#3.

to fit the experimental data was a major challenge. In some cases, one gets very unsatisfactory graphical representations of the models. By the application of several fatigue models, at least one suitable model for each material could be found. By the implementation of statistical analysis also the large outliers could be included in the parameter estimation.

JMP is a powerful tool for statistical analysis. Especially, the analysis of the failure probability is a very important feature. The estimate in JMP within the experimental data range leads to slightly different prediction than that of the fatigue models. Extrapolation with JMP out of range is highly dependent on the quality of the measurement data and therefore, does not always leads to plausible results.

List of symbols

A, B, C, D material parameter

n number of cycles

$\Delta\sigma$ loading amplitude

$\Delta\sigma_0$ loading amplitude at endurance limit

$\Delta\sigma_{st}$ loading amplitude at ultimate strength

σ standard deviation

β_0 positional parameter in JMP

β_1 shape parameter in JMP

μ estimation for the mean value

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