

Preliminary Determination of Propeller Aerodynamic Characteristics for Small Aeroplanes

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This paper deals with preliminary determination of propeller thrust and power coefficients depending on the advance ratio by means of some representative geometric parameters of the blade at a specific radius: propeller blade chord and blade angle setting at 70 % of the top radius, airfoil thickness at the radius near the tip and the position of the maximum blade width. A rough estimation of the non-linear influence of propeller blades number is included.

The published method is based on Lock's model of the characteristic section and the Bull-Bennett lift and drag propeller blade curves. Lock's integral decomposition factors and the loss factor were modified by the evolution of the experimental propeller characteristics. The numerical-obtained factors were smoothed and expressed in the form of analytical functions depending on the geometric propeller blade parameters and the advance ratio.

Keywords: propeller, propeller aerodynamics, thrust coefficient, power coefficient, propeller efficiency, propeller design.

1 Introduction

One of the main tasks during the introduction stage of aeroplane design is to determine the basic aeroplane performance. One of the input is the thrust curve of the power plant – available thrust versus flight velocity. The optimisation procedure requires combinations of suitable engines and propellers offered on the market to compare different thrust curves and, consequently, aircraft performance. Designers of small sport aircraft very often have only the shape and the number of propeller blades without any aerodynamic characteristics.

It is evident that very sophisticated and precise numerical methods (helix vortex surfaces or sophisticated solutions by means of FEMs of the real flow around the rotating lift surfaces) require large input data files. These conclusions have led the author to present an easy and sufficiently precise procedure for calculating the integral propeller aerodynamic characteristics with minimum demands on geometric and aerodynamic propeller input data. An inspection of various aerodynamic propeller theories indicated that a suitable method can be gained by enhancing Lock's model of the

referential section connected with Bull-Bennett mean lift and drag propeller blade curves.

2 Lock's propeller model of the referential section

Lock's model [1] considers a referential section on a propeller blade located at 70 % of the tip radius to be representative of the total aerodynamic forces acting on the blade (thrust T_{bl} and tangential force Q_{bl} or lift L_{bl} and drag D_{bl}) – see Fig. 1. It is assumed that these forces are configured according to the local relative wind W determined by the incoming flow W_0 at this section (composed of tangential speed U and flight speed V) and induced speed v_i . The induced speed is related to lifting line theory.

The propeller blade lift and drag expressed by means of the lift and drag coefficient:

$$L_{bl} = \frac{1}{2} \rho W^2 c_l(\alpha) b_{0.7} r_{0.7} \quad (1)$$

$$D_{bl} = \frac{1}{2} \rho W^2 c_d(\alpha) b_{0.7} r_{0.7} \quad (2)$$

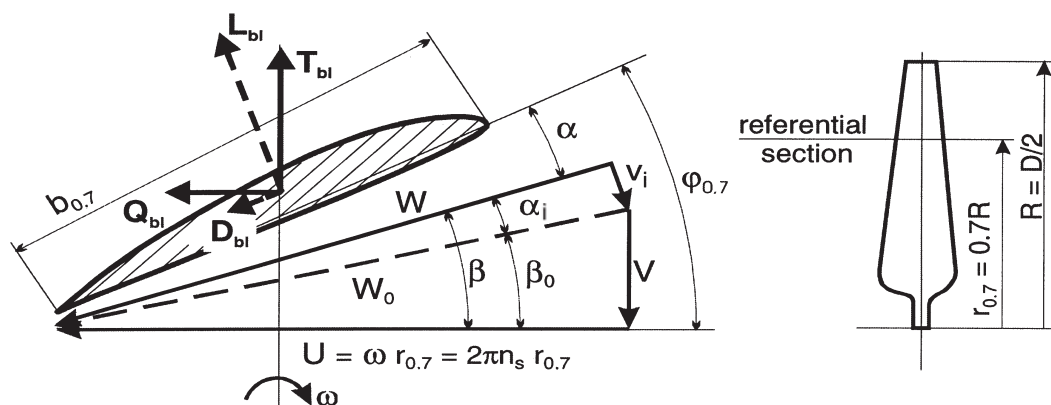


Fig. 1: Lock's scheme of the referential blade section

enables us to write the total thrust of a propeller with z blades as:

