Acta Polytechnica **53**(2):98–102, 2013

available online at http://ctn.cvut.cz/ap/

MULTIGROUP APPROXIMATION OF RADIATION TRANSFER IN SF_6 ARC PLASMAS

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Abstract. The first order of the method of spherical harmonics (P1-approximation) has been used to evaluate the radiation properties of arc plasmas of various mixtures of SF_6 and PTFE ($(C_2F_4)n$, polytetrafluoroethylene) in the temperature range $(1000 \div 35\,000)$ K and pressures from 0.5 to 5 MPa. Calculations have been performed for isothermal cylindrical plasma of various radii $(0.01 \div 10)$ cm. The frequency dependence of the absorption coefficients has been handled using the Planck and Rosseland averaging methods for several frequency intervals. Results obtained using various means calculated for different choices of frequency intervals are discussed.

Keywords: SF_6 and PTFE plasmas, radiation transfer, mean absorption coefficients, P1-approximation.

1. Introduction

An electric (switching) arc between separated contacts is an integral part of a switching process. For all kinds of high power circuit breakers, the basic mechanism is to extinguish the switching arc at the natural current zero by gas convection.

The switching arc is responsible for proper disconnection of a circuit. In the mid and high voltage region, SF_6 self-blast circuit breakers are widely used. Radiation transfer is the dominant energy exchange mechanism during the high current period of the switching operation. Due to the extreme conditions, experimental work only gives global information instead of local information, which may be important for determining the optimum operating conditions; theoretical modelling is then of great importance. Several approximate methods for radiation transfer in arc plasma have been developed (isothermal net emission coefficient method [7, 1, 8], partial characteristics method [2, 11], P1-approximation [9], discrete ordinates method [9], etc.). In this paper, the P1-approximation has been used to predict radiation processes in various mixtures of SF_6 and PTFE plasmas.

2. P1-approximation

If diffusion of light is neglected and local thermodynamic equilibrium is assumed, the radiation transfer equation can be written as

$$\mathbf{\Omega} \cdot \nabla I_\nu(\mathbf{r}, \mathbf{\Omega}) = \kappa_\nu (B_\nu - I_\nu), \tag{1}$$

where I_ν is the spectral intensity of radiation, $\mathbf{\Omega}$ is a unit direction vector, κ_ν is the spectral absorption coefficient, and B_ν is the Planck function – the spectral density of equilibrium radiation. In P1-approximation, the angular dependence of the specific intensity is assumed to be represented by the first two terms in a spherical harmonics expansion

$$I_\nu(\mathbf{r}, \mathbf{\Omega}) = \frac{c}{4\pi} U_\nu(\mathbf{r}) + \frac{3}{4\pi} \mathbf{F}_\nu(\mathbf{r}) \cdot \mathbf{\Omega}\,, \tag{2}$$

where U_ν denotes the radiation field density, $\mathbf{F}_\nu$ is the radiation flux, and c is the speed of light. Combining this expression with Eq. 1, one finds for radiation flux

$$\mathbf{F}_\nu(\mathbf{r}) = -\frac{c}{3\kappa_\nu} \nabla U_\nu(\mathbf{r}) \tag{3}$$

and a simple elliptic partial differential equation for the density of radiation U_ν

$$\nabla \cdot \left[\frac{-c}{3\kappa_\nu(T)} \nabla U_\nu(\mathbf{r}) \right] + \kappa_\nu(T) c U_\nu(\mathbf{r}) = \kappa_\nu(T) 4\pi B_\nu(T)\,. \tag{4}$$

Integrating over frequency, the total density of the radiation and the total radiation flux are obtained

$$U(\mathbf{r}) = \int_0^\infty U_\nu(\mathbf{r})\,\mathrm{d}\nu,$$
$$\mathbf{F}(\mathbf{r}) = \int_0^\infty \mathbf{F}_\nu(\mathbf{r})\,\mathrm{d}\nu. \tag{5}$$

3. Absorption coefficients

Prediction of both radiation emission and absorption properties requires knowledge of the spectral coefficients κ_ν of absorption as a function of radiation frequency. These coefficients are proportional to the concentration of the chemical species occurring in the plasma, and depend on the cross sections of various radiation processes.

