The Kriging method of ionospheric parameter f_0F_2 instantaneous mapping

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Abstract
This paper discusses the spatial variation of the ionospheric characteristic f_0F_2 in the PRIME area (35°-55°N, – 10°-20°E) expressed in terms of the variogram and the Kriging mapping method as modelling of an instantaneous experimental situation. It is shown that by applying this method to the real data it is possible to estimate f_0F_2 at any unsampled location within the restricted area with satisfactory accuracy.

Key words *ionosphere – ionospheric mapping*

1. Introduction

The proper estimation of the ionospheric f_0F_2 parameter in the considered area of the ionosphere is the basic job of instantaneous ionospheric mapping. A simple averaging of samples of the randomly distributed vertical incidence soundings is not an appropriate simplification because these measurements are region-dependent variables. One of the mapping methods that regards these circumstances of measurements is Kriging.

The Kriging method is based on the characteristic variability demonstrated using the semivariogram, *i.e.*, the function that illustrates differentiation of the parameter value depending on the distance between the measurements. If the measurements are randomly distributed, the points which define the empirical variogram are grouped along the horizontal line. Kriging consists in giving to the particular measurements the weighting factors that assure the most accurate estimation of the unknown parameter (*e.g.*, Davis, 1986; Oliver and Webster, 1990).

When applied to region-dependent variables the Kriging method (Matheron, 1971; Nieć and Kokesz, 1992) has minimum variances. A computer contouring technique based upon this method is a useful tool in the evaluation and interpretation of ionospheric data (Samardjiev *et al.*, 1993; Bradley *et al.*, 1994). The analysis consists of the following steps: identification of data set periods, constructing and modelling the empirical semivariogram, obtaining the results of the interpolation procedure and comparisons with values obtained by a commercial SURFER program that also uses Kriging interpolation and measured values.

2. Semivariogram constructing and modelling

It is well known that the semivariogram summarizes the general form of the variation, its magnitude and spatial scale. The precision

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of the estimated semi-variances depends on the sample size and on the distribution of the data. If the semivariogram is anisotropic, *i.e.*, it is systematically different in different geographical azimuths, the coordinate-dependent scaling factor to the latitude should be introduced (Bradley *et al.*, 1994). To estimate this factor the cross correlation coefficients R between the measurements at all available PRIME stations was calculated for the 1989 year and for May 1989 separately (see table I for station list). The calculations were made independently for 10 quiet (Q) and for 5 disturbed (D) days for every month of the year (Gulyaeva, 1993).

The axial ratio of correlation ellipses (b/a = latitudinal/longitudinal distance) is shown in table II. These results agree with the correlation ellipse parameters in Gibson and Bradley (1991).

In figs. 1 and 2a,b we plot coefficient R according to difference in latitude ($\delta\phi$) and longitude ($\delta\lambda$) separately for 1989 and May 1989, respectively. The coefficients of regression lines are given. Different slopes at the regression lines for $\delta\lambda$ and $\delta\phi$ are the clear indication of the anisotrophy in the behaviour of f_0F_2. Apparently especially during May 1989 quiet and disturbed days are very well separated. Note, however, that the anisotropy is more pronounced when shorter data segments are analysed. In this case the semivariogram is anisotropic and the scaling factor in the Kriging method is determined by the axial ratio of the correlation ellipse.

Table I. The list of the stations.

Station	Lat.	Long.	Station	Lat.	Long.
Uppsala	59.8	17.6	Pruhonice	49.5	14.6
South Uist	57.4	−7.3	Lannion	48.8	−3.5
Kaliningrad	54.7	20.6	Poitiers	46.6	0.3
Juliusruh	54.6	13.4	Rome	41.9	12.5
St. Peter Ording	54.3	8.6	Lisbon	38.8	−9.2
Dourbes	50.1	4.6	Gibilmanna	37.6	14.0

Table II. Axial ratio of the correlation distance (b/a).

