Acta Polytechnica **62**(4):488–497, 2022
https://doi.org/10.14311/AP.2022.62.0488
© 2022 The Author(s). Licensed under a CC-BY 4.0 licence
Published by the Czech Technical University in Prague

A COMPARATIVE STUDY OF FERROFLUID LUBRICATION ON DOUBLE-LAYER POROUS SQUEEZE CURVED ANNULAR PLATES WITH SLIP VELOCITY

Niru C. Patel[a], Jimit R. Patel[a,*], Gunamani M. Deheri[b]

[a] *Charotar University of Science and Technology (CHARUSAT), P. D. Patel Institute of Applied Sciences, Department of Mathematical Sciences, CHARUSAT campus, Changa 388 421, Gujarat, India*

[b] *Sardar Patel University, Department of Mathematics, V. V. Nagar 388 120, Anand, Gujarat, India*

[*] corresponding author: patel.jimitphdmarch2013@gmail.com

Abstract.

This article makes an effort to present a comparative study on the performance of a Shliomis model-based ferrofluid (FF) lubrication of a porous squeeze film in curved annular plates taking slip velocity into account. The modified Darcy's law has been adopted to find the impact of the double-layered porosity, while the slip velocity effect has been calculated according to Beavers and Joseph's slip conditions. The modified Reynolds equation for the double-layered bearing system is solved to compute a dimensionless pressure profile and load-bearing capacity (LBC). The graphical results of the study reveal that the LBC increases in the case of magnetization, volume concentration and upper plate's curvature parameter while it decreases with other parameters for both the film thickness profile. A comparative study suggests that the exponential film thickness profile is more suitable to enhance LBC for the annular plates lubricated by ferrofluid, including the presence of a slip. The study shows that the slip model performed quite well and there is a potential for improving the performance efficiency. Besides, multiple methods have been presented to enhance the performance of the above mentioned bearing system by selecting various combinations of parameters governing the system.

Keywords: Shliomis model, curved annular plates, double-layered porous, slip velocity, exponential and hyperbolic film profile, ferrofluid.

1. Introduction

Porous materials seem to be ubiquitous and play a notable role in many aspects of day-to-day life. They are extensively used in various areas, such as energy management, vibration automotive, heat insulation, processes of sound, turbine industries and fluid filtration.

Due to the phenomenon known as self-lubrication, the porous bearing has a porous film filled with some amount of lubricants so that it does not require more lubrication throughout the life period of the bearing. The lubricant comes out of the porous layer and is deposited between the annular plates to inhibit friction and wear, as well as withstanding the original load applied to the annular plates. Therefore, the impact of lubrication due to the double porous layer is better than that of the single porous layer. Due to the remarkable mechanical properties and wide applications of the annular plates, many researchers have been focused on analysing annular bearing systems, such as Lin [1], Shah and Bhat [2], Bujurke et al. [3], Deheri et al. [4], Fatima et al. [5] and Hanumangowda et al. [6].

Also, numerous studies (Ting [7], Gupta et al. [8], Bhat and Deheri [9], Shah et al. [10], Shimpi and Deheri [11], Patel and Deheri [12], Rao and Agarwal [13], Vasanth et al. [14] and Shah et al. [15]) have been carried out to examine the impact of porosity on the effectivity of annular plates.

A synthetic fluid, namely "ferrofluid", is a mixture of colloidal dispersions containing ferromagnetic particles in a liquid carrier. Besides being used in elastic dampers to reduce noise, FFs are used in cooling and heating cycles, long-term sealing of rotating shafts, and reducing unwanted resonances in loudspeakers. In the last four decades, several investigators (Kumar et al. [16], Sinha et al. [17], Shah and Bhat [18], Patel and Deheri [19], Shah and Shah [20] and Munshi et al. [21]) have worked on a FF lubrication theory to examine the behaviour of various bearing systems.

Alternative physical boundary conditions were proposed in the advanced study of Beavers and Joseph [22] that allowed a non-zero tangential velocity (called slip velocity) at the surfaces and uncovered that slip velocity had a broad effect on the bearing performance. Several studies have been documented in the literature about slip velocities for different conditions in bearing systems (Chattopadhyay and Majumdar [23], Shah and Parsania [24], Shah and Patel [25], Venkata et al. [26], Deheri and Patel [27], Patel and Deheri [28], Shah et al. [29], and Mishra et al. [30]).

In their studies, Fragassa et al. [31], Janevski et al. [32] and Geike [33] analysed the theory of static-dynamic load and lubricated contacts, respectively. These investigations confirm that the load profile remains crucial for the bearing design. Patel and Deheri [34] examined the influence of variations of viscosity of the ferrofluid on long bearings. It was noticed that the viscosity variation does not help to increase the LBC in the case of long bearings. Patel and Deheri [35] presented a comparison of a porous structure on shliomis-model-based ferrofluid lubrication of a squeeze film between rotating rough-curved circular plates. It has been ascertained that the Kozeny-Carman model has an edge over the Irmay's model in improving the LBC. A study of thin film lubrication at nanoscale appears in Patel and Deheri [36], where a ferrofluid-based infinitely long rough porous slider bearing has been considered. It has been found that the magnetic fluid induced a higher load and showed a further improvement when the thin film lubrication at nanoscale took place.

Very few studies have been made regarding the ferrofluid lubrication in multi-layered porous plates in the presence of slip velocity. And even lesser amount of studies has been done concerning the comparative studies on the performance of ferrofluid lubricated porous squeeze film in the multi-layered bearing system considering slip velocity. Thus, it was thought proper to put forward a comparative study regarding the performance of a ferrofluid-based squeeze film in two-layered porous annular plates when the slip velocity is taken into account. To what extent can the ferrofluid lubrication counter the adverse effect of porosity and slip velocity? This fundamental question has been addressed while presenting the comparison.

2. Analysis

Figure 1 involves two annular disks (inner and outer radius b and a, respectively ($b < a$)) with curved (exponential and hyperbolic film) upper surface and flat lower surface.