In the mixture of SF_6 and PTFE (C_2F_4) we assume the following species: SF_6 molecules, S, F, C atoms,

S^+, S^{+2}, S^{+3}, F^+, F^{+2}, C^+, C^{+2}, C^{+3} ions and electrons. The equilibrium concentrations of each species in various SF_6 + PTFE mixtures were taken from [3].

Spectral coefficients of absorption were calculated using semi-empirical formulas to represent both continuum and line radiation. The continuum spectrum is formed by bound-free transitions (photo-recombination, photo-ionization) and free-free transitions (bremsstrahlung). The photo-ionization cross sections for neutral atoms were calculated by the quantum defect method of Seaton [12], the cross sections of the photo-ionization of ions and free-free transitions were treated using Coulomb approximation for hydrogen-like species [6]. In the discrete radiation calculations, spectral lines broadening and their complex shapes have to be carefully considered. The lines are broadened due to numerous phenomena. The most important are Doppler broadening, Stark broadening, and resonance broadening. For each line, we have calculated the values of half-widths and spectral shifts. The line shape is given by convolution of the Doppler and Lorentz profiles, resulting in a simplified Voigt profile. The lines that overlap have also been taken into account. Due to lack of data, from molecular species we have only considered SF_6 molecules with their experimentally measured absorption cross sections [5].

4. Absorption means

One of the procedures for handling the frequency variable in the radiation transfer equation is the multigroup method [10, 4]. It is based on a simplified spectral description with only some spectral groups assuming grey body conditions within each group with a certain average absorption coefficient value, i.e. for the k-th spectral group

$$\kappa_\nu(\mathbf{r},\nu,T) = \overline{\kappa}_k(\mathbf{r},T); \quad \nu_k \le \nu \le \nu_{k+1}. \tag{6}$$

The mean absorption coefficient values are generally taken as either the Rosseland mean or the Planck mean.

The Planck mean is appropriate in the case of an optically thin system. The Planck mean absorption coefficient is given by

$$\overline{\kappa}_P = \frac{\int_{\nu_k}^{\nu_{k+1}} \kappa_\nu B_\nu \,\mathrm{d}\nu}{B_k}, \tag{7}$$

where

$$B_k = \int_{\nu_k}^{\nu_{k+1}} B_\nu \,\mathrm{d}\nu.$$

The Rosseland mean is appropriate when the system approaches equilibrium (almost all radiation is reabsorbed). The Rosseland mean is given by

$$\overline{\kappa}_R^{-1} = \frac{\int_{\nu_k}^{\nu_{k+1}} \kappa_\nu^{-1} \frac{\mathrm{d}B_\nu}{\mathrm{d}T}\,\mathrm{d}\nu}{\int_{\nu_k}^{\nu_{k+1}} \frac{\mathrm{d}B_\nu}{\mathrm{d}T}\,\mathrm{d}\nu}. \tag{8}$$

The total radiation density value is then given by

$$U(\mathbf{r}) = \sum_k U_k(\mathbf{r}), \tag{9}$$

where U_k are solutions of Eq. 4 with frequency independent $\overline{\kappa}_k(T)$ and $B_k(T)$.

5. Net emission coefficients

Assuming local thermodynamic equilibrium, coefficient of absorption κ_ν is related to the coefficient of emission ε_ν by Kirchhoff's law

$$\varepsilon_\nu = B_\nu \kappa_\nu. \tag{10}$$

Strong self-absorption of radiation in the plasma volume occurs, and this must be taken into account in the calculations. The net emission coefficient of radiation, $\varepsilon_{\mathrm{N}\nu}$ is defined by Lowke [7] as

$$\varepsilon_{\mathrm{N}\nu} = \varepsilon_\nu - J_\nu \kappa_\nu, \tag{11}$$

where J_ν is an average radiation intensity, which is a function of temperature. For an isothermal plasma sphere at radius R (the results are approximately the same as for the isothermal cylinder), it is defined as

$$J_\nu = B_\nu[1 - \exp(-\kappa_\nu R)]. \tag{12}$$

A combination of Eqs. 10–12 gives the expression for the net emission coefficient

$$\varepsilon_\mathrm{N} = \int_0^\infty B_\nu \kappa_\nu \exp(-\kappa_\nu R)\,\mathrm{d}\nu. \tag{13}$$