$$T = zT_{bl} = z(L_{bl} \cos(\beta) - D_{bl} \sin(\beta)) \quad (3)$$

in the form:

$$T = \frac{1}{2} \rho W^2 b_{0.7} r_{0.7} (c_l \cos(\beta) - c_d \sin(\beta)) \quad (4)$$

Substitution of apparent relations from Fig. 1 for the resultant velocities W :

$$W = W_0 \cos(\alpha_i) = \sqrt{V^2 + (2\pi n_s r_{0.7})^2} \cos(\alpha_i) = (2n_s R)^2 \sqrt{\lambda^2 + (\pi \bar{r}_{0.7})^2} \cos(\alpha_i), \quad (5)$$

and the angle of the real incoming flow β :

$$\beta = \varphi_{0.7} - \alpha = \varphi_{0.7} - \beta_0 - \alpha_i = \varphi_{0.7} - \arctg \frac{\lambda}{\pi \bar{r}_{0.7}} - \alpha_i \quad (6)$$

(λ is the advanced ratio: $\lambda = V/n_s D$ and $\varphi_{0.7}$ is the blade angle setting) into the expression for the thrust (4) and the use of non dimensional geometry ($\bar{b}_{0.7} = b_{0.7}/R$, $\bar{r}_{0.7} = r_{0.7}/R = 0.7$) the expression of the propeller thrust coefficient:

$$c_T = \frac{T}{\rho n_s^2 D^4} \quad (7)$$

is achieved in the final form:

$$c_T = z \frac{1}{8} \bar{b}_{0.7} (\lambda^2 + \pi \bar{r}_{0.7}) \bar{r}_{0.7} \cos^2 \left(\varphi_{0.7} - \arctg \frac{\lambda}{\pi \bar{r}_{0.7}} \right) \times [c_l \cos(\varphi_{0.7} - \alpha) - c_d \sin(\varphi_{0.7} - \alpha)]. \quad (8)$$

Calculation of the thrust coefficient at given advance ratios requires not only the lift $c_l(\alpha)$ and drag $c_d(\alpha)$ blade curves but also the relationship to determine angle of attack α . Lock [1] developed the induced equation as the dependence of the lift coefficient on the tip loss factor χ (function of advance ratio λ and angle β of the real incoming flow):

$$s_{0.7} c_l = 4\chi \sin(\beta) \text{tg}(\alpha_i) \quad (9)$$

where $s_{0.7}$ is the propeller solidity factor related to the referential section:

$$s_{0.7} = \frac{z \bar{b}_{0.7}}{2\pi \bar{r}_{0.7}}, \quad (\bar{r}_{0.7} = 0.7). \quad (10)$$

The relations among thrust T_{bl} and tangential force Q_{bl} acting on the propeller blade and the equivalent blade lift L_{bl} and drag D_{bl} forces were designed by Lock [1] in a de-

composition form based on the angle of the real incoming flow β at the referential section. This resolution is corrected by integration factors E and F :

$$c_l s_{0.7} = E c_T \cos(\beta) + F c_M \sin(\beta) \quad (11)$$

$$c_d s_{0.7} = F c_M \cos(\beta) + E c_T \sin(\beta). \quad (12)$$

Torque coefficient c_M represents the tangential force due to

$$c_M = \frac{M_k}{\rho n_s^2 D^5} = \frac{z Q r_{0.7} \omega}{\rho n_s^2 D^5}. \quad (13)$$

