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Mid-point electron density profiles from oblique ionograms

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Abstract
A computationally efficient technique for the inversion of oblique ionograms into mid-point electron density profiles is described. The profile is given as the sum of quasi-parabolic functions suitable for ray tracing. The CPU time for a 486 desk top computer is 30 s.

Key words *ionospheric radio – electron density profiles – oblique sounding*

1. Introduction

Bistatic oblique ionograms can provide information on the mid-point vertical electron density profiles, that can serve as additional fix points for ionospheric mapping over large regions. The routine use of oblique sounding is handicapped by the non-availability of autoscaling techniques for Oblique Incidence (OI) ionograms. While reasonable progress has been made in the automatic processing of Vertical Incidence (VI) ionograms in terms of autoscaling and profile inversion, no operational procedures exist for the autoscaling of OI oblique incidence ionograms. An iterative approach is under development by the authors that scales the oblique traces in a variational process assuming a multi-parabolic mid-point profile $N_c(h)$. One building block in this procedure is the calculation of $N_c(h)$ from oblique ionogram traces; this inversion is described in this paper.

2. Models and assumptions

Operational real time applications require computationally efficient algorithms suitable for the PC environment. It is therefore necessary to make some simplifying assumptions.

1) The ionosphere is spherically stratified.

2) Each ionospheric layer has a quasi-parabolic electron density distribution.

3) The geomagnetic field can be neglected when using the ordinary traces (Krasheninnikov *et al.*, 1994).

The electron density profile at the midpoint

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can then be written in the form (Croft and Hoogasian, 1968):

$$N(r) = \begin{cases} N_m\left\{1 - \left[\dfrac{r-r_m}{y}\right]^2 \left[\dfrac{r_b}{r}\right]^2\right\}, & \text{for } r_b < r < \dfrac{r_m r_b}{r_b - y} \\ 0 & \text{elsewhere} \end{cases} \tag{2.1}$$

where r is the geocentric radius, N_m is the maximum density at the layer peak r_m, and r_b is the base radius. The semi-thickness of the layer is then given by $y = r_m - r_b$. The index of refraction for the O-trace is given by

$$\mu_0^2 = 1 - f_N^2/f^2 \tag{2.2}$$

where f_N is the plasma frequency and f the sounding frequency.

Figure 1 shows a Composite Quasi-Parabolic (CQP) profile for a ionosphere with E, F_1 and F_2 layers. The objective of the inversion process is to find N_m (or the maximum plasma frequency f_m), r_m, and r_b for each layer from the oblique ionogram traces. The forward process of calculating the oblique ionogram for a distance D and a given mid-point electron density profile using eqs. (3.1a) and (3.1b) given below. For D = 2000 km and the $f_N(h)$ profile of fig. 1, the resultant one-hop and two-hop oblique traces are shown in fig. 2. The inverse process is described in the next section.

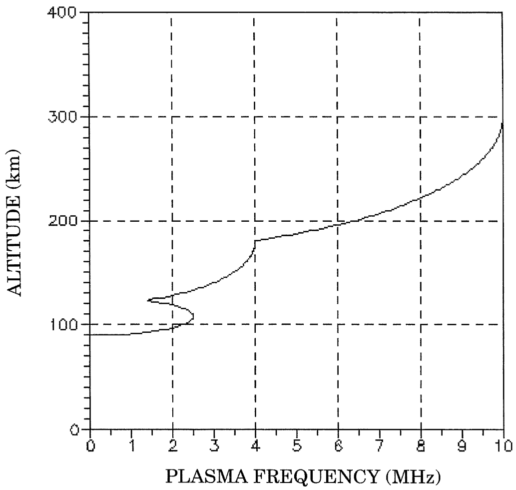


Fig. 1. Composite quasi-parabolic profile.

