The Amount of Regenerated Heat Inside the Regenerator of a Stirling Engine

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The paper deals with analytical computing of the regenerated heat inside the regenerator of a Stirling engine. The total sum of the regenerated heat is constructed as a function of the crank angle in the case of Schmidt's idealization.

Keywords: Stirling engine, regenerator, regenerated heat, Schmidt's idealization.

1 Introduction

Stirling's engine (Fig. 1) is a volumetric engine, in which work is done by changing the volume, the pressure and the temperature of the working gas. The working gas is moved by pistons on the hot and cold side through the regenerator. The motion of the pistons is regular, and the pistons are mechanically connected. The pistons are frequently moved by a crank shaft. The power output is through the crank shaft of the engine. One cycle of the Stirling engine is one turn of the crank shaft.

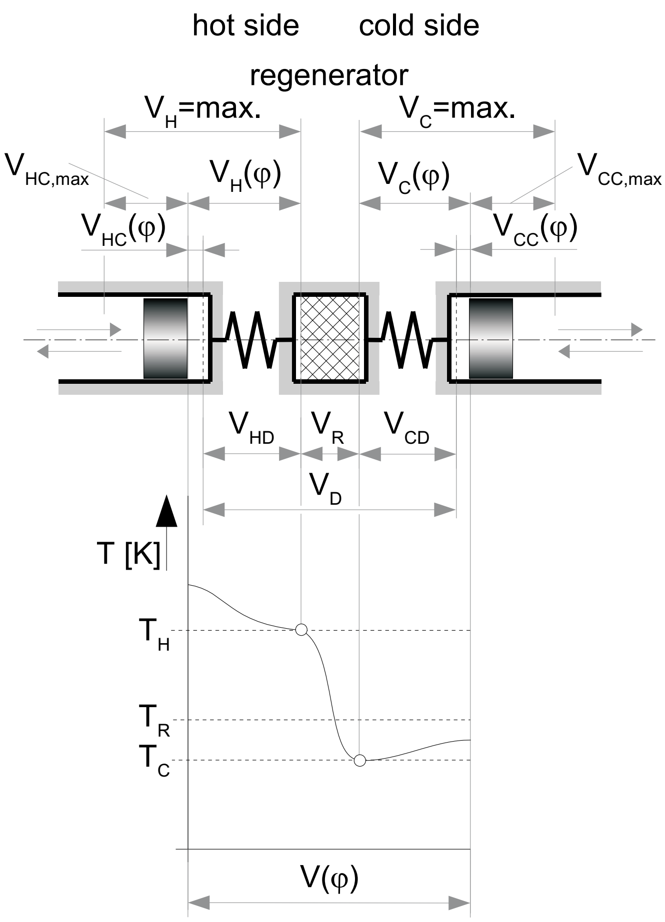


Fig. 1: Scheme of a Stirling engine with dead volumes and progress of the temperature of the working gas for crank angle φ

The regeneration of the heat inside the regenerator is perfect according to Schmidt's idealization. If the working gas is flowing from the hot side to the cold side, the working gas is cooled in the regenerator from temperature T_H to temperature T_C. Heat is saved to the matrix of the regenerator during cooling. If the working gas is flowing from the cold side to the hot side, the working gas is heated from temperature T_C to temperature T_H inside the regenerator. The heat is taken from the matrix of the regenerator during heating. This process is called regeneration of the heat. The temperature of the working gas on the hot side and on the cold side is not constant during the cycle in a real Stirling engine (Fig. 1).

The amount of regenerated heat inside the regenerator of the Stirling engine is much greater than the heat put into the engine (Urieli and Berchowitz – simply adiabatic analysis [1]). However no an analytical computing method for the amount of regenerated heat inside the regenerator has existed untill now. The amount of regenerated heat has an elemental action on the thermal efficiency, the performance and the dimensions of the engine.

Schmidt's idealization [2] is a comparative cycle of the engine with outer transfer of the heat (Stirling engine), when isothermal processes are taking place in all volumes of the engine Fig. 3. Schmidt's idealization has been the only analytic computing method for thermodynamic design of Stirling engine untill now. The heat flows, the work autput and the thermal efficiency of the cycle without wastage of energy are computed on the basis of the assumptions of Schmidt's idealization. The assumptions of Schmidt's idealization are stated below.