The isothermal net emission coefficient corresponds to the fraction of the total power per unit volume and unit solid angle irradiated into a volume surrounding the axis of the arc plasma and escaping from the arc column after crossing thickness R of the isothermal plasma. It is often used for predicting the energy balance, since the net emission of radiation (the divergence of the radiation flux) can be written as

$$\nabla \cdot \mathbf{F}_R = 4\pi\varepsilon_\mathrm{N}. \tag{14}$$

In multigroup P1-approximation, the net emission coefficient can be determined from Eq. 4. In the case of cylindrically symmetrical isothermal plasma, Eq. 4 has constant coefficients $\overline{\kappa}_k$ and B_k, and depends only on one variable – radial distance r. It represents the modified Bessel equation, and can be solved analytically. Taking into account the boundary condition (no radiation enters into the plasma cylinder from outside)

$$\mathbf{n} \cdot \mathbf{F}_k(R) = -\frac{c\,U_k(R)}{2} \tag{15}$$

the net emission over the volume of the arc for the k-th frequency group is

$$(w_\mathrm{avg})_k = \frac{2\pi}{\pi R^2}\int_0^R r\,\nabla\cdot\mathbf{F}_k(r)\,\mathrm{d}r = \\ = \frac{2}{R}\,\frac{4\pi B_k}{2I_1(\sqrt{3}\,\overline{\kappa}_k R) + \sqrt{3}\,I_0(\sqrt{3}\,\overline{\kappa}_k R)}\,I_1(\sqrt{3}\,\overline{\kappa}_k R) \tag{16}$$

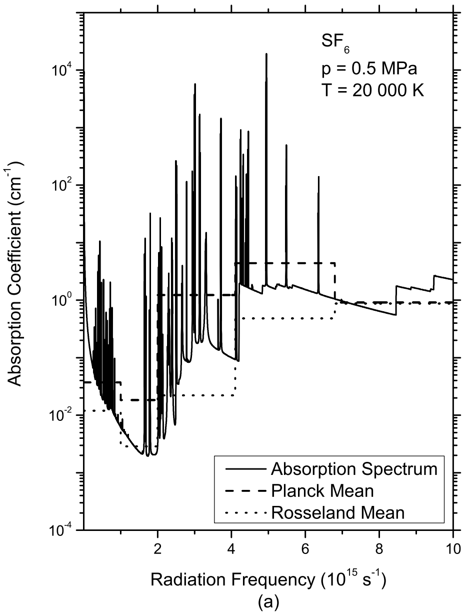

FIGURE 1. The real absorption spectrum of SF_6 plasma at $p = 0.5\,\text{MPa}$ and $T = 20\,000\,\text{K}$ compared with the Planck and Rosseland means.

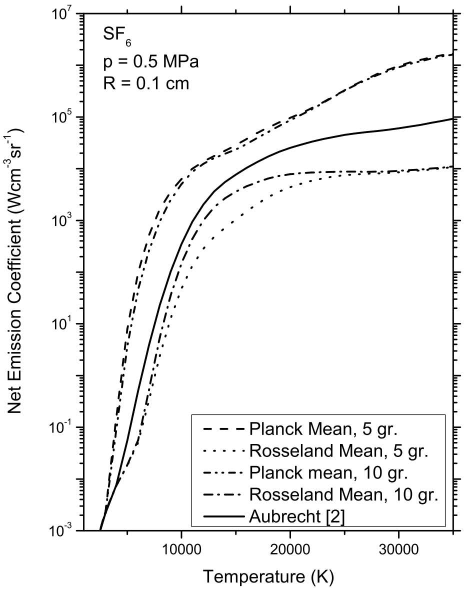

FIGURE 2. Net emission coefficients of SF_6 plasma with radius 0.1 cm as a function of temperature for two different cuttings of the frequency interval and various absorption means; comparison with results of Aubrecht [1].

