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Neural networks and dynamical system techniques for volcanic tremor analysis

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Abstract
A volcano can be seen as a dynamical system, the number of state variables being its dimension N. The state is usually confined on a manifold with a lower dimension f, manifold which is characteristic of a persistent «structural configuration». A change in this manifold may be a hint that something is happening to the dynamics of the volcano, possibly leading to a paroxysmal phase. In this work the original state space of the volcano dynamical system is substituted by a pseudo state space reconstructed by the method of time-delayed coordinates, with suitably chosen lag time and embedding dimension, from experimental time series of seismic activity, *i.e.* volcanic tremor recorded at Stromboli volcano. The monitoring is done by a neural network which first learns the dynamics of the persistent tremor and then tries to detect structural changes in its behaviour.

Key words *neural networks – dynamical systems – time series analysis – volcanic tremor*

1. Introduction

Stromboli, one of the Aeolian Islands, located north of Sicily, Italy, presents a persistent volcanic activity which can be recorded by a seismometer both in terms of explosion-quakes, several times per hour, and of volcanic tremor, which changes its amplitude and spectral content but it is always present. The same kind of behaviour has been observed for hundreds of years without significant variations. This suggests the idea of considering the source of the seismic activity as a *dynamical system* (Birkhoff, 1927), *i.e.* a system which evolves with time but maintains some sort of «structural stability» (Carniel, 1993). Moreover, the persistence of such seismic activity gives volcanologists the opportunity to obtain many valuable data in a relatively short period of time. Data coming from a fixed seismic station installed by our Department near the summit of Stromboli (Beinat *et al.*, 1994) were extensively used in the developing stage of the method described in this paper. Although Stromboli is a very peculiar volcano, the method may be used to analyze seismic or even other kinds of data recorded at other volcanoes all over the world.

2. Dynamical systems

A dynamical system is characterized by several *state variables*, which are the components of its *state vector* $\boldsymbol{x}$. Denoted by N the dimension of the dynamical system, *i.e.* the dimension of the state vector, we generally observe that the dynamics is confined on a manifold (*attractor*) which has a lower, usually fractal (Mandelbrot, 1977) dimension f.

Of course the ground motion is not the only state variable of the volcano dynamical system, but it is the only one we suppose available. We therefore have to reconstruct a multidimensional pseudo state space using the well known time-delayed coordinates method (Packard *et al.*, 1980): delayed values of the same time series $\{R_i\}$ are used as different coordinates to build n-dimensional vectors

$$x_i(R_i\,,\,R_{i+\tau,}\,R_{i+2\tau,\,\ldots,}\,R_{i+(n-1)\tau}).$$

A trajectory of reconstructed points is guaranteed to be the image of the original trajectory under a map which is differentiable and invertible, if the condition $n > 2f$ holds (Takens, 1981). Of course Takens' theorem cannot be applied in practice because the dimension of the stable manifold f is as unknown as the dimension N of the original state space. All we know is that the so-called *embedding dimension n* must be kept as low as possible in order to reduce the computing time of subsequent analyses.

Secondly, we must choose the value of the delay τ used in the reconstruction. This choice, although not mentioned in Takens' theorem, may seriously affect the goodness of the reconstruction.

3. Neural networks

A neural network (see fig. 1) is a computational tool composed of a set of neurones – the circles of the graph – connected by synapses – the arcs. The definition goes back to the 40's (McCullogh and Pitts, 1943), although it was not practically used until the advent of modern computers. Each neurone reads its input through the synapses, combines them linearly with weights associated to each of them and passes to its output synapses the result of the application of a suitably defined non-linear activation function to the linear combination.

In the training phase the neural network has to «learn its task» with a set of examples. The interconnection of nodes and arcs being fixed, this can only be done by changing the weights associated to the different synapses. A back-propagation updating algorithm is often used, which goes back from the output to the input changing the weights according to the derivative of the error signal with respect to the weight under update, *i.e.* descending the error surface in the direction opposite to the gradient.

Several architectures exist for neural networks, which can be supervised or not, *i.e.* trained with a set of input-output couples or just with a set of inputs. A particular kind of supervised neural network is the auto-associative one, whose target is to reproduce at its output just the data received as input.