2 Notation

symbol	description	unit
c_p	specific heat capacity for working gas at constant pressure	$\mathrm{J{\cdot}kg^{-1}{\cdot}K^{-1}}$
I	enthalpy of working gas	J
m	total amount of working gas in engine	kg
p	pressure	Pa
Q	energy balance (heat)	J
r	individual gas constant of working gas	$\mathrm{J{\cdot}kg^{-1}{\cdot}K^{-1}}$
T	absolute temperature	K
V	volume	$\mathrm{m^3}$
α	phase angle, of the hot side volume variation to the cold side volume variations	rad
η	thermal efficiency	-

φ	crank angle	rad
κ	adiabatic index	-
τ	temperature rate	-

subscripts

C	cold side
Car	Carnot
CC	volume of cylinder of cold side
D	death (death volume)
H	hot side
HC	volume of cylinder of hot side
max	maximum
min	minimum
R	regenerator
reg	regenerated

3 The amount of regenerated heat inside the regenerator

The amount of regenerated heat inside the regenerator of the Stirling engine can be found from an energy balance cycle.

This energy balance can be applied to the Stirling engine (Fig. 1) under the following assumptions:

1) The working gas is an ideal gas.
2) There is no pressure loss, and the pressure is the same on all the volume.
3) The engine is totally sealed.
4) There is no heat transfer between the matrix of the regenerator and the structure of the engine.
5) Steady state conditions are assumed for overall operation of the engine so that the pressures, temperatures, etc. are subject to cyclic variations only.

The energy balance of the cycle on a part of a cycle 1-2 in the T-s chart (Fig. 2) is described by the following this equation

$$\int_{S_1}^{S_2} \mathrm{d}Q = Q_{1-2} = Q_{\mathrm{H},1-2} + Q_{\mathrm{C},1-2} + Q_{\mathrm{R},1-2}. \tag{1}$$

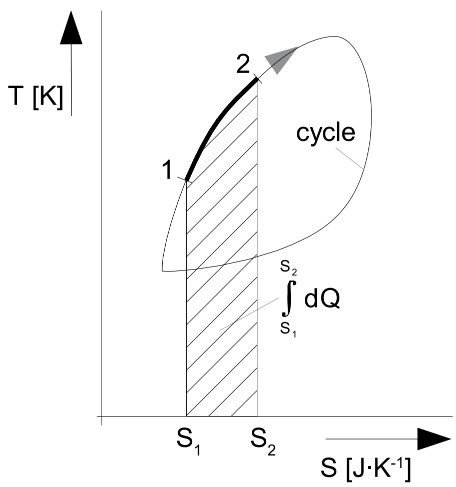


Fig. 2: General thermal cycle in the T-s chart

The thermal flows and the entropy are functions of the crank angle, so equation (1) is

$$\int_{\varphi_1}^{\varphi_2} \mathrm{d}Q = Q_{1-2} = Q_{\mathrm{H},1-2} + Q_{\mathrm{C},1-2} + Q_{\mathrm{R},1-2}. \tag{2}$$

The energy balance of the working gas inside the regenerator on a part of cycle 1-2 can be found using the equation (2)

$$Q_{\mathrm{R},1-2} = \int_{\varphi_1}^{\varphi_2} \mathrm{d}Q - Q_{\mathrm{H},1-2} - Q_{\mathrm{C},1-2}. \tag{3}$$

The energy balance of the working gas on the hot side of the engine on a part of cycle 1-2

$$Q_{\mathrm{H},1-2} = \int_{\varphi_1}^{\varphi_2} \mathrm{d}I_{\mathrm{H}} - \int_{\varphi_1}^{\varphi_2} V_{\mathrm{H}}(\varphi)\mathrm{d}p = [I_{\mathrm{H}}(\varphi)]_{\varphi_1}^{\varphi_2} - \int_{\varphi_1}^{\varphi_2} V_{\mathrm{H}}(\varphi)\mathrm{d}p. \tag{4}$$