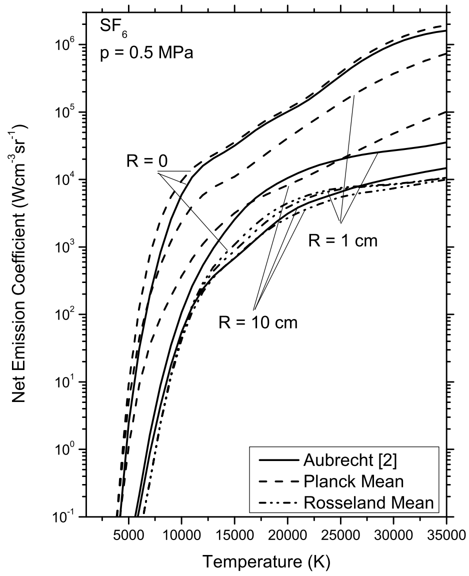

FIGURE 3. Net emission coefficients of SF_6 plasma as a function of temperature for various thicknesses of the plasma and various absorption means.

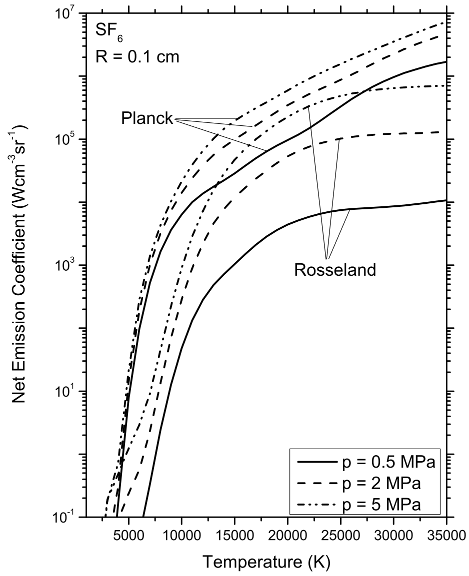

FIGURE 4. Net emission coefficients of SF_6 plasma with radius 0.1 cm as a function of temperature for various pressures.

where $I_0(x)$ and $I_1(x)$ are modified Bessel functions. Summing over all frequency groups gives the net emission of radiation

$$\nabla \cdot \mathbf{F}_R = \sum_k (w_{\text{avg}})_k = 4\pi\varepsilon_{\text{N}}. \tag{17}$$

6. RESULTS

The mean absorption coefficient values depend on the choice of the frequency interval cutting. The cutting frequencies are mainly defined by the steep jumps of the evolution of the continuum absorption coefficients that correspond to individual absorption edges. However, the number of groups should be minimized to decrease the computation time. In this work, the frequency interval $(10^{12} - 10^{16})\,\text{s}^{-1}$ was cut into

(a) five frequency groups with cutting frequencies

$$(0.001, 1, 2, 4.1, 6.8, 10) \times 10^{15}\,\text{s}^{-1}\,, \tag{18}$$

(b) ten frequency groups with cutting frequencies

$$\begin{aligned}(0.001, 1, 1.4, 1.77, 2, 2.2, 2.5, 3,\\ 4.1, 6.8, 10) \times 10^{15}\,\text{s}^{-1}\,.\end{aligned} \tag{19}$$