Correlation coefficients	Axial ratio			
	1989		May	
	Quiet	Dist.	Quiet	Dist.
0.4	0.773	0.749	0.669	0.734
0.5	0.742	0.717	0.667	0.711
0.6	0.703	0.678	0.664	0.684
0.7	0.651	0.628	0.658	0.654
0.8	0.580	0.562	0.647	0.618
0.9	0.476	0.471	0.605	0.575
0.95	0.403	0.411	0.480	0.550
0.96	0.386	0.397	0.379	0.545
0.97	0.368	0.383	0.064	0.540

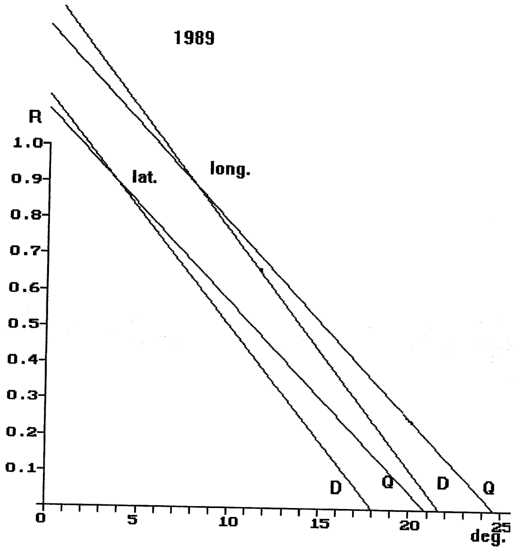

Fig. 1. R dependence on longitude difference and latitude difference between stations for 1989 year. The coefficients of regression lines are: longitude, quiet (Q): 1.33, – 0.05, disturbed (D): 1.42, – 0.07, latitude, quiet (Q): 1.10, – 0.05, disturbed (D): 1.14, – 0.06.

Figure 3 shows the data from about ten ionospheric stations for constructing the instantaneous semivariogram. Each point represents

$$X = \text{ionospheric distance between two stations } (A) \text{ and } (B) \qquad (2.1a)$$

$$Y = (f_0F_2(A) - f_0F_2(B))^2 \qquad (2.1b)$$

where the «ionospheric distance» is defined as follows

$$d = \sqrt{(\mathrm{Lon}\,(A) - \mathrm{Lon}\,(B))^2 + (SF \times (\mathrm{Lat}\,(A) - \mathrm{Lat}\,(B)))^2} \qquad (2.2)$$

SF is the scale factor. Figure 3 is typical in the sense that the pictures for all other moments also have very random distribution.

Figure 4 shows the data of the same type, but for about 700 moments of time. Dots represent real data and the circles represent averages over 10 intervals in $\delta\lambda$ and $\delta\phi$. The linear approximation to the averages

$$\text{Semivariogram} = c_0 + c_1 \times d \qquad (2.3)$$

is determined by the coefficients c_0 and c_1. Table III shows the dependence of the correlation coefficient R and coefficient c_0 on the scaling factor SF. We conclude that instantaneous semivariograms are useless while averaged data give a reasonable semivariogram that can be approximated by a linear function. For our station set, the scaling factor corresponding to the highest correlation coefficient is $SF \simeq 2$ (see table III). Then the semivariogram coefficient c_0 can be neglected and the approximation takes the form

$$\text{Semivariogram} = c \times d \qquad (2.4)$$

where d is the ionospheric distance and c is a constant. In the Kriging method the values from the semivariogram enter the system of linear equations from which the weights can be obtained (Nieć and Kokesz, 1992). The resulting weights do not depend on the value of c (see eq. 3.2a), so finally d itself will be used instead of a semivariogram.

3. Example of Kriging interpolation

The input data set for the Kriging interpolation consists of N points. For each i-th point the coordinates X_i, Y_i are given and Z_i – the value of the ionospheric parameter in the point (X_i, Y_i). On output the method gives Z_0 – the interpolated value in a given point (X_0, Y_0). The result is a weighted average of all input values Z_i