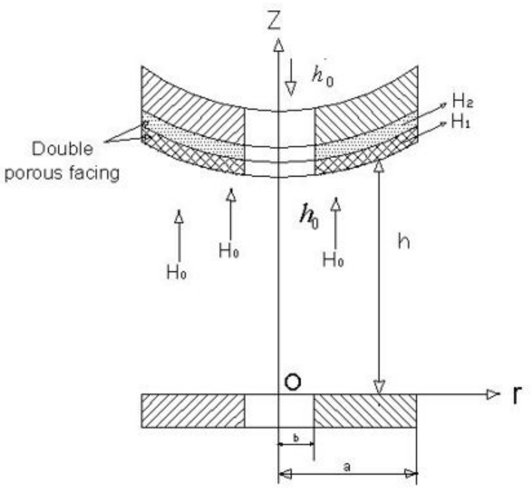

Figure 1. Diagram of the porous annular bearing

In view of Murti [37], Shah and Bhat [2] and Patel and Deheri [38], the thickness profile h of the film is assumed as

$$h(r) = h_0 e^{-\beta r^2},\ b \le r \le a$$

for the exponential and

$$h(r) = \frac{h_0}{1+\beta r},\ b \le r \le a \tag{1}$$

for the hyperbolic profile.

As per the discussions of Shliomis [39] and Kumar [40], and neglecting the assumptions of Shukla and Kumar [41], the governing equations for the flow of FF as suggested by Shliomis [39] are

$$-\nabla p + \eta \nabla^2 \overline{q} + \mu_0 (\overline{M}\cdot\nabla)\overline{H} + \frac{1}{2\tau_s}\nabla\times(\overline{S} - I\overline{\Omega}) = 0 \tag{2}$$

$$\overline{S} = I\overline{\Omega} + \mu_0 \tau_s (\overline{M}\times\overline{H}), \tag{3}$$

$$\overline{M} = M_0 \frac{\overline{H}}{H} + \frac{\tau_B}{I}(\overline{S}\times\overline{M}), \tag{4}$$

with the continuity equation ($\nabla\cdot\overline{q} = 0$), equations of Maxwell $\nabla\times\overline{H} = 0$, $\nabla\cdot(\overline{H}+\overline{M}) = 0$ and $\overline{\Omega} = \frac{1}{2}\nabla\times\overline{q}$ (Bhat [42]).

Above mentioned equation(2) reduces to

$$-\nabla p + \eta\nabla^2\overline{q} + \mu_0(\overline{M}\cdot\nabla)\overline{H} + \frac{1}{2}\mu_0\nabla\times(\overline{M}\times\overline{H}) = 0 \tag{5}$$

and

$$\overline{M} = \frac{M_0}{H}\left[\overline{H} + \overline{\tau}(\overline{\Omega}\times\overline{H})\right],$$

where

$$\overline{\tau} = \frac{\tau_B}{1 + \frac{\mu_0\tau_B\tau_s}{I}M_0 H}$$

with the help of equations (3) and (4), as given in Shliomis [39].

Equation(5) takes the following form, as discussed in Bhat [42] and Patel and Patel [43] with u, $\overline{H} = (0,0,H_0)$ and an axially symmetric flow,

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{\eta(1+\tau)}\frac{\mathrm{d}p}{\mathrm{d}r} \tag{6}$$

where

$$\tau = \frac{3}{2}\phi\frac{\xi - \tan h\xi}{\xi + \tan h\xi}.$$

Because of Beavers and Joseph's [22] slip boundary conditions

$$u(z=0) = 0,$$
$$u(z=h) = -\frac{1}{s}\frac{\partial u}{\partial z},$$
$$\eta_a = \eta(1+\tau),$$

the solution of equation (6) can be transformed to

$$u = -\frac{1}{\eta_a}\frac{z^2 - zsh(z-h)}{2(1+sh)}\frac{\mathrm{d}p}{\mathrm{d}r}. \tag{7}$$

With the help of the above expression (7), one can find the continuity equation $\left(\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\int_0^h ru\,dz + w_h - w_0 = 0\right)$, as

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{h^3 r(2+sh)}{1+sh}\frac{\mathrm{d}p}{\mathrm{d}r}\right) = 12\eta_a(w_h - w_0). \tag{8}$$

Assuming the upper surface having a double layered porous facing and the lower flat surface being solid in the annular plate bearing.

In the present study, P_1 and P_2 of the porous region satisfy the following equations, respectively (Bhat [42]),

$$\begin{aligned}&\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial P_1}{\partial r}\right) + \frac{\partial^2 P_1}{\partial z^2} = 0 \quad \text{and}\\ &\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial P_2}{\partial r}\right) + \frac{\partial^2 P_2}{\partial z^2} = 0.\end{aligned} \tag{9}$$

Using the Morgan-Cameron approximation, one gets

$$\begin{aligned}\left(\frac{\partial P_1}{\partial z}\right)_{z=h_1} &= \frac{H_1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}p}{\mathrm{d}r}\right),\\ \left(\frac{\partial P_2}{\partial z}\right)_{z=h_2} &= \frac{H_2}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}p}{\mathrm{d}r}\right).\end{aligned} \tag{10}$$

Since the lower surface is solid and the upper surface has a double layer porous facing, the velocity component along z-direction is

$$\begin{aligned}w_0 &= 0\\ w_h &= \dot{h}_0 - \left[\frac{k_1}{\eta_a}\left(\frac{\partial P_1}{\partial z}\right)_{z=h_1}\right.\\ &\quad \left. + \frac{k_2}{\eta_a}\left(\frac{\partial P_2}{\partial z}\right)_{z=h_2}\right].\end{aligned} \tag{11}$$

Incorporating equation (9) and (10), equation (11) turns into

$$\begin{aligned}w_0 &= 0\\ w_h &= \dot{h}_0 - \left[\frac{k_1}{\eta_a}\left(\frac{H_1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}p}{\mathrm{d}r}\right)\right)\right.\\ &\quad \left. + \frac{k_2}{\eta_a}\left(\frac{H_2}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}p}{\mathrm{d}r}\right)\right)\right].\end{aligned} \tag{12}$$