The introducing of a propeller power P and power coefficient c_P :

$$c_P = \frac{P}{\rho n_s^3 D^5} \quad (14)$$

provides a constant relation between the power and torque coefficient: $c_P = 2\pi c_M$. Integration factors E and F were developed by Lock in dependence on the advance ratio:

$$E = \frac{3.276}{4.336 + \lambda^2} \quad (15)$$

$$F = \frac{2E}{\bar{r}_{0.7}}, \quad (\bar{r}_{0.7} = 0.7). \quad (16)$$

Decomposition equations (11) and (12) can be used for an inverse procedure to calculate the thrust and power coefficient by means of the integration factors and blade lift and drag curves:

$$c_T = \frac{s_{0.7}}{E \cos(\beta)} \left(c_l - \text{tg}(\beta) \frac{c_d + c_l \text{tg}(\beta)}{1 + \text{tg}^2(\beta)} \right) \quad (17)$$

$$c_P = 2\pi c_M = 2\pi \frac{s_{0.7} (c_d + c_l \text{tg}(\beta))}{F \cos(\beta) (1 + \text{tg}^2(\beta))}. \quad (18)$$

3 Lift and drag of the propeller blade

Bull and Bennett [2] published the lift and drag propeller blade curve gained by an applying Lock's scheme (11) and (12) to a set of experimental propeller aerodynamic characteristics. To calculate the induced values the set of Lock's induced equation (9) and the decomposition relation (11) for the lift was used. The results of the calculations were presented in [2] and are shown in Fig. 2. These curves represent different RAF-6 section propellers covering a wide range of

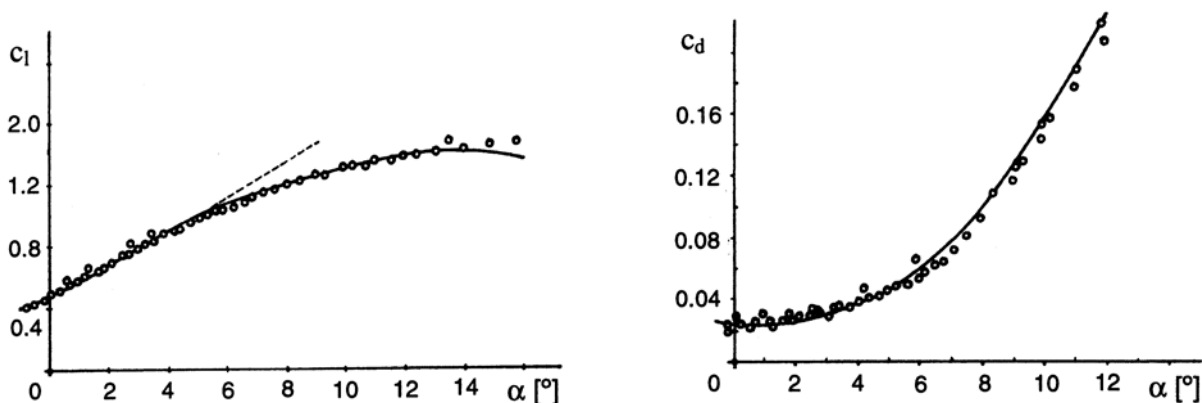


Fig. 2: Propeller blade lift and drag curve

setting angles and advance ratios. The tip Mach number never reached 0.7.

The mean values of the lift and drag blade curve are described by simple linear and quadratic forms – [2]:

$$c_l = 0.4996 + 0.1096\alpha \quad \dots \quad \alpha < 4.98^\circ \quad (19)$$

$$c_l = 0.9867 - 0.0001\alpha + 0.0024\alpha^2 \quad \dots \quad \alpha \geq 4.98^\circ$$

$$c_d = 0.0258 - 0.00318\alpha + 0.00173\alpha^2 \quad (20)$$

4 Modification of Lock's method

There are at least three reasons for improving Lock's procedure to obtain a more effective and accurate method for quick preliminary calculations of integral propeller aerodynamic characteristics.