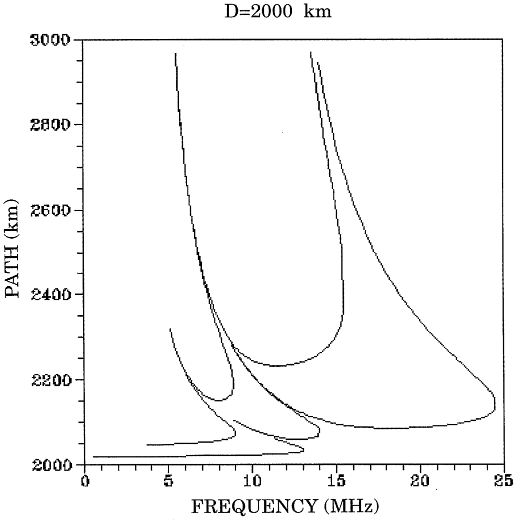


Fig. 2. Calculated one- and two-hop traces for a 2000 km ground distance for the profile in fig. 1.

3. Inversion procedure

A multi-variable minimization process is used to determine the CQP parameters from the oblique O-echo traces. It is assumed that trace data points (f_k, P'_k) are identified as belonging to a low or high angle trace section of a specific layer. The E layer parameters are determined first using data points only from the E trace, then the F_1 parameters, then the F_2 parameters.

The advantage of using the CQP model is that the ground distance D and the group path $P' = c \cdot t_g$ (c = speed of light, t_g = group travel time) can be expressed by analytical functions.

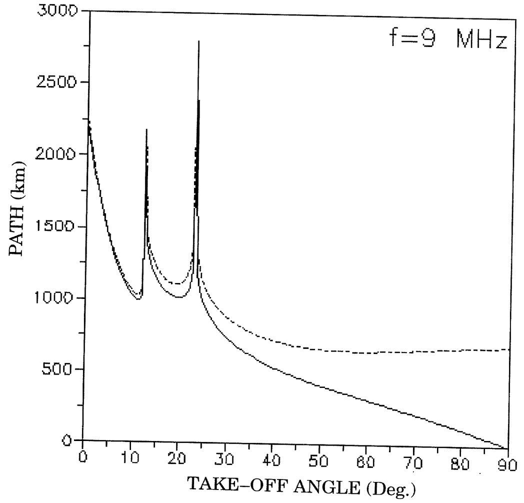


Fig. 3. Distance D (solid line) and group path P' as function of take-off angle for a 9 MHz signal and the profile of fig. 1.

For a signal reflected in the F_2 layer:

$$D = F_1\,(f,\ \beta, f_0E,\ r_{mE},\ r_{bE}, f_0F_1,\ r_{mF_1},\ r_{bF_1}, f_0F_2,\ r_{mF_2},\ r_{bF_2}) \qquad (3.1\text{a})$$

$$P' = F_2\,(f,\ \beta, f_0E,\ r_{mE},\ r_{bE},\ f_0F_1,\ r_{mF_1},\ r_{bF_1}, f_0F_2,\ r_{mF_2},\ r_{bF_2}) \qquad (3.1\text{b})$$

where f is the operating frequency, β the take-off angle of the ray, and f_0E to r_{bF_2} are the model parameters. Starting with the E layer, the parameters f_0E, r_{mE} and r_{bE} must be determined for each take-off angle during the homing process. The three-dimensional parameter surface $S(f_0,\ r_m,\ r_b)$ can be reduced to a two-dimensional surface $S(r_m,\ r_b)$ if the junction frequency f_j has been scaled from the OI ionogram. In that case the critical frequency f_m of the parabolic layer can be calculated as function of f_j, r_m and r_b

$$f_0 = f_0(f_j,\ r_m,\ r_b). \qquad (3.2)$$

The method of steepest decent is then used to find the optimal (r_m, r_b) values (in the least-squares sense) that best reproduce the measured oblique traces. The fitting process allows for partial overlapping of the parabolas.