$$Z_0 = \Sigma\,(W_i \times Z_i); \quad i = 1 \ldots N \qquad (3.1)$$

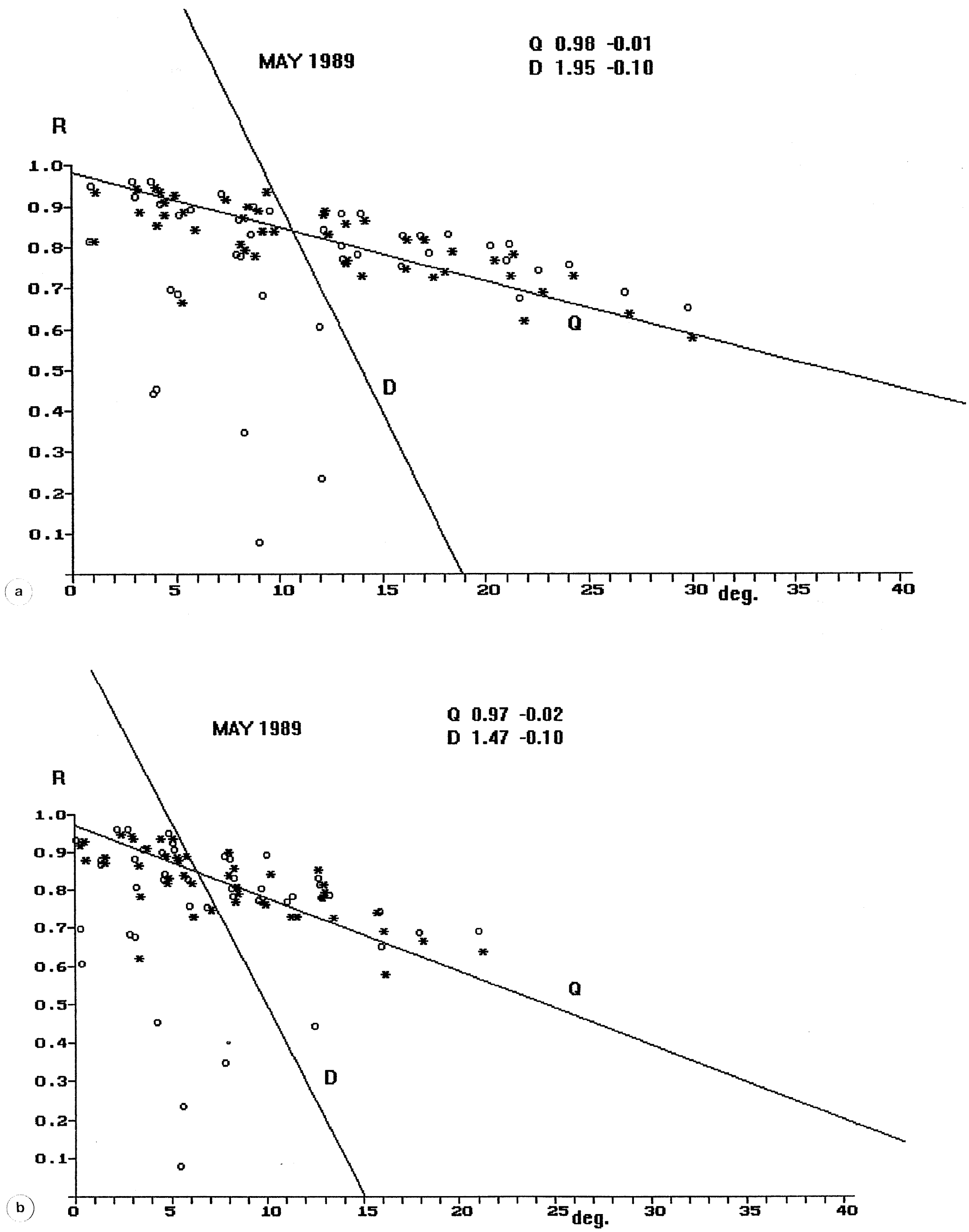

Fig. 2a,b. a) *R* dependence on longitude difference between stations for May 1989. The coefficients of regression lines are inclosed in the upper right corner of each figure (Q = quiet, D = disturbed conditions); b) *R* dependence on latitude difference.