Using equation (12), and $\eta_a = \eta_0\left(1+\frac{5}{2}\phi\right)(1+\tau)$, equation (8) yields

$$\begin{aligned}&\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(\left\{\frac{h^3(2+sh)}{1+sh} + 12k_1H_1 + 12k_2H_2\right\}r\frac{\mathrm{d}p}{\mathrm{d}r}\right)\\ &\qquad = 12\eta_0\left(1+\frac{5}{2}\phi\right)(1+\tau)\dot{h}_0.\end{aligned} \tag{13}$$

Upon introduction of the non-dimensional measures:

$$\begin{aligned}R &= \frac{r}{b},\\ \overline{h} &= \frac{h}{h_0},\\ \overline{\beta} &= \beta b^2\,\text{(exponential)},\\ \overline{\beta} &= \beta b\,\text{(hyperbolic)},\\ P &= -\frac{{h_0}^3 p}{\eta_0 b^2 \dot{h}_0},\\ \overline{s} &= sh_0,\\ \psi_1 &= \frac{k_1H_1}{{h_0}^3},\\ \psi_2 &= \frac{k_2H_2}{{h_0}^3},\end{aligned} \tag{14}$$

and using above equation (14), equation (13) transforms to

$$\begin{aligned}&\frac{1}{R}\frac{\mathrm{d}}{\mathrm{d}R}\left\{\left[\frac{\overline{h}^3(2+\overline{s}\overline{h})}{1+\overline{s}\overline{h}} + 12(\psi_1+\psi_2)\right]R\frac{\mathrm{d}P}{\mathrm{d}R}\right\}\\ &\qquad = -12\left(1+\frac{5}{2}\phi\right)(1+\tau).\end{aligned} \tag{15}$$

Considering boundary conditions of annular plates,

$$P(1) = P(k) = 0, \tag{16}$$

one can find the dimensionless P as

$$P = \int_1^R \left(-\frac{6ER}{G}\right)dR + C_1\int_1^R\left(\frac{1}{GR}\right)dR, \tag{17}$$

where

$$\begin{aligned}C_1 &= \frac{\int_1^k\left(\frac{6ER}{G}\right)dR}{\int_1^k\left(\frac{1}{GR}\right)dR},\\ G &= \frac{\overline{h}^3(2+\overline{s}\overline{h})}{1+\overline{s}\overline{h}} + 12(\psi_1+\psi_2),\\ E &= \left(1+\frac{5}{2}\phi\right)(1+\tau),\end{aligned}$$

while the non-dimensional LBC of the annular plates can be found as,

$$\begin{aligned}\overline{W} &= -\frac{{h_0}^3 W}{2\pi\eta_0 b^4\dot{h}_0} = \int_1^k RP\,dR\\ &= -\frac{1}{2}\left(\int_1^k -\frac{6ER^3}{G}\,dR + C_1\int_1^k\frac{R}{G}\,dR\right)\end{aligned} \tag{18}$$

3. Result and discussion

The results for the double-layered porous medium and slip velocity on exponential and hyperbolic film profiles of the annular bearing are discussed in this section. Equation (17) establishes the non-dimensional pressure, while equation (18) represents a dimensionless LBC. In addition, expression (18) is linear in terms of the magnetization parameter, which indicates an improvement of the LBC of annular plates mathematically.

As far as LBC is concerned, a comparison of the film profile is exhibited graphically in Figures 2–13 with double-porous facing in the occurrence of a slip. The first figure demonstrates the exponential film shape and the second figure depicts the impact of the hyperbolic profile. The performance of appearance of the Shliomis' FF lubricated double porous medium annular plates is based on the foundation of several non-dimensional parameters like magnetization, upper plate's curvature, porosity, volume concentration and slip velocity. It can be observed that the exponential film fares better.

It is noticed that equation (18) suggests the LBC of a single layer porous medium when $\psi_2 \to 0$. With $\psi_1 \to 0$ and $\psi_2 \to 0$, this investigation transfers to the non-porous FF based annular bearing system with the slip velocity. Also, this study reduces to a study of a traditional annular bearing by removing the effect of magnetization in the absence of the slip.

For the range of the parameters, one can refer below: τ: 0.1–0.5, ϕ: 0.01–0.05, $\overline{\beta}$: 1.5–1.9, $1/\overline{s}$: 0.02–0.1, ψ_1: 0.001–0.005 and ψ_2: 0.001–0.005.

The dispensation of LBC about the τ, for numerous values of ϕ, $\overline{\beta}$, ψ_1, ψ_2 and $1/\overline{s}$ shown in Figures 2–5, recommends that the LBC rises strictly due to the FF lubricant. A closer examination of the figures emphasizes that the functionality of bearing systems as well as the increase in load is connected with all the parameters for both the film profiles. Exponential film profile registers a higher load as compared to the hyperbolic shape in Figures 2–5.

The behaviour of the volume concentration parameter concerning various parameters of LBC is illustrated in Figures 6–8, respectively. Due to the rise of the volume concentration parameter, the effect of LBC decreases with porosity (in Figure 7) and slip velocity (in Figure 8), while the effect of $\overline{\beta}$ (in Figure 6) increases the LBC. Moreover, Figure 8 suggests a marginally improved effect of the slip velocity in an exponential film bearing, which indicates an enhancement of the overall annular bearing's performance up to some extent.

The profile of a non-dimensional LBC with respect to the $\overline{\beta}$ is described in Figures 9–11. If we increase the $\overline{\beta}$, then the capacity of the load is growing sharply in the case of the hyperbolic function, while a reverse behaviour is observed with porosity and slip velocity. However, the LBC increases slightly for the exponential profile and follows the same trends for the parameters mentioned above. One can visualize an identical scenario for the curvature of exponential and hyperbolic functions, which is shown in Figures 9–11.

The effect of the porosity on the load distribution of the bearing is shown in Figures 12 and 13. In Figure 12, the effect of slip velocity is negligible for the exponential profile. However, Figure 13 suggests that the trends of both the porosity parameters are almost the same. Both layers help to improve the lifespan of the system by creating a film layer between the surfaces.

The graphical representation makes it clear that the following takes place.

(1.) The nature of both porous facings is almost the same with the same values ($\psi_1 = \psi_2$), however, that does not apply when the porosity values differ. (meaning $\psi_1 > \psi_2$ or $\psi_1 < \psi_2$).