The energy balance of the working gas on the cold side of the engine on a part of cycle 1-2

$$\begin{aligned} Q_{\mathrm{C},1-2} &= c_p \int_{\varphi_1}^{\varphi_2} m_{\mathrm{C}}(\varphi)\mathrm{d}T_{\mathrm{C}} - \int_{\varphi_1}^{\varphi_2} V_{\mathrm{C}}(\varphi)\mathrm{d}p \\ &= [I_{\mathrm{C}}(\varphi)]_{\varphi_1}^{\varphi_2} - \int_{\varphi_1}^{\varphi_2} V_{\mathrm{C}}(\varphi)\mathrm{d}p. \end{aligned} \tag{5}$$

For the energy balance of the cycle on a part of cycle 1-2, we use the first law of thermodynamics

$$\begin{aligned} \int_{\varphi_1}^{\varphi_2} \mathrm{d}Q &= c_p \cdot m \int_{\varphi_1}^{\varphi_2} \mathrm{d}T - \int_{\varphi_1}^{\varphi_2} V(\varphi)\mathrm{d}p \\ &= c_p \cdot m\, [T(\varphi)]_{\varphi_1}^{\varphi_2} - \int_{\varphi_1}^{\varphi_2} V(\varphi)\mathrm{d}p. \end{aligned} \tag{6}$$

The energy balance of the working gas inside the regenerator on a part of cycle 1-2 can be found by substituting equations (4), (5), (6) into equation (3)

$$\begin{aligned} Q_{\mathrm{R},1-2} &= c_p \cdot m\, [T(\varphi)]_{\varphi_1}^{\varphi_2} - \int_{\varphi_1}^{\varphi_2} V(\varphi)\mathrm{d}p \\ &\quad - [I_{\mathrm{H}}(\varphi)]_{\varphi_1}^{\varphi_2} + \int_{\varphi_1}^{\varphi_2} V_{\mathrm{H}}(\varphi)\mathrm{d}p \\ &\quad - [I_{\mathrm{C}}(\varphi)]_{\varphi_1}^{\varphi_2} + \int_{\varphi_1}^{\varphi_2} V_{\mathrm{C}}(\varphi)\mathrm{d}p, \end{aligned} \tag{7}$$

whereas for a medium temperature working gas in the engine (from the state equation) is

$$T(\varphi) = \frac{p(\varphi) \cdot V(\varphi)}{r \cdot m}. \tag{8}$$

The total volume is the sum of all working volumes of the engine

$$V(\varphi) = V_{\mathrm{H}}(\varphi) + V_{\mathrm{R}} + V_{\mathrm{C}}(\varphi). \tag{9}$$

The energy balance of the working gas inside the regenerator on a part of cycle 1-2 can be found by substituting equations (8), (9) into equation (7)

$$Q_{R,1-2} = \frac{\kappa}{\kappa-1}\left[p(\varphi)\cdot V(\varphi)\right]_{\varphi_1}^{\varphi_2} - V_R\left[p(\varphi)\right]_{\varphi_1}^{\varphi_2} - \left[I_H(\varphi)\right]_{\varphi_1}^{\varphi_2} - \left[I_C(\varphi)\right]_{\varphi_1}^{\varphi_2}. \tag{10}$$

This equation is the energy balance of the regenerator on a part of cycle 1-2.

The total sum of the regenerated heat inside the regenerator in the state $\varphi_1 = 0$ can be found from equation (10)

$$\begin{aligned} Q_{R,0-\varphi} &= \frac{\kappa}{\kappa-1}\left[p(\varphi)\cdot V(\varphi)\right]_0^{\varphi} - V_R\left[p(\varphi)\right]_0^{\varphi} - \left[I_H(\varphi)\right]_0^{\varphi} - \left[I_C(\varphi)\right]_0^{\varphi} \\ &= \frac{\kappa}{\kappa-1}\left[p(\varphi)\cdot V(\varphi) - p(0)\cdot V(0)\right] - V_R\left[p(\varphi) - p(0)\right] - \left[I_H(\varphi) + I_C(\varphi)\right]_0^{\varphi}. \end{aligned} \tag{11}$$

The pressure $p(\varphi)$ and the volume of the engine $V(\varphi)$ for a specific crank angle is calculated or measured. Fig. 4 describes the progress of the total sum of the regenerated heat inside the regenerator (11) as a function of the crank angle. From equations (4), (5) and (11) we can calculate the heat input to the engine, the heat output from the engine and the regenerated heat inside the regenerator on a part of cycle 1-2.