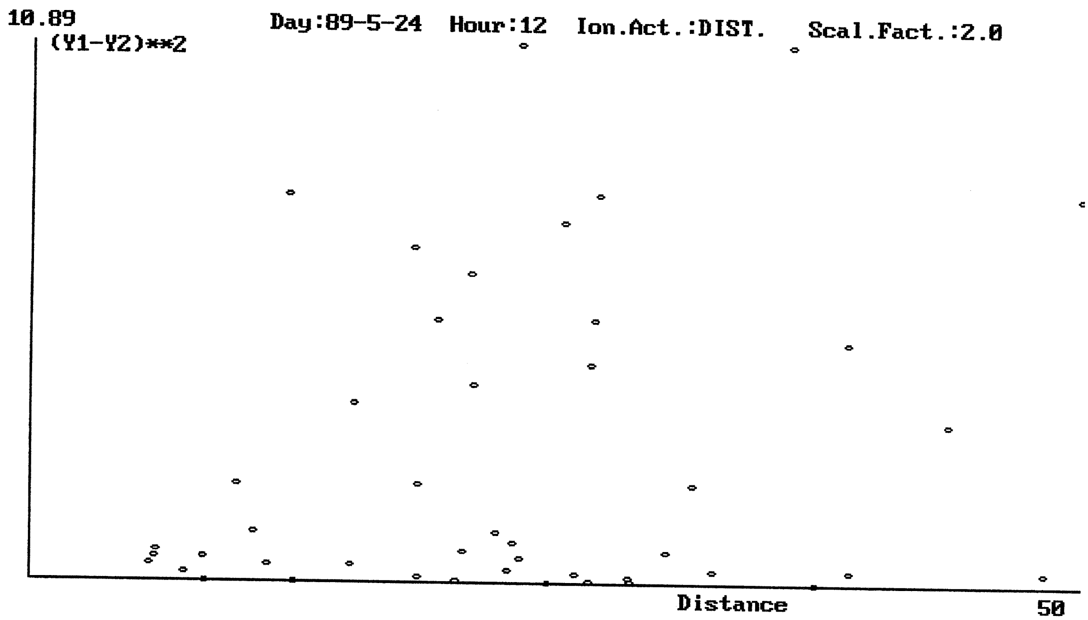

Fig. 3. Data for constructing of semivariogram for a single moment of time.

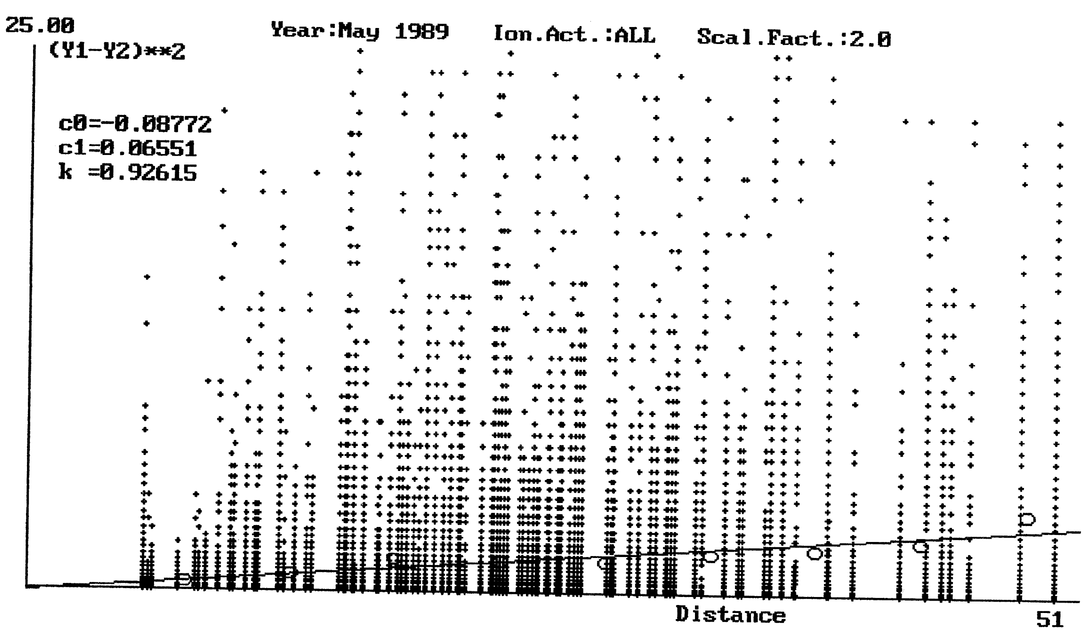

Fig. 4. Semivariogram made on the data for about 700 moments of time.

Table III. c_0 and R as a function of *SF*.

SF	*R*	c_0
1.0	0.78	− 0.5
1.1	0.78	− 0.3
1.2	0.81	− 0.5
1.3	0.81	− 0.4
1.4	0.80	− 0.4
1.5	0.72	− 1
1.6	0.71	− 1
1.7	0.72	− 1
1.8	0.87	− 0.01
1.9	0.88	− 0.02
2.0	0.93	− 0.09
2.1	0.96	+ 0.03
2.2	0.96	+ 0.05
2.3	0.95	+ 0.05
2.4	0.84	+ 0.04
2.5	0.84	+ 0.04
2.6	0.83	+ 0.6
2.7	0.64	+ 0.7
2.8	0.65	+ 0.6
2.9	0.73	+ 0.6
3.0	0.72	+ 0.6

Weights W_i are obtained from the system of N+1 linear equations

$$\sum (V_{ij} \times W_i) = V_{j0} - \text{lambda};$$
$$i, j = 1 \ldots N \text{ (} j\text{-th equation)} \qquad (3.2a)$$

$$\sum W_i = 1; \quad i = 1 \ldots N \text{ (} N\text{+1 -th equation)} \qquad (3.2b)$$

where: V_{ij} – value of the semivariogram for the ionospheric distance between i-th and j-th points which is in our case the distance itself instead of the semivariogram; lambda – Lagrange factor.