(2.) Figure 13 displays the maximum load among all the figures, which means the double porous layer improves the LBC in annular plates bearing systems.

(3.) Higher values of curvature parameter have a negligible effect on the LBC in the case of the exponential profile, but it does reflect on the LBC in the case of the hyperbolic film profile.

(4.) The effect of slip velocity is satisfactory for the exponential surface profile as compared to the hyperbolic surface.

(5.) Finally, this study helps to improve the LBC by considering the proper selection of all parameters and film shapes while designing the bearing system of annular plates.

4. Conclusion

The effect of the double-porous layered on MF lubricated curved annular bearing is investigated theoretically with the theory of Shliomis' flow model of FF, modified Darcy's law for double layer, and Beavers and Joseph's more realistic slip conditions. In view of the bearing's life period, it is evident that some of the parameters appear to have an opposite effect on the performance of the bearing system. Hence, this investigation clarifies that while designing the bearing system, the porosity in two layers, and the slip velocity must be considered. Interestingly, numerous factors (like porosity and slip velocity) disturb the system adversely even though the bearing can support a load without flow, this does not apply in the case of traditional lubricants. Even the upper plate's curvature, either exponential or hyperbolic, may significantly impact the performance of this bearing system, considering the moderate values of volume concentration, slip velocity, and porosity. Lastly, the exponential film profile exhibits a higher load-bearing capacity, in the case of the double-layered porosity with Shliomis' magnetic fluid flow when slip is in place. A pertinent question is to elevate this analysis by incorporating the effect of surface roughness and deformation. An immediate concern is to carry out this analysis for some other types of bearing systems including the circular ones with slip velocity.

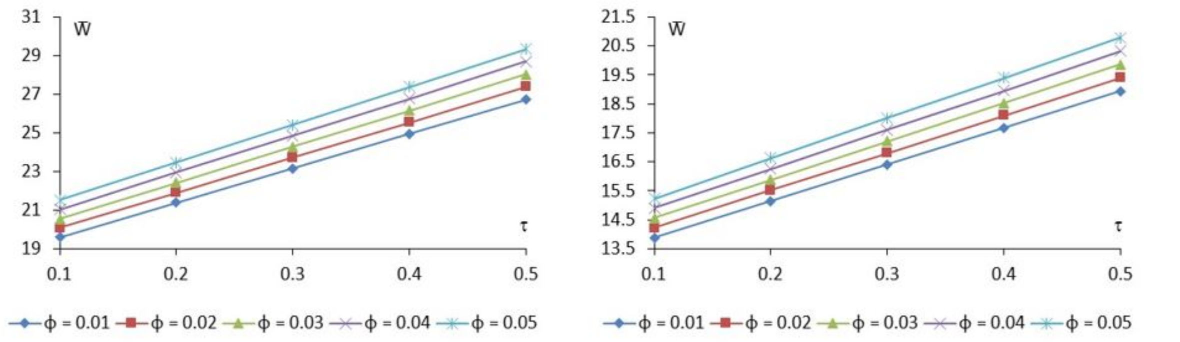

Figure 2. Variation of LBC as regards of τ and ϕ.

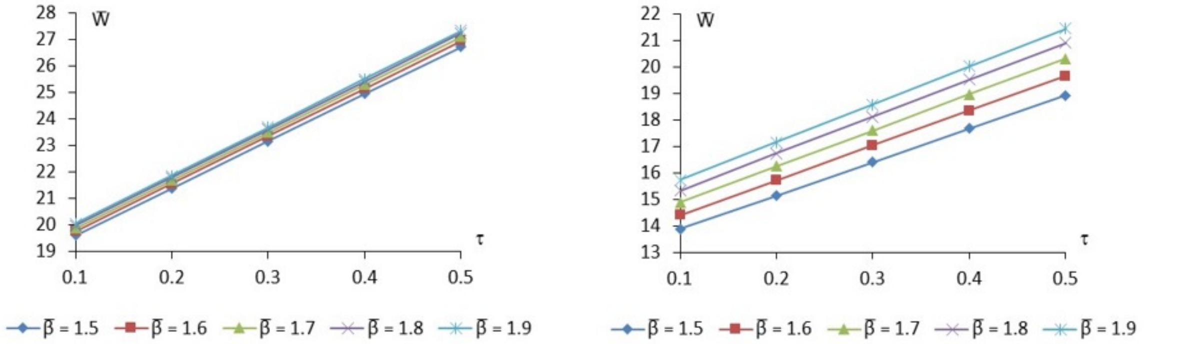

Figure 3. Variation of LBC as regards of τ and $\overline{\beta}$.

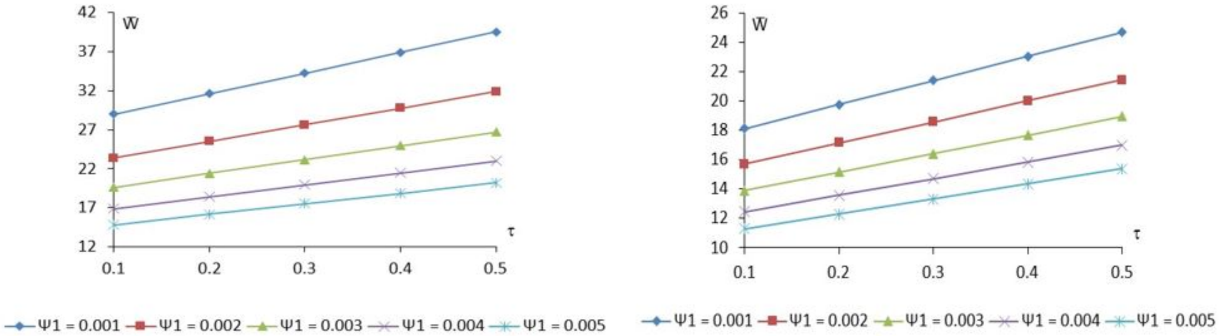

Figure 4. Variation of LBC as regards of τ and ψ_1.