- 1) Lock's loss factor $\chi(\lambda, \beta)$ is given only in tabular form, and requires interpolation procedures.
- 2) The blade geometry represented by the referential section ($b_{0.7}$ and $\varphi_{0.7}$) is too reduced to affect the entire propeller acceptably.
- 3) Lock's method involves a number of blades in linear form.

Use was made of experimental thrust and power coefficients and a presumption of the Bull-Bennett lift and drag blade curve independence of the geometry and flight regime (fixed curves in Fig. 2 for all types of propellers) to meet the above outlined requirements. Eleven two-blade propellers with RAF-6 sections were involved in the calculations. Two other geometric parameters were added: blade thickness at 90 % of the propeller tip radius – $t_{0.9}$ and the position (radius) of the maximum blade chord – r_{\max} . All of the geometric parameters and the tip Mach numbers M are presented in Table 1, where $\bar{t}_{0.9} = (t_{0.9}/b_{0.9})100$ and $\bar{r}_{\max} = r_{\max}/R$.

Table 1: Set of experimental propellers

	$s_{0.7}$ [1]	$\bar{t}_{0.9}$ [%]	\bar{r}_{\max} [1]	$\varphi_{0.7}$ [°]	M	Ref.
M 337	0.0620	10.5	0.313	14.60	0.45 0.5 0.55	[3]
M60-180	0.0568	13.4	0.355	18.77	0.5 0.6 0.7	[3]
M60-130	0.0606	13.5	0.320	15.74	0.5 0.6 0.7	[3]
M30-011	0.0576	11.7	0.365	12.66	0.5 0.6 0.7	[3]
R503-2V	0.0502	13.8	0.300	10.59	0.5 0.6 0.7	[3]
M30-04A	0.0605	12.0	0.335	11.53	0.5 0.6 0.7	[3]
N5868-15	0.0596	8.3	0.500	16.06	0.46	[4]
N5868-25	0.0596	8.3	0.500	21.06	0.46	[4]
N3647-15	0.0888	8.3	0.500	16.06	0.46	[4]
N3647-25	0.0888	8.3	0.500	21.06	0.46	[4]
VR 411	0.0947	6.4	0.680	9.50	0.6	[5]

5 Induced velocity

Lock's expression for the thrust coefficient (8) with Bull-Bennett lift (19) and drag (20) blade curves was equal to the experimental thrust and numerically solved the unknown

induced angle α_{iexp} for the corresponding advance ratio and blade geometry:

$$c_T = c_T(z=2, \varphi_{0.7}, \bar{b}_{0.7}, \bar{r}_{0.7}, \lambda, c_y(\alpha), c_x(\alpha), \alpha) \equiv c_{Texp} \quad (21)$$

$$\alpha = \varphi_{0.7} - \arctg \frac{\lambda}{\pi \bar{r}_{0.7}} - \alpha_i.$$

The calculated induced angles α_{iexp} were then used to determine of the loss factor directly from Lock's induced equation (9):

$$\chi = \frac{s_{0.7} c_l(\alpha)}{4 \sin(\beta) \operatorname{tg}(\alpha_i)}. \quad (22)$$

A three-step procedure was used to simulate the influence of the blade geometric parameters on the induced values. The first step smoothed the loss factor only as a function of the advance ratio and induced angle. Subsequently this analytical expression of the loss factor was used to calculate induced angles α_i by solving the induced equation (9) for all the experimental propellers. Finally, these induced angles α_i were correlated with the experimental set α_{iexp} .