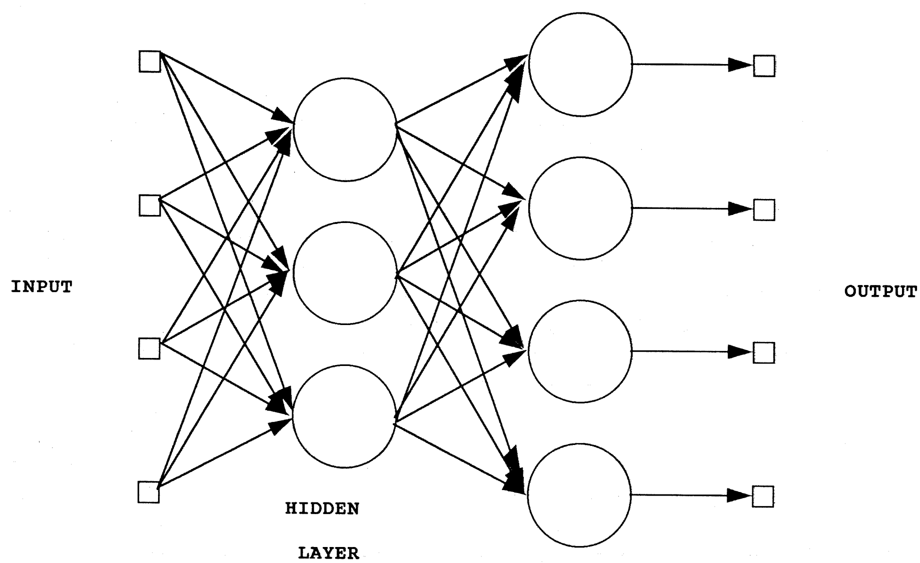

Fig. 1. Architecture of a neural network.

A recent work (Romeo and Taccetti, 1994) highlighted the possibility of using an auto-associative neural network as a trigger mechanism for a seismic station. The neural network is trained to mimic the seismic «normal activity», recorded by the instrumentation in absence of interesting seismic events; the data acquisition is then triggered each time there is an increase in the error signal of the neural network, *i.e.* a divergence between the «predicted» and the observed values. The authors show good results also when there is a need to detect unevident patterns such as small or far earthquakes, which would not be triggered by classical threshold or STA/LTA algorithms.

The drawback of the method is that, as the error function does not change abruptly, it cannot be used to determine the first arrival of a seismic wave precisely; however, it is highly suitable as a pre-trigger algorithm in order to recognize events in the presence of a low signal-to-noise ratio, leaving to other methods the task of picking up the different phases of the seismic event.

4. A combined dynamical – neural method

In the following, the feasibility of the application of a similar idea to a quite different problem is investigated, *i.e.* the analysis of a pseudo state space of the dynamical system governing the activity of a volcano.

A time series representing a persistent volcanic tremor, *i.e.* the one continuously recorded at Stromboli, is considered as a state variable of a dynamical system governing the behaviour of the whole volcanic system (Carniel, 1993). From this single variable a pseudo-state space is reconstructed using the method of time-delayed coordinates. The lag time is chosen in correspondence of the first minimum of the mutual information or redundancy function (Fraser and Swinney, 1986), while the choice of the embedding dimension is made according to the method of false nearest neighbours (Kennel *et al.*, 1992). Details of the embedding procedure for tremor data will be given in the next section.

It can easily be understood that any «change» in the characteristics of the dynamics of the reconstructed system may suggest a corresponding variation in the dynamics of the real volcano, a variation which could be not evident with the «classical» analysis tools, either in time or in frequency domain. An auto-associative neural network is therefore used to investigate those changes.

Given the architecture of the neural network, which is the one shown in fig. 1 with outputs forced to be the same as inputs, the first thing to be fixed is the number of inputs, outputs and hidden nodes, these latter being obviously the nodes in the first layer which are not connected to the output.

The idea is to study the relation of each reconstructed point to the attractor of the volcano dynamical system. Therefore the embedding dimension n is used to fix the number of inputs and outputs of the neural network.