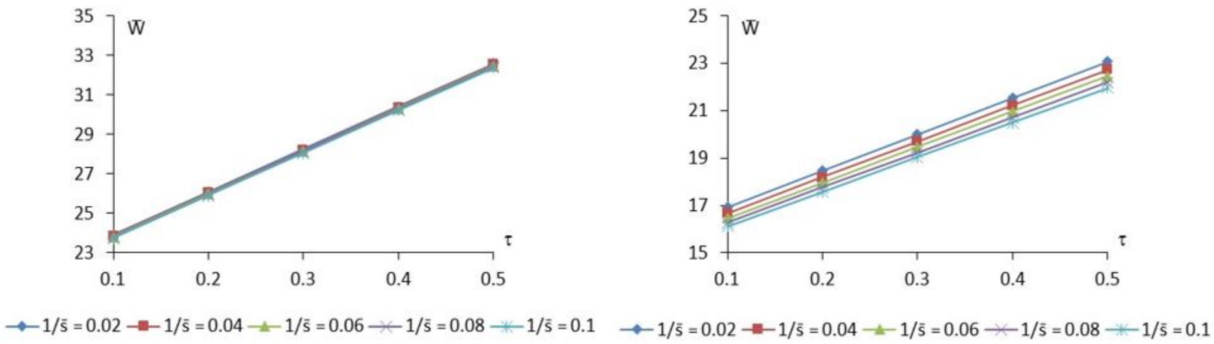

Figure 5. Variation of LBC as regards of τ and $1/\overline{s}$.

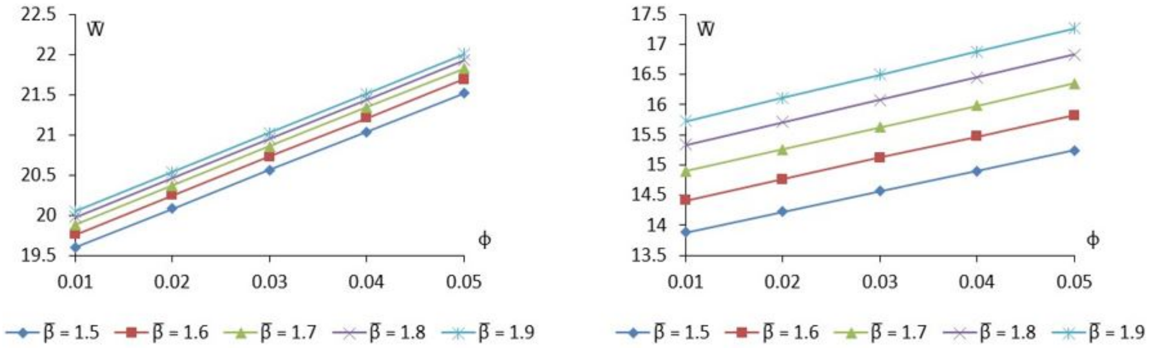

FIGURE 6. Variation of LBC as regards of ϕ and $\overline{\beta}$.

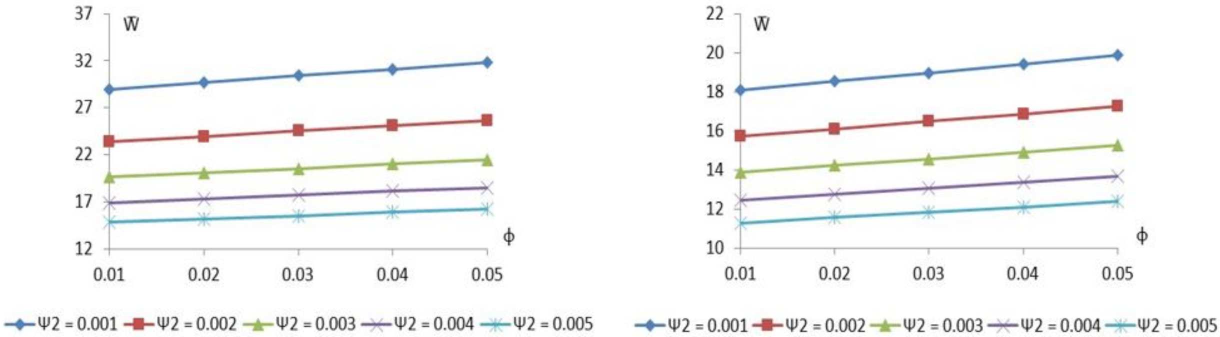

FIGURE 7. Variation of LBC as regards of ϕ and ψ_2.

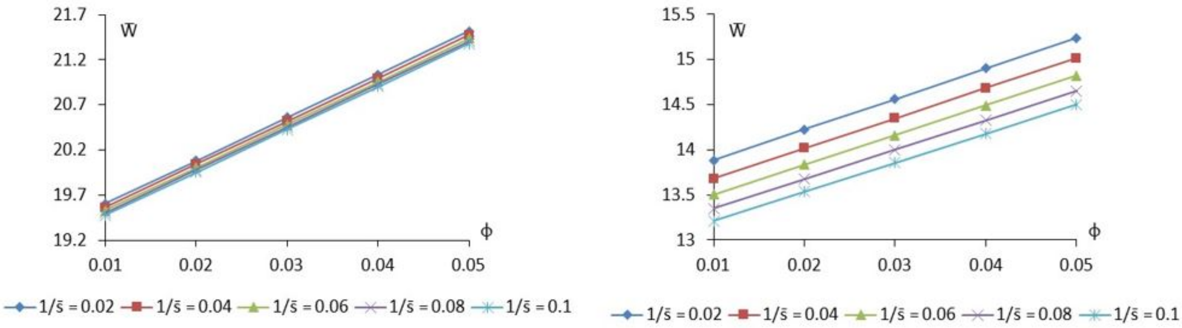

FIGURE 8. Variation of LBC as regards of ϕ and $1/\overline{s}$.