The function of the smooth loss factor that approximates the numerical results was stated in the form:

$$\chi = \frac{1}{a_\chi \sqrt{\alpha_i} + b_\chi \alpha_i} \quad (23)$$

with coefficients:

$$a_\chi = 0.3254\lambda^2 + 0.3529\lambda + 0.4449 \quad (23a)$$

$$b_\chi = 0.8213\lambda^2 - 0.0854\lambda + 0.0628. \quad (23b)$$

A comparison of experimental set of the induced angles α_{iexp} with induced angles α_i calculated by means the smooth loss factor showed differences that were evaluated by regression analysis into the final linear correction function:

$$\alpha_{iexp} \cong \alpha_{icor} = A\alpha_i + B \quad (24)$$

$$A = 1.088 - 0.0149\varphi_{0.7} - 1.74 s_{0.7} + 0.462 \bar{r}_{\max} \quad (24a)$$

$$B = 1.286 - 0.113 \bar{t}_{0.9}. \quad (24b)$$

This linear expression gives a good approximation in the region of higher angles. In order to keep the simple linear correlation through all the angles, a slightly different form based on coefficients A (24a) and B (24b) is used for a range of small induced angles:

$$\alpha_i \leq 0.5: \quad \alpha_{iexp} \cong \alpha_{icor} = 1.3\alpha_i + 0.5A + B - 0.65 \quad (25)$$

6 Integral factors

The calculated experimental induced angles α_{iexp} can also be used to express more precisely the integral factors E and F of Lock's decomposition equations (11) and (12). Such modified factors are necessary for more accurate calculations of the thrust and power (torque) coefficients directly with use of Lock's decomposition equations (17) and (18). The integral factors were explicitly derived from decomposition equations (11) and (12):

$$E = \frac{2\pi s_{0.7} c_l - F c_p \sin(\beta)}{2\pi c_T \cos(\beta)} \quad (26)$$

$$F = \frac{2\pi s_{0.7} (c_d + c_l \operatorname{tg}(\beta))}{c_p (\cos(\beta) + \sin(\beta) \operatorname{tg}(\beta))}. \quad (27)$$

The numerical dependences of the two factors on the blade geometry and flight regime were obtained by using the set of experimental induced angles $\alpha_{i\text{exp}}$ in the expressions (6) for angles α and β and introducing these angles with the corresponding experimental values of thrust and power coefficients into equations (26) and (27). The numerical results were smoothed by the following functions:

$$E = a_E + b_E \lambda + c_E \lambda^2 \quad (28)$$

$$F = \frac{1}{a_F \lambda + b_F + c_F \lambda} \quad (29)$$

with parameters describing the influence of the blade geometry and also partly the flight regime:

$$\begin{aligned} a_E &= 0.565, & b_E &= -0.0825, & c_E &= -0.0375 \\ a_F &= 0.639 - 1.8189 \bar{b}_{0.7}, & b_F &= 0.965 F_m \bar{b}_{0.7} - a_F \lambda_r, \\ F_m &= 0.393 - 0.9731 \bar{b}_{0.7} + 0.027 \varphi_{0.7} - 0.414 \lambda_m + \\ &\quad - 0.182 \bar{r}_{\max} + 0.0234 \bar{t}_{0.9} \\ \lambda_m &= 0.0475 \varphi_{0.7} - 1.0777 \bar{b}_{0.7} - 0.1, & (29a) \\ \lambda_r &= \lambda_m - 0.165 \\ \lambda \leq \lambda_r: & \quad c_F = 0 \\ \lambda \geq \lambda_r: & \quad c_F = (9.8676 \bar{b}_{0.7} - 2.542)(\lambda - \lambda_r)^2 + \\ &\quad - (1.9144 \bar{b}_{0.7} + 1)(\lambda - \lambda_r)^3 \end{aligned}$$

The analysis confirmed the independence of the E factor from the propeller geometry in acceptance with Lock's original model.

7 Number of blades

Lock's scheme considers the linear dependence on the number of blades with the use of the solidity factor (10) both in the tip loss factor (22) and in the relations for thrust (17) and power (18) coefficients derived from the decomposition equations. The linear model gives thrust and power coefficients that are higher than they are in reality, and the propeller propulsive efficiency does not depend on the number of blades.