The choice of the number of hidden nodes requires some discussion. Although no theoretical results exist to make this choice objectively, methodologies are being developed to estimate the optimal number (see *e.g.*, Vysniauskas *et al.*, 1993); trial-and-error procedures are however still the most used in order to find the best configuration for the network.

In this work we will limit ourselves to a number of hidden nodes smaller than the number of inputs and outputs; there are several reasons for this choice. The first is that the points we want to simulate «live» in an n-dimensional state space but are supposed to belong to a manifold with a smaller, fractal dimension f. The dynamics therefore should not span the entire n-dimensional space and a data compression of the points should be possible. Such a data compression while passing from input to output would also avoid the risk of weights being changed trivially in order to copy the input data to the output; in this case the network, although showing good performance in the prediction, would not «learn» much about the way the mechanism under study is working.

For the first reason discussed above, an integer approximation of f would be a good choice to fix the number of nodes in the hidden layer, both forcing a compression of the data representing the tremor observations and trying to respect the real dynamics of the system. The first method to determine the value of f was published some ten years ago (Grassberger and Procaccia, 1983); several others were developed by other authors in order to solve the lack of objectivity of the original method, which could result in very different results if applied by different researchers. Nevertheless, the problem is still quite difficult, especially in the presence of a low signal-to-noise ratio (Casdagli *et al.*, 1991) and too short datasets (Ramsey and Yuan, 1990). It is useful to remember that the demand for data and the related difficulty of determination of f increase exponentially with its value.

The final reason to justify our choice of the number of hidden nodes is found directly on the procedure we want to carry out. In fact, as we will discuss in the following, we are not interested in the *absolute* value of the error function but in the *ratio* of the error functions obtained using different weights. We have therefore to use the same architecture for all the tremor samples we analyze, although this may be not the optimal choice for some tremor regimes.

Therefore, the number of nodes in the hidden layer in this work is fixed to be just one less than the number of inputs and outputs. This assures the data compression suggested by the embedding of an f-dimensional manifold in an n-dimensional space and experimentally offers sufficiently good results.

5. Tremor data pre-processing and embedding

The method described in this paper was developed and tested using a set of tremor data recorded at Stromboli by a three component station (Beinat *et al.*, 1994) before and after one of the paroxysmal phases of 1993. Of course this is no more than a case study, as the change in the dynamics of the volcano had already been evidenced with a more conventional approach (Carniel *et al.*, 1994; Carniel and Iacop, 1996). In any case, the method has to be tested where a change definitely happened; a long term analysis can then be planned using the results of the case study in order to find other structural changes.

Two very strong explosions were felt at 1:10 GMT on October 16, 1993, with large blocks and spatter up to 2 m in diameter ejected 500 m from the crater, injuring one woman (GVN, 1993). Besides being a much larger manifestation than usual of the persistent strombolian activity, this marked a significant discontinuity in the external and seismic behaviour of the volcano, as can be argued looking both at the mean tremor amplitude and at its spectral content (Carniel *et al.*, 1994; Carniel and Iacop, 1996).

As outlined above, the first step in the application of the combined neural – dynamical procedure is to embed tremor data, *i.e.* to reconstruct the pseudo-state space.

The first choice is that of the lag time, *i.e.* the time separating the time series values taken as different coordinates of the reconstructed points. These values should be as independent as possible; while the first zero of the autocorrelation function assures linear independence, the first minimum of the redundancy function, an application of Shannon's information theory to time series, guarantees a more general independence. Without entering into too many details, which can be found in (Fraser and Swinney, 1986), we can say that the mutual information or redundancy function for a given lag time τ measures the number of bits of the time series value $R_{i+\tau}$ which we can predict on the average, given the value of R_i. We therefore look for values of τ for which the redundancy is minimal; in practice, we simply choose the first minimum of the mutual information as a function of τ. The method described was used in order to determine the optimal lag time for the tremor data. Figure 2 shows the value of the first minimum of the redundancy function for the radial, tangential and vertical component using tremor samples recorded at the same hour (around 12 GMT) of different days of October, both before and after the strong explosions.

Although the picture may suggest quite different values for the various samples and/or components, fig. 3 shows that as a matter of fact the behaviour of the redundancy function,