To reduce the calculation time the search range for the take-off angle $\beta(f)$ is limited to only those values that are physically possible for the iteratively assumed profiles. In fig. 3, the functions $D(\beta)$ (solid lines) and $P'(\beta)$ (dotted lines) for a sounding frequency of 9 MHz are plotted for the profile given in fig. 1. The maximum take-off angles for E and F_1 layer propagation $\beta_{\max E}$ and $\beta_{\max F_1}$ (12.34° and

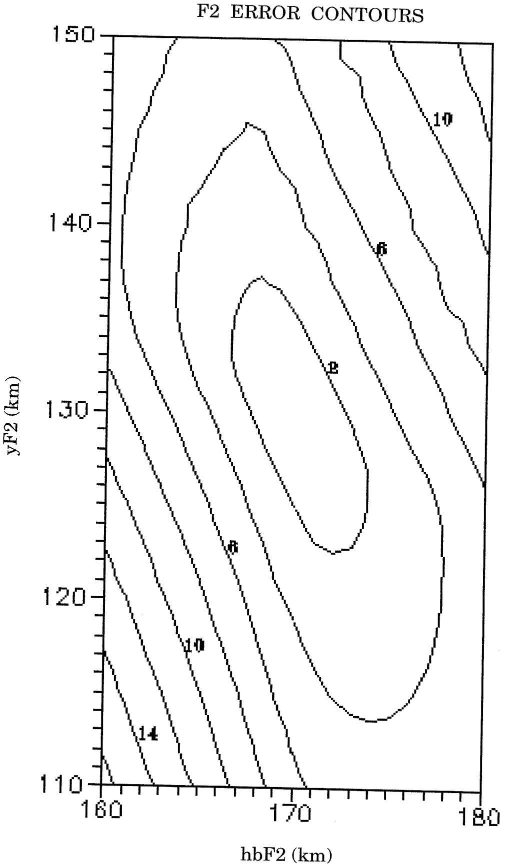


Fig. 4. The error $|P'_{\text{calc}} - P'_{\text{obs}}|$ as function of h_{bF_2} (horizontal) and y_{F_2} (vertical).

22.93° in this example) can be determined analytically. The homing process of finding the take-off angles for a given distance, for example 2000 km, can then be limited to values just above zero and just below $\beta_{\max E}$ for low and high angle E layer rays, just above $\beta_{\max E}$ and just below β_{maxF_1} for the F_1 layer rays, and just above $\beta_{\max F_1}$ for the F_2 layer ray. This qualified homing procedure reduces computation time by a factor of 40 compared to a brute-force homing approach. A typical homing process for the F_2 layer parameters is illustrated in fig. 4, where the error contours $|P' - P''_{\text{observed}}|$ are plotted versus r_{bF_2} (horizontal axis) and y_{F_2} (vertical axis). The total CPU time for calculation of the mid-point CQP on a 486 PC is only about 30 s.

The CQP algorithm has been applied to the oblique ionogram in fig. 5 between Wallops Island and Millstone Hill (D = 629 km) using GPS synchronized DPSs (Digisonde Portable Sounder; Reinisch *et al.*, 1992; Haines, 1994). The resulting mid-point profile $N_c(h)$ is shown in the lower part of fig. 6. The critical frequency, peak height and half width for each layer are listed on the right side. The upper plot in fig. 6 compares the observed ranges P'_{obs} (dots) with the calculated P' values (solid lines) for the profile $N_c(h)$. The agreement is very good.

Rcvr. ULCAR - MILLSTONE HILL, WESTFORD, MASSACHUSETTS
LAT. 42.6, LONG. 288.5 DIP 72.9 fH 1.4
Xmtr: ULCAR - WALLOPS ISLAND, VA
LAT. 37.9, LONG. 255.5 DIP 69.9 fH 1.4
Lowell Digisonde Portable Sounder VS. 04.93 SS# 70