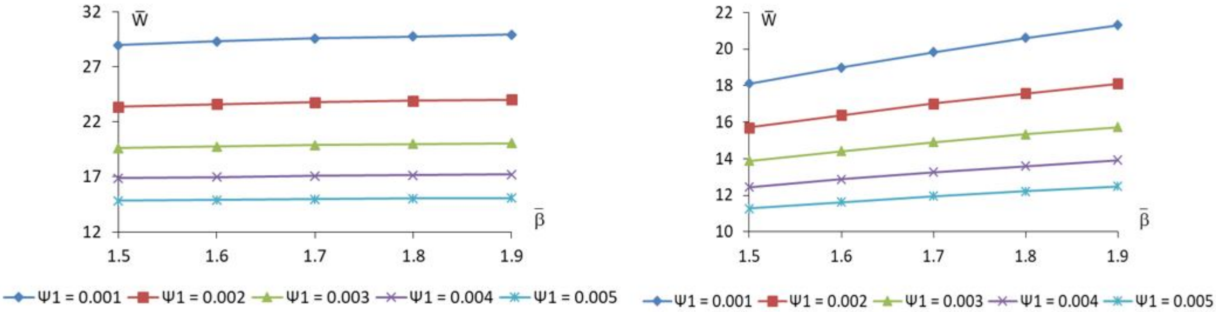

FIGURE 9. Variation of LBC as regards of $\overline{\beta}$ and ψ_1.

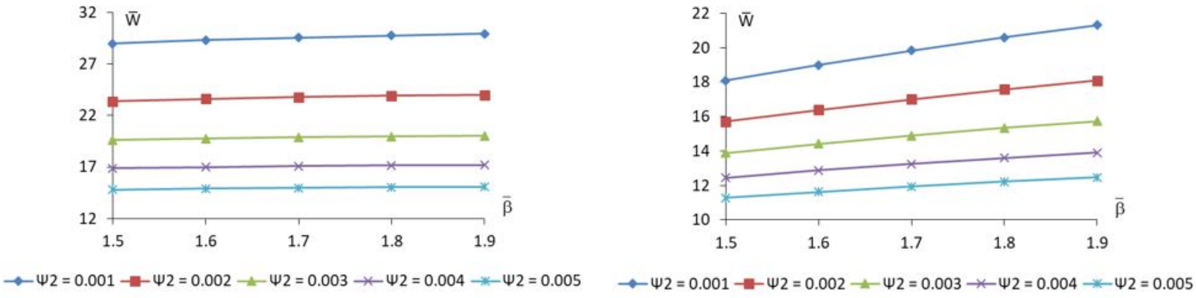

Figure 10. Variation of LBC as regards of $\overline{\beta}$ and ψ_2.

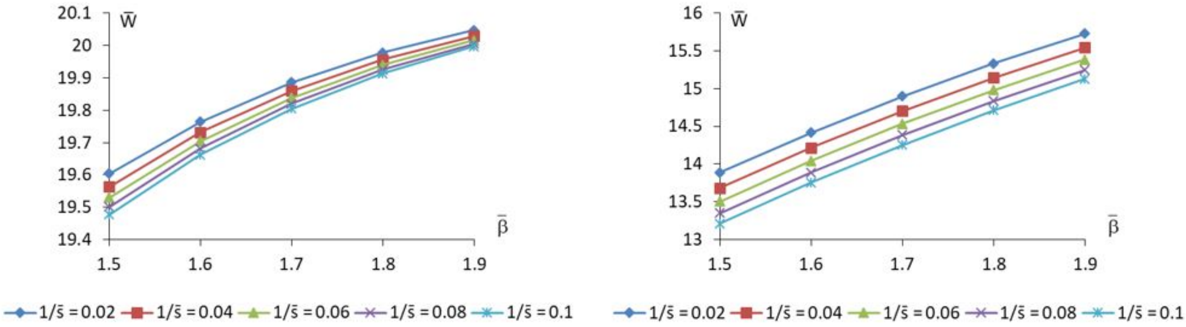

Figure 11. Variation of LBC as regards of $\overline{\beta}$ and $1/\overline{s}$.

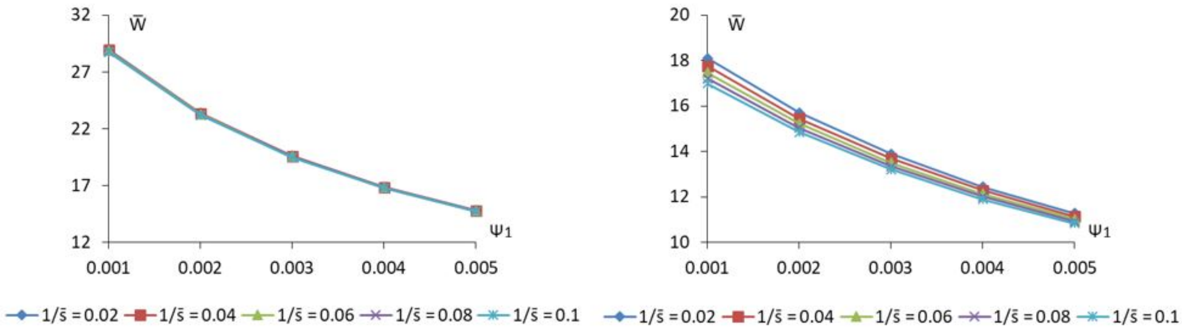

Figure 12. Variation of LBC as regards of ψ_1 and $1/\overline{s}$.

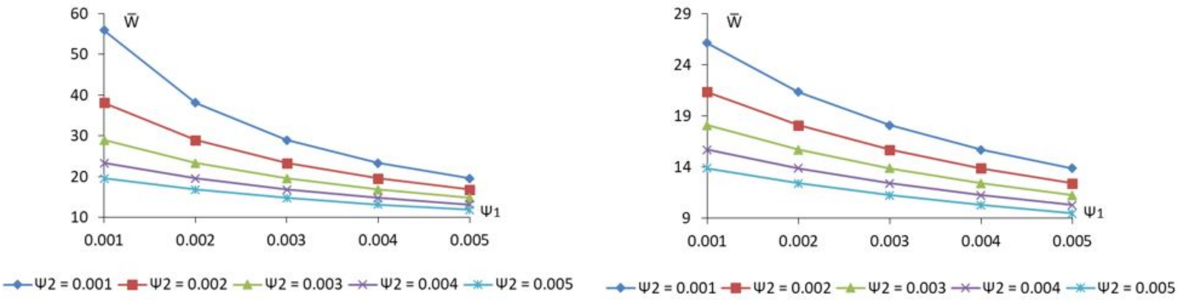

Figure 13. Variation of LBC as regards of ψ_1 and ψ_2.