To preserve the simplicity of the developed procedure for two-blade propellers, an initial correction of the linear model was designed on the basis of the evolution of experimental thrust and power coefficients. By comparing different blade propeller number having the same blade geometry [4], it was found that the mean value of the rate between the thrust (power) per blade of a two-blade propeller and a z-blade propeller systematically increases from 1 ($z = 2$) to higher values ($z > 2$). The analytical expressions of the mean thrust K_T and power K_P ratio are as follows:

$$K_T = \frac{c_T(z=2)/2}{c_T(z)/z} = 0.837 + \quad (30)$$

$$+ 0.08583z - 1.5 \times 10^{-3} z^2 - 3.33333 \times 10^{-4} z^3$$

$$K_P = \frac{c_P(z=2)/2}{c_P(z)/z} = 0.764 + \quad (31)$$

$$+ 0.16533z - 27 \times 10^{-3} z^2 - 0.16666 \times 10^{-4} z^3.$$

The ratio $K_T(z)$ and $K_P(z)$ can therefore be used as conversion factors between two-blade and z-blade propeller thrust and power coefficients:

$$c_T(z) \cong \frac{c_T(z=2) \cdot z}{2K_T} \quad (32)$$

$$c_P(z) \cong \frac{c_P(z=2) \cdot z}{2K_P} \quad (33)$$

In order to ensure the correct internal calculation of a two-blade propeller even if the right solidity factor (10) of the z-blade propeller is given, an effective solidity factor must be considered during the calculation:

$$s_{0.7\text{ef}} = s_{0.7} \frac{2}{z} \quad (34)$$

8 Calculation procedure

- 1) Geometric input data:

referential section	
(chord, setting angle)	$-b_{0.7}/R$ [1], $\varphi_{0.7}$ [°]
relative thickness	$-(t_{0.9}/b_{0.9})100$ [%]
position of the max. blade chord	$-r_{\max}/R$ [1]
number of blades	$-z$
- 2) Flight regime input:

advance ratio	$-\lambda$ [1]
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- 3) Calculation of the effective solidity factor – (34)
- 4) Solution of the induced angle – the root of the transcendental induced equation (22) with the modified loss factor (23), (blade lift curve (19) is required)
- 5) Linear correction of the induced angle – (24) and (25)
- 6) Calculation of the modified integral factors E and F – (28) and (29)
- 7) Calculation of the thrust and power coefficient with the effective solidity factor – (17) and (18), (blade lift and drag curves (19) and (20) are required)
- 8) Conversion of the gained thrust and power coefficients by means of the K_T and K_P factors with respect to the number of blades – (32) and (33)
- 9) Calculation of the propulsive efficiency: $\eta = (c_T/c_P) \lambda$ [1] (or $\eta = 0.8(c_T)^{3/2}/c_P$ in case of $\lambda = 0$)

9 Validity

It was proved by systematic reversal calculations of the experimental propeller set, Table 1, that the maximum relative error of both thrust and power coefficients is less than 10 % and the mean error is about 5 %. These differences are valid from the start regime up to flight regimes with maximum propeller efficiency. The range of geometric parameters that ensures a relative error limit of 10 % can therefore be directly estimated from the data in Table 1:

blade width $b_{0.7}/R = (0.09 - 0.22)$,
 blade angle setting $j_{0.7} = (9 - 23)^\circ$,
 maximum blade width $r_{\max}/R = (0.3 - 0.7)$ and
 airfoil thickness $(t_{0.9}/b_{0.9})100 = (6 - 14) \%$.

The tip Mach number should not exceed 0.75. All analyses were performed with RAF-6 blade airfoil propellers.