The difference between the maximum and minimum value of the function $Q_{R,0-\varphi}$ is the amount of regenerated heat inside the regenerator in the cycle

$$Q_{reg} = Q_{R,0-\varphi,\max} - Q_{R,0-\varphi,\min}. \tag{12}$$

The amount of regenerated heat can be computed exactly if the change enthalpy of the working gas on the hot side and the cold side $\left[I_H(\varphi) + I_C(\varphi)\right]_0^{\varphi}$ is known (more about this problem in [3], [4]). If the thermodynamic processes are isothermal on the hot side and the cold side, then the enthalpy of the working gas does not change ($\left[I_H(\varphi) + I_C(\varphi)\right]_0^{\varphi} = 0$) and we can using equation (13). The thermodynamic processes are not isothermal in real Stirling engines. These changes of the temperature (enthalpy) of the working gas on the hot side and the cold side cannot be computed without measurement. However the amount of the regenerated heat inside the regenerator can be computed from equation (13) for the assumption

$$\left[I_H(\varphi) + I_C(\varphi)\right]_0^{\varphi} \ll \frac{\kappa}{\kappa-1}\left[p(\varphi)\cdot V(\varphi)\right]_{\varphi_1}^{\varphi_2} - V_R\left[p(\varphi)\right]_{\varphi_1}^{\varphi_2}$$

$$Q_{R,0-\varphi} = \frac{\kappa}{\kappa-1}\left[p(\varphi)\cdot V(\varphi) - p(0)\cdot V(0)\right] - V_R\left[p(\varphi) - p(0)\right]. \tag{13}$$

The value of the regenerated heat computed from equation (13) is greater than the value of the regenerated heat computed from equation (11). The regenerator designed by equation (13) is therefore greater.

The derivation of equation (13) was obtained using differential equations by a prof. Uriel, but only for conditions: $T_H = \text{const}$, $T_C = \text{const}$.

4 A calculation of the amount regenerated heat inside the regenerator for a Stirling engine fulfilling the assumptions of Schmidt's idealization

From equation (11) or (13) we can compute the amount of regenerated heat inside the regenerator of the Stirling engine if computed or measured the functions p(φ) and V(φ) are known. This section presents a method for calculation the amount of regenerated heat inside the regenerator in the cycle of the Stirling engine, fulfilling the assumptions of Schmidt's idealization.

Schmidt's idealization can be applied to the engine in Fig. 3, under the following assumptions:

1) The temperature of the working gas which flows from the regenerator on the hot side is T_H, and the temperature of working gas which flows from the regenerator on the cold side is T_C.
2) There is no pressure loss, and the pressure is the same throughout the volume.
3) The working gas is an ideal gas.
4) The engine is totally close.
5) Sinusoidal motion of the pistons.
6) The temperature of working gas on the hot side is constant and equal to T_H, and the temperature of working gas on the cold side is constant and equal to T_C. The mean temperature of working gas in the regenerator is constant and equal to T_R.
7) Heat enters the engine only through the walls on the hot and cold sides of the engine.
8) There is perfect regeneration.

The summary equations for Schmidt's idealization are evolved from the following assumptions [2]:

Pressure

$$p(\varphi) = \frac{r\cdot T_H\cdot m}{\frac{1}{2}V_{HC,\max}\left(A + B\cdot\cos(\varphi-\beta)\right)}, \tag{14}$$

$$A = 1 + 2\frac{1}{V_{HC,\max}}T_H\left(\frac{V_{HD}}{T_H} + \frac{V_R}{T_R} + \frac{V_{CD}}{T_C}\right) + k_1\cdot\tau,$$

$$T_R = \frac{T_C - T_H}{\ln\left(\frac{T_C}{T_H}\right)},$$

$$k_1 = \frac{V_{CC,\max}}{V_{HC,\max}},$$

$$\tau = \frac{T_H}{T_C},$$

$$B = -\sqrt{x^2 + z^2}$$

$$x = 1 + k_1\cdot\tau\cdot\cos\alpha$$

$$z = k_1\cdot\tau\cdot\sin\alpha,$$

$$\beta = \arctan\left(\frac{z}{x}\right) = \arctan\left(\frac{k_1\cdot\tau\cdot\sin\alpha}{1 + k_1\cdot\tau\cdot\cos\alpha}\right).$$