List of symbols

a Outer radius of annular plates
b Inner radius of annular plates
h Film thickness
r Radial coordinates
p Pressure of fluid
u x – component of $\overline{q}$
w z – component of $\overline{q}$
H Magnitude of $\overline{H}$
I A sum of moments of inertia of the particles per unit volume
$\overline{s}$ Slip velocity
$\overline{q}$ Fluid velocity in the film region
$\overline{M}$ Magnetization vector
$\overline{H}$ An identical magnetic field
$\overline{S}$ Internal angular momentum
h_0 Central film thickness
w_0, w_h Values of w at $z = 0$, h respectively
h_1 Thickness of lubricant in the inner layer
h_2 Thickness of lubricant in the outer layer
k_1 Permeability of inner layer of the porous region
k_2 Permeability of outer layer of the porous region
M_0 Equilibrium magnetization
H_0 Constant magnetic field
H_1 Thickness of the inner layer adjacent to lubricant layer
H_2 Thickness of the outer layer adjacent to solid wall
P_1 Pressure of inside layer in the porous region
P_2 Pressure of outside layer in the porous region
η Viscosity of suspension
β Curvature of the upper plate
ξ Langevin's parameter
τ Magnetization parameter
ϕ Volume concentration
τ_B Brownian relaxation time parameter
τ_S Relaxation time parameter
μ_0 Permeability of free space
η_0 Carrier fluid viscosity
ψ_1 Inner layer porous structure parameter
ψ_2 Outer layer porous structure parameter

References

[1] J. Lin. Magneto-hydrodynamic squeeze film characteristics between annular disks. *Industrial Lubrication and Tribology* **53**(2):66–71, 2001. https://doi.org/10.1108/00368790110384028.

[2] R. Shah, M. Bhat. Ferrofluid squeeze film between curved annular plates including rotation of magnetic particles. *Journal of Engineering Mathematics* **51**(4):317–324, 2005. https://doi.org/10.1007/s10665-004-1770-9.

[3] N. Bujurke, N. Naduvinamani, D. Basti. Effect of surface roughness on the squeeze film lubrication between curved annular plates. *Industrial Lubrication and Tribology* **59**(4):178–185, 2007. https://doi.org/10.1108/00368790710753572.

[4] G. Deheri, R. Patel, N. Abhangi. Magnetic fluid-based squeeze film behavior between transversely rough curved annular plates: A comparative study. *Industrial Lubrication and Tribology* **63**(4):254–270, 2011. https://doi.org/10.1108/00368791111140477.

[5] S. Fatima, T. Biradar, B. Hanumagowda. Magneto-hydrodynamics couple stress squeeze film lubrication of rough annular plates. *International Journal of Current Research* **9**(9):58007–58014, 2017.

[6] G. Hanumagowda, B.and Savitramma, A. Salma, Noorjahan. Combined effect of piezo-viscous dependency and non-newtonian couple stresses in annular plates squeeze-film characteristics. *Journal of Physics: Conference Series* **1000**(1):1–8, 2018. https://doi.org/10.1088/1742-6596/1000/1/012083.

[7] L. Ting. Engagement behavior of lubricated porous annular disks. part i: Squeeze film phase - surface roughness and elastic deformation effects. *Wear* **34**(2):159–172, 1975. https://doi.org/10.1016/0043-1648(75)90062-9.

[8] J. Gupta, K. Vora, M. Bhat. The effect of rotational inertia on the squeeze film load between porous annular curved plates. *Wear* **79**:235–240, 1982. https://doi.org/10.1016/0043-1648(82)90171-5.

[9] M. Bhat, G. Deheri. Squeeze film behaviour in porous annular discs lubricated with magnetic fluid. *Wear* **151**(1):123–128, 1991. https://doi.org/10.1016/0043-1648(91)90352-u.

[10] R. Shah, S. Tripathi, M. Bhat. Magnetic fluid based squeeze film between porous annular curved plates with the effect of rotational inertia. *Pramana - Journal of Physics* **58**(3):545–550, 2002. https://doi.org/10.1007/s12043-002-0064-x.

[11] M. Shimpi, G. Deheri. A study on the performance of a magnetic fluid based squeeze film in curved porous rotating rough annular plates and deformation effect. *Tribology International* **47**:90–99, 2012. https://doi.org/10.1016/j.triboint.2011.10.015.

[12] J. Patel, G. Deheri. Theoretical study of shliomis model based magnetic squeeze film in rough curved annular plates with assorted porous structures. *FME Transactions* **42**(1):56–66, 2014. https://doi.org/10.5937/fmet1401056p.

[13] P. Rao, S. Agarwal. Couple stress fluid-based squeeze film between porous annular curved plates with the effect of rotational inertia. *Iranian Journal of Science and Technology, Transactions A: Science* **41**(4):1171–1175, 2017. https://doi.org/10.1007/s40995-017-0295-9.

[14] K. Vasanth, J. Hanumagowda, J. Santhosh Kumar. Combined effect of piezoviscous dependency and non-newtonian couple stress on squeeze-film porous annular plate. *Journal of Physics: Conference Series* **1000**(1):1–8, 2018. https://doi.org/10.1088/1742-6596/1000/1/012080.

[15] R. Shah, D. Patel, D. Patel. Ferrofluid-based annular squeeze film bearing with the effects of roughness and micromodel patterns of porous structures. *Tribology - Materials, Surfaces & Interfaces* **12**(4):208–222, 2018. https://doi.org/10.1080/17515831.2018.1542192.

[16] P. Kumar, D.and Sinha, P. Chandra. Ferrofluid squeeze film for spherical and conical bearings. *International Journal of Engineering Science* **30**(5):645–656, 1992. https://doi.org/10.1016/0020-7225(92)90008-5.

[17] P. Sinha, P. Chandra, D. Kumar. Ferrofluid lubrication of cylindrical rollers with cavitation. *Acta Mechanica* **98**:27–38, 1993. https://doi.org/10.1007/bf01174291.