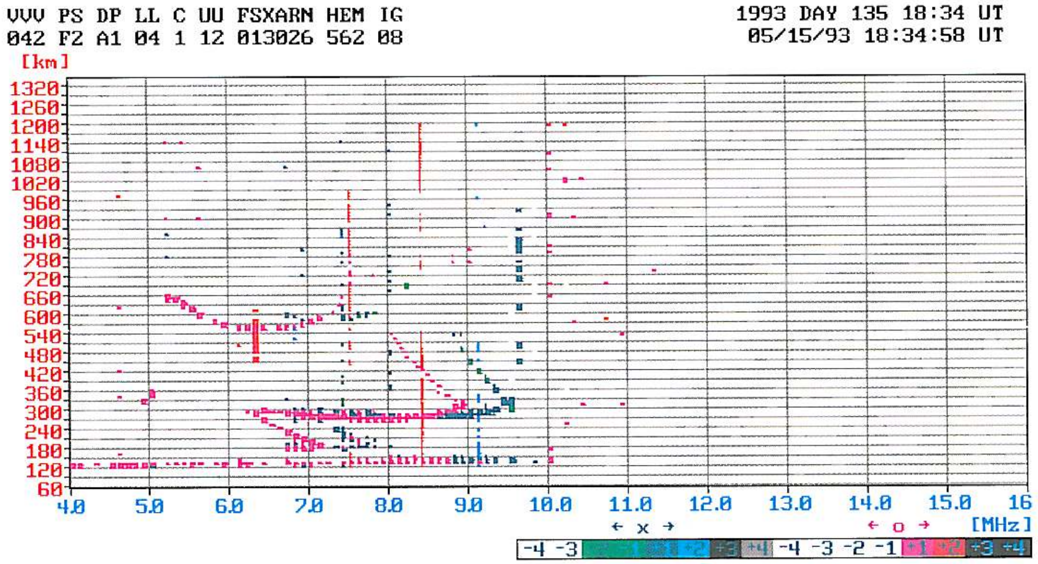
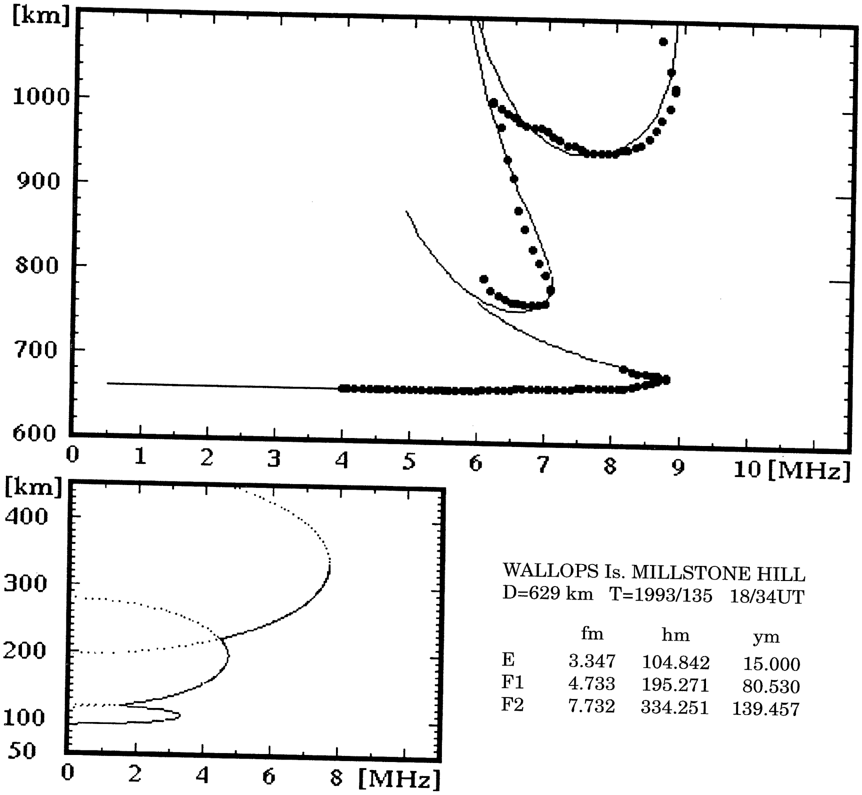


Fig. 5. Digital OI ionogram between Wallops Island and Millstone Hill (D = 629 km).



WALLOPS Is. MILLSTONE HILL
D=629 km T=1993/135 18/34UT

	fm	hm	ym
E	3.347	104.842	15.000
F1	4.733	195.271	80.530
F2	7.732	334.251	139.457

Fig. 6. Calculated mid-point profile (bottom) for the ionogram in fig. 5. The top shows the recalculated ionogram with the measured points (dots) overlaid.

4. Summary

A computationally efficient algorithm has been developed that calculates a composite quasi-parabolic mid-point profile from oblique ionogram traces. The CPU time required to calculate the profile is typically half a minute on a 486 PC.

Acknowledgements

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