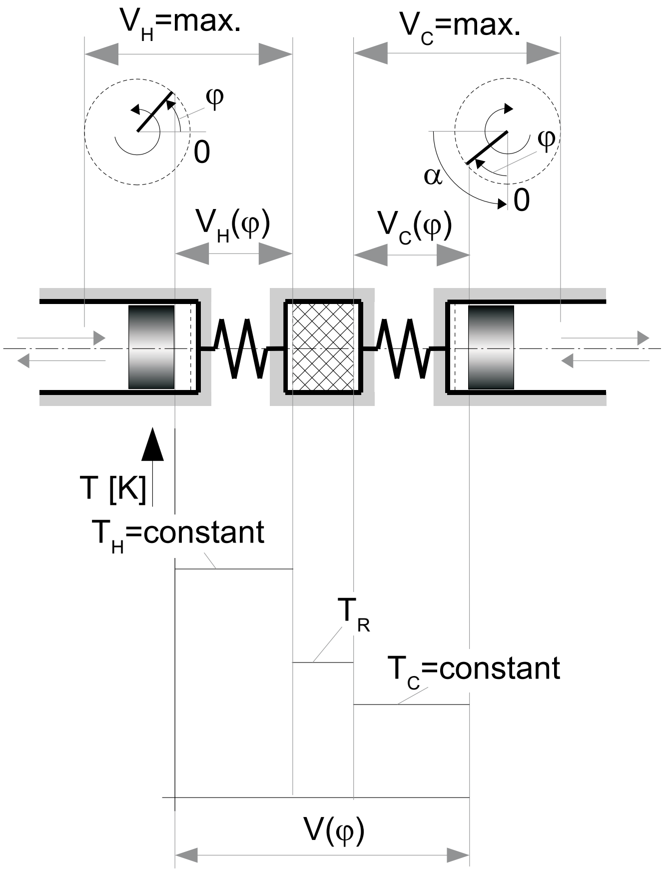


Fig. 3: Simplified scheme of the Stirling engine, fulfilling the assumptions of Schmidt's idealization, and the progress of the temperature of the working gas in volumes of the engine

Volume of the hot side

$$V_H(\varphi) = V_{HC}(\varphi) + V_{HD}. \tag{15}$$

Volume of the cold side

$$V_C(\varphi) = V_{CC}(\varphi) + V_{CD}. \tag{16}$$

Heat input to the engine

$$Q_H = -\tau \cdot r \cdot T_H \cdot m \cdot k_1 \cdot \sin(\beta - \alpha) \frac{2\pi}{B}\left(1 - \frac{A}{\sqrt{A^2 - B^2}}\right). \tag{17}$$

Heat output from the engine

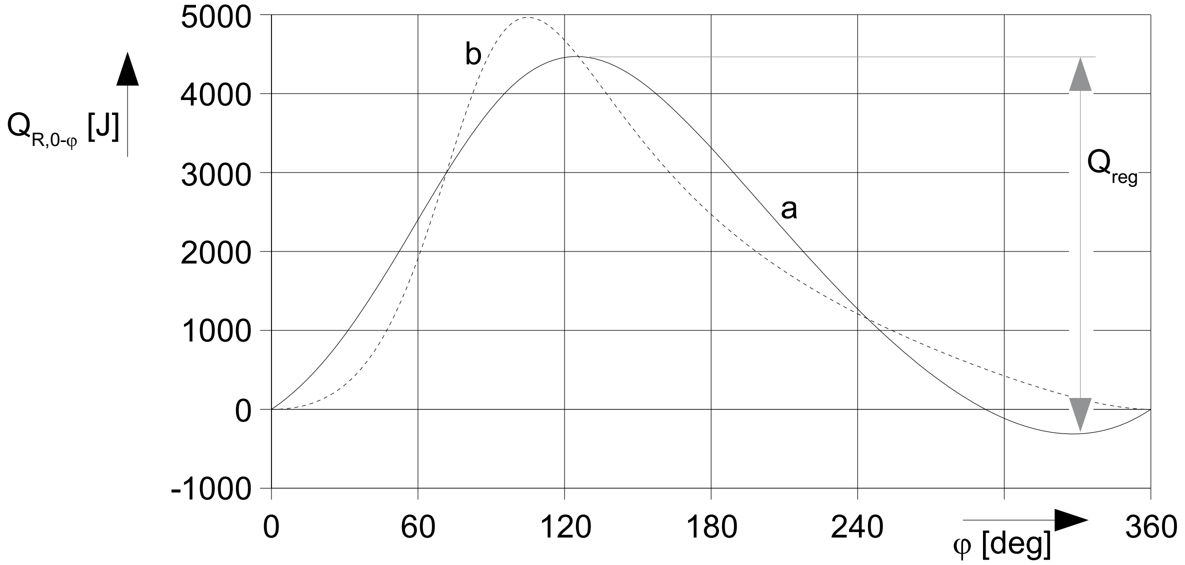
$$Q_C = r \cdot T_H \cdot m \cdot k_1 \cdot \sin(\beta - \alpha) \frac{2\pi}{B}\left(1 - \frac{A}{\sqrt{A^2 - B^2}}\right). \tag{18}$$