[18] R. Shah, M. Bhat. Ferrofluid squeeze film in a long journal bearing. *Tribology International* **37**:441–446, 2004. https://doi.org/10.1016/j.triboint.2003.10.007.

[19] J. Patel, G. Deheri. Performance of a ferrofluid based rough parallel plate slider bearing: A comparison of three magnetic fluid flow models. *Advances in Tribology* **2016**:1–9, 2016. https://doi.org/10.1155/2016/8197160.

[20] R. Shah, R. Shah. Derivation of ferrofluid lubrication equation for slider bearings with variable magnetic field and rotations of the carrier liquid as well as magnetic particles. *Meccanica* **53**(4-5):857–869, 2017. https://doi.org/10.1007/s11012-017-0788-9.

[21] M. Munshi, A. Patel, G. Deheri. Lubrication of rough short bearing on shliomis model by ferrofluid considering viscosity variation effect. *International Journal of Mathematical, Engineering and Management Sciences* **4**(4):982–997, 2019. https://doi.org/10.33889/ijmems.2019.4.4-078.

[22] G. Beavers, D. Joseph. Boundary conditions at a natural permeable wall. *Journal of Fluid Mechanics* **30**(1):197–207, 1967. https://doi.org/10.1017/s0022112067001375.

[23] A. Chattopadhyay, B. Majumdar. Steady state solution of finite hydrostatic porous oil journal bearings with tangential velocity slip. *Tribology International* **17**(6):317–323, 1984. https://doi.org/10.1016/0301-679x(84)90095-1.

[24] R. Shah, M. Parsania. Ferrofluid lubrication equation for non-isotropic porous squeeze film bearing with slip velocity. *Mathematics today* **28**(2):43–49, 2012.

[25] R. Shah, D. Patel. Squeeze film based on ferrofluid in curved porous circular plates with various porous structure. *Applied Mathematics* **2**(4):121–123, 2012. https://doi.org/10.5923/j.am.20120204.04.

[26] J. Venkata, R. Murthy, M. Kumar. Effect of slip parameter on the flow of viscous fluid past an impervious sphere. *International Journal of Applied Science and Engineering* **12**(3):203–223, 2014. https://doi.org/10.6703/IJASE.2014.12(3).203.

[27] G. Deheri, S. Patel. Combined effect of slip velocity and surface roughness on a magnetic squeeze film for a sphere in a spherical seat. *Indian Journal of Materials Science* **2015**:1–9, 2015. https://doi.org/10.1155/2015/159698.

[28] J. Patel, G. Deheri. Numerical modeling of jenkins model based ferrofluid lubrication squeeze film performance in rough curved annular plates under the presence of slip velocity. *Facta Universitatis, Series: Mathematics and Informatics* **31**(1):11–31, 2016.

[29] N. Shah, R.and Patel, R. Kataria. Some porous squeeze film-bearings using ferrofluid lubricant: A review with contributions. *Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology* **230**(9):1157–1171, 2016. https://doi.org/10.1177/1350650116629096.

[30] M. Mishra, S.and Barik, G. Dash. An analysis of hydrodynamic ferrofluid lubrication of an inclined rough slider bearing. *Tribology - Materials, Surfaces & Interfaces* **12**(1):17–26, 2018. https://doi.org/10.1080/17515831.2017.1418280.

[31] C. Fragassa, G. Minak, A. Pavlovic. Measuring deformations in the telescopic boom under static and dynamic load conditions. *Facta Universitatis, Series: Mechanical Engineering* **18**(2):315–328, 2020. https://doi.org/10.22190/FUME181201001F.

[32] G. Janevski, P. Kozic, A. Pavlovic, S. Posavljak. Moment lyapunov exponents and stochastic stability of a thin-walled beam subjected to axial loads and end moments. *Facta Universitatis, Series: Mechanical Engineering* **19**(2):209–228, 2021. https://doi.org/10.22190/FUME191127014J.

[33] T. Geike. Bubble dynamics-based modeling of the cavitation dynamics in lubricated contacts. *Facta Universitatis, Series: Mechanical Engineering* **19**(1):115–124, 2021. https://doi.org/10.22190/FUME210112027G.

[34] J. Patel, G. Deheri. Influence of viscosity variation on ferrofluid based long bearing. *Reports in Mechanical Engineering* **3**(1):37–45, 2022. https://doi.org/10.31181/rme200103037j.

[35] J. Patel, G. Deheri. Effect of various porous structures on the shliomis model based ferrofluid lubrication of the film squeezed between rotating rough curved circular plates. *Facta Universitatis, Series: Mechanical Engineering* **12**(3):305 – 323, 2014.

[36] J. Patel, G. Deheri. A study of thin film lubrication at nanoscale for a ferrofluid based infinitely long rough porous slider bearing. *Facta Universitatis, Series: Mechanical Engineering* **14**:89–99, 2016. https://doi.org/10.22190/FUME1601089P.

[37] P. Murti. Squeeze films in curved porous circular plates. *Journal of Lubrication Technology* **97**(4):650–652, 1975. https://doi.org/10.1115/1.3452699.

[38] J. Patel, G. Deheri. Jenkins model based magnetic squeeze film in curved rough circular plates considering slip velocity: A comparison of shapes. *FME Transactions* **43**(2):144–153, 2015. https://doi.org/10.5937/fmet1502144p.

[39] M. Shliomis. Effective viscosity of magnetic suspensions. *Soviet Physics – JETP* **34**(6):1291–1294, 1972.

[40] D. Kumar. *Lubrication with a Magnetic fluid.* Ph.D. thesis, IIT Kanpur, 1991.

[41] J. Shukla, D. Kumar. A theory for ferromagnetic lubrication. *Journal of Magnetism and Magnetic Materials* **65**(2-3):375–378, 1987. https://doi.org/10.1016/0304-8853(87)90075-8.

[42] M. Bhat. *Lubrication with a Magnetic fluid.* Team Spirit (India) Pvt. Ltd., India, 2003.

[43] N. Patel, J. Patel. Magnetic fluid-based squeeze film between curved porous annular plates considering the rotation of magnetic particles and slip velocity. *Journal of Serbian Society for Computational Mechanics* **14**(2):69–82, 2020. https://doi.org/10.24874/jsscm.2020.14.02.05.