10 Examples

The first example presents calculations of the two-blade propeller VLU 001 [6] with upper geometric limits of the blade chord and angle setting. The tip Mach number $M=0.45$. Numerical results are compared with experimental values. The input geometric parameters are as follows:

- blade angle setting at 70 % of the propeller diameter – $\varphi_{0.7}=24.4$ [°],
- relative chord of the blade at 70 % of the propeller diameter – $b_{0.7}/R=0.227$ [1],

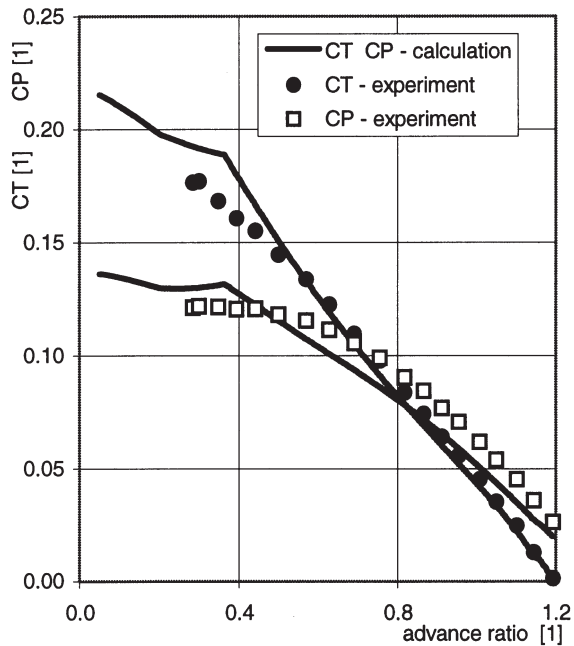


Fig. 3: Thrust and power coefficients of two-blade propeller VLU 001

- relative position of the maximum blade width – $r_{\max}/R=0.535$ [1],
- relative airfoil thickness at 90 % of the propeller diameter – $(t_{0.9}/b_{0.9})100=8.5$ [%].

The thrust and power coefficients are shown in Fig. 3. The propeller efficiency is presented in Fig. 4. The relative error of the power coefficient c_P reached about 10 %. The thrust coefficient gives better results.

The second example shows the possibilities of a parametric study. Propeller efficiency in a static regime ($\lambda=0$, non-forward movable propeller $V=0$ – see Fig. 1) defined as $\eta=0.8(c_T)^{3/2}/c_P$ is calculated for the case a two-blade propeller