Thermal efficiency of the cycle

$$\eta = \frac{A}{Q_H} = 1 - \frac{1}{\tau} = \eta_{car}. \tag{19}$$

As is evident from equation (19), an engine the fulfilling assumptions of Schmidt's idealization has thermal efficiency equal to the thermal efficiency of Carnot's for temperature rate T_H/T_C.

The temperature of the working gas on the hot and cold sides of the engine is constant. Therefore the enthalpy of the working gas on these sides does not change ($[I_H(\varphi) + I_C(\varphi)]_0^{\varphi} = 0$)) and we can use the equation (13).



(a) $-Q_{R,0-\varphi}$ diagram for the Stirling engine United Stirling V-160 (working gas - helium, $\kappa = 1.67$, $r = 2077.22$ J/kg·K, $T_H = 900$ K, $T_C = 330$ K, $p_{mean} = 15$ MPa, $V_{HC,max} = 160\ \text{cm}^3$, $V_{HD} = 191\ \text{cm}^3$, $V_{CD} = 110\ \text{cm}^3$, $V_R = 28\ \text{cm}^3$, $\alpha = 105°$)

(b) $-Q_{R,0-\varphi}$ diagram for the same Stirling engine but in the case of $V_{HD} = V_{CD} = V_R = 0\ \text{cm}^3$

Q_{reg} amount of regenerated heat inside the regenerator in case (a). Other results of these cycles are shown in Table 1

Fig. 4: Progress of the sum total of regenerated heat (13) as a function of the crank angle for a Stirling engine fulfilling the assumptions of Schmidt's idealization

Table 1: Other results of the Stirling engine cycle fulfilling the assumptions Schmidt's idealization for the parameters presented in the legend of Fig. 4.

parameter	a	b
Q_H [J]	969.1	3129.1
Q_H [J]	–355.4	–1147.4
A [J]	613.8	1981.8
Q_H [J]	4784.1	4963.8
Q_{reg}/Q_D [-]	4.94	1.59
η [-]	0.63	0.63

5 Conclusion

The amount of regenerated heat inside the regenerator of a Stirling engine can be computed analytically if functions $p(\varphi)$ and $V(\varphi)$ from equations (13) are known.

The progress of the total sum of the regenerated heat $Q_{R,0-\varphi}$ as a function of the crank angle for a Stirling engine fulfilling the assumptions of Schmidt's idealization is shown in Fig. 4a. This figure shows the amount of regenerated heat inside the regenerator Q_{reg}.

From equation (17) and (12), we can compute the ratio of the regenerated heat to the heat input to the engine. This ratio is 4.94 (for an engine with the technical specifications in Fig. 4), and it corresponds perfectly with the ratio computed by Urieli and Berchowitz [1]. The larger this ratio is, the greater the impact on the efficiency of regeneration of the heat input to the engine.

The progress of the total sum of regenerated heat $Q_{R,0-\varphi}$ as a function of the crank angle for the Stirling engine fulfilling the assumptions of Schmidt's idealization without dead volumes is shown in Fig. 4b. The ratio of the regenerated heat to the heat input to the engine (1.59) is smaller than for the previous case. Thus, if the dead volumes in the engine decrease, the influence of the regenerated heat inside the regenerator on the thermal efficiency of cycle also decreases.

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