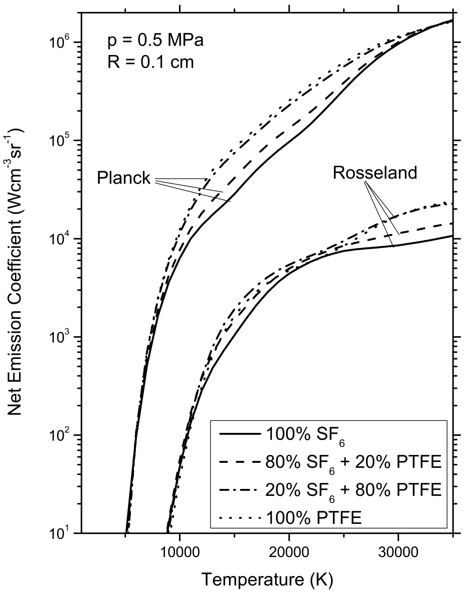

Figure 5. Net emission coefficients of different mixtures of SF_6 and PTFE plasmas as a function of temperature at pressure 0.5 MPa for various absorption means.

The two cuttings differ in the frequency interval $(1 \div 4.1) \times 10^{15}\,\mathrm{s}^{-1}$, which is split into two groups in case (a), and in greater detail into seven groups in case (b). The absorption spectrum evaluated at 20 000 K compared with various averaged versions for five groups cutting is shown in Fig. 1.

The net emission coefficients were calculated by combining Eqs. 16 and 17. Results for an isothermal plasma cylinder of radius $R = 0.1$ cm for two different cuttings Eqs. 18, 19 of the frequency interval are given in Fig. 2. In the case of the Planck averaging method, cutting the spectrum into more frequency groups influences the resulting net emission coefficients only slightly. A comparison is also provided with the values of Aubrecht [1], which were obtained by direct integration from Eq. 13. It can be seen that the Planck mean leads to an overestimation of the emitted radiation, while the Rosseland approach underestimates it.

An example of the calculated temperature dependence of the net emission coefficients for various thicknesses of pure SF_6 plasma at a pressure of 0.5 MPa is presented in Fig. 3. The strong effect of plasma thickness can be seen both for direct frequency integration Eq. 13 and for Planck means; Rosseland averages are influenced only slightly. As can be expected from the definition of the Planck and Rosseland means, by omitting self-absorption ($R = 0$) the Planck means give good agreement with the results of direct integration, while for thick plasma ($R = 10$ cm) the Rosseland mean is a good approach.

The influence of the plasma pressure on the net emission coefficient values is shown in Fig. 4. Net emission coefficients increase with increasing pressure, mainly for Rosseland means.

The influence of an admixture of PTFE on the values of the net emission coefficients of SF_6 plasma is given in the Fig. 5 for plasma thickness 0.1 cm. The differences between net emission coefficients are very small. This can be explained by the approximately equivalent role of sulphur and carbon species. Sulphur and carbon atoms and ions have similar radiation emission behavior.

7. Conclusions

Net emission coefficients for various mixtures of SF_6 and PTFE plasmas have been calculated using P1-approximation for an isothermal plasma cylinder. Multigroup approximation for handling the frequency variable has been used. Both Planck and Rosseland averaging methods have been applied to obtain mean values of absorption coefficient values. A comparison with the net emission coefficients calculated by direct frequency integration has been provided. It has been shown that Planck means generally overestimate the emission of radiation, while Rosseland means underestimate it. Planck means give good results only for a very small plasma radius (omitting self-absorption). The Rosseland mean is a suitable approach for thick plasma (absorption dominated system). In reality, neither mean is correct in general. The simplest procedure for improving the accuracy is to use the Planck mean for frequency groups with low absorption coefficient values and the Rosseland mean for groups with high absorption coefficient values.

Another approach was suggested in [9], where each group based on original frequency splitting was further divided according to the absorption coefficient values, and Planck averaging for these new groups was calculated. This procedure partially solves the problem of overestimation of the role of lines in the Planck averaging method. Another correction of the influence of lines on Planck means was presented in [4], where the escape factor was introduced.

Acknowledgements

This work has been supported by the Czech Science Foundation under project No. GD102/09/H074 and by the European Regional Development Fund under projects Nos. CZ.1.05/2.1.00/01.0014 and CZ.1.07/2.3.00/09.0214.

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