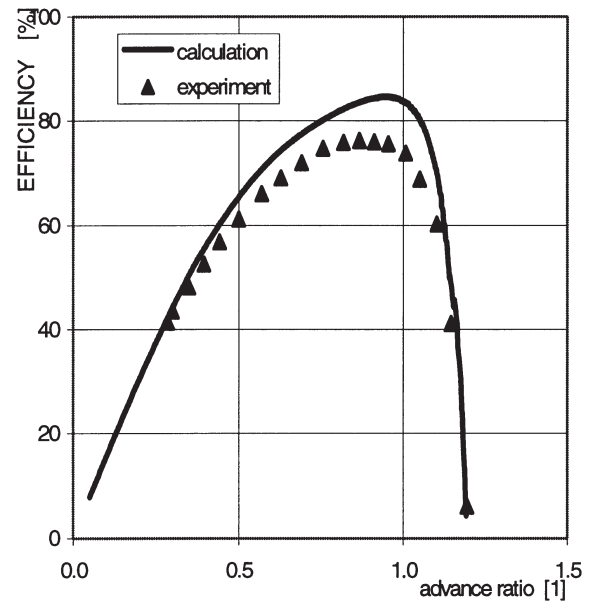


Fig. 4: Efficiency of two-blade propeller VLU 001

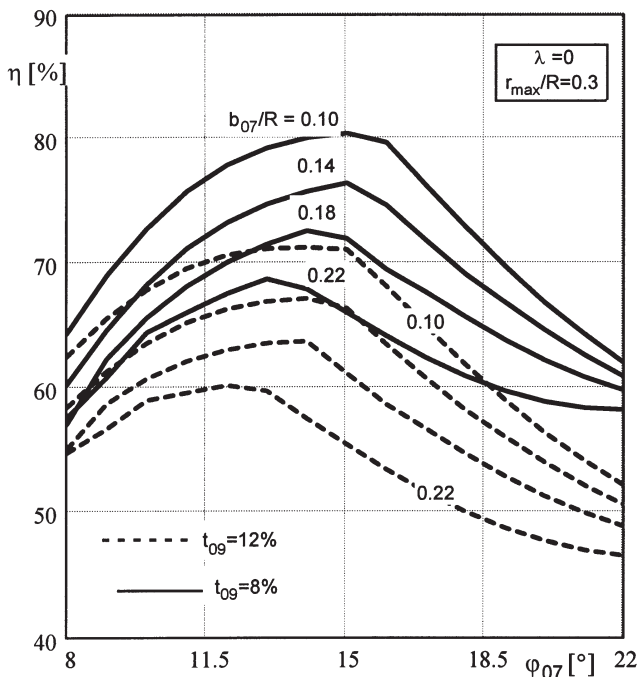


Fig. 5: Efficiency of a two-blade propeller at $\lambda=0$ with different geometric parameters

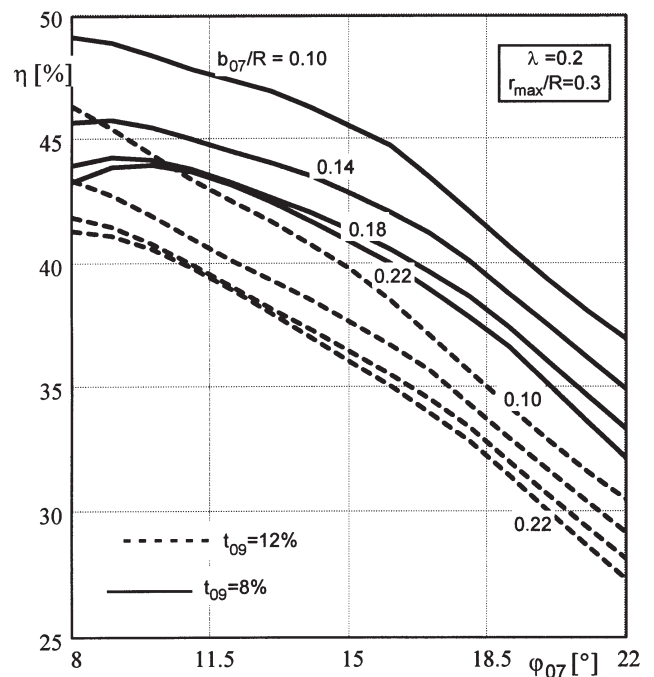


Fig. 6: Efficiency of a two-blade propeller at $\lambda=0.2$ with different geometric parameters

ler with fixed r_{\max}/R , two thickness parameters ($t_{0,9}/b_{0,9}$) and four $b_{0,7}/R$. The results are plotted in Fig. 5 as a function $\varphi_{0,7}$. Fig. 6 depicts the propulsive efficiency of the same propeller at the advance ratio $\lambda = 0.2$.

11 Conclusion

The published method presents a simple and quick calculation procedure for thrust and power propeller coefficients based on Lock's 2D scheme of the referential section. The numerical demands are restricted to the solution of a non-linear algebraic equation to obtain the induced angle. The thrust and power coefficients are consequently calculated directly by explicit analytical algebraic formulae.

The aerodynamics characteristics of the propeller are obtained with an acceptable error for preliminary aircraft performance analyses: the maximum relative deviation of both the thrust and the power coefficient does not exceed 10 % from the start regime up to flight regimes with maximum propeller efficiency. The mean error is about 5 %. The range of blade geometric parameters was set to keep the calculations within this error limit.

In addition to applications in the small aeroplane industry the presented method is also suitable for student study projects at technical universities with aerospace study programmes. Parametrical input data of the propeller blade geometry and the number of blades enables easy studies of the influence of propeller geometry on the aerodynamic characteristics.

This procedure can be further enhanced by considering the tip Mach number effect, the aerodynamics of a blade airfoils and by a more detailed analysis of the influence of blade numbers. Systematic use of FEMs (e.g. FLUENT) can supply the experimental basis.

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