Lets assume the following notation for these equations

$$A \times W = B \qquad (3.3)$$

where

$A - (N+1) \times (N+1)$ array,
W, $B - (N+1)$ vectors.

Vector W consists of N weights W_i and lambda.

An example called «Kriging Report» of the Kriging interpolation follows. In the report after the line «Input *X Y Z*» there are 3 columns:

Table IV. Results of Kriging for 16 cases.

Date/hour		Dourbes	Rome	Uppsala	Lisbon
1989-05-18/01	(our Kriging)	7.85	7.50	6.67	8.02
	(SURFER Kriging)	7.85	7.50	6.64	8.02
	(measured value)	(7.7)	(8.5)	(6.9)	(9.4)
/12	(our Kriging)	8.84	8.41	8.33	7.70
	(SURFER Kriging)	8.84	8.41	8.31	7.69
	(measured value)	(8.2)	(10.5)	(8.2)	(8.7)
1989-05-24/01	(our Kriging)	5.81	5.68	5.77	5.84
	(SURFER Kriging)	5.81	5.68	5.84	5.77
	(measured value)	(5.8)	(6.9)	(4.6)	(7.2)
/12	(our Kriging)	6.57	6.72	6.36	6.58
	(SURFER Kriging)	6.57	6.72	6.32	6.58
	(measured value)	(6.5)	(9.1)	(5.8)	(8.6)

Kriging report

Day 1989-5-18 Hour 1 Data type f_0F_2

Input X Y Z

20.6	54.7	6.50
−3.3	48.8	8.10
−9.3	38.7	9.40
0.3	46.6	8.40
12.5	41.9	8.50
−0.6	51.5	7.90
9.3	36.0	7.40
17.6	59.8	6.90

Matrix A

0.0000	26.6543	43.7951	25.9717	26.8509	22.1450	39.0698	10.6320	1.00
26.6543	0.0000	21.0723	5.6851	20.9781	6.0374	28.5328	30.3449	1.00
43.7951	21.0723	0.0000	18.4878	22.7200	27.0379	19.3680	50.0445	1.00
25.9717	5.6851	18.4878	0.0000	15.4013	9.8412	23.0313	31.5634	1.00
26.8509	20.9781	22.7200	15.4013	0.0000	23.2433	12.2262	36.1614	1.00
22.1450	6.0374	27.0379	9.8412	23.2433	0.0000	32.5424	24.6333	1.00
39.0698	28.5328	19.3680	23.0313	12.2262	32.5424	0.0000	48.3182	1.00
10.6320	30.3449	50.0445	31.5634	36.1614	24.6333	48.3182	0.0000	1.00
1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.00

Asked point 4.6 50.1 Result = 7.85 (7.7)

Vector B

18.4564	8.3169	26.7030	8.2152	18.2036	5.9059	28.5890	23.3529	1.00

Weights

0.1378	0.0225	−0.0445	0.2900	0.1048	0.4976	−0.0148	0.0067

longitude, latitude and measured f_0F_2. In the line «Asked point» the first two numbers are the longitude and latitude of the point in which interpolation is made. After «Result =» there is: result of interpolation and in brackets the measured value. Matrix A and vectors B and W are also shown. The summary of 16 interpolations with comparison to the measured values is presented in the table IV. This table also contains the results from the commercial (Golden Software, Inc.) SURFER version of Kriging with introduced scaling factor $SF = 2$.

There are practically no differences between the results of our method and the SURFER program. A reasonable agreement between Kriging results and measurements is obtained.

4. Conclusions

Many ionospheric properties vary in an apparently random yet spatially correlated way. It seems profitable to analyze them using the method based on the theory of region-dependent variables. Using Kriging for interpolation of the ionospheric parameter f_0F_2 the results obtained fit the measured values well. The simplicity and accuracy are the main advantages of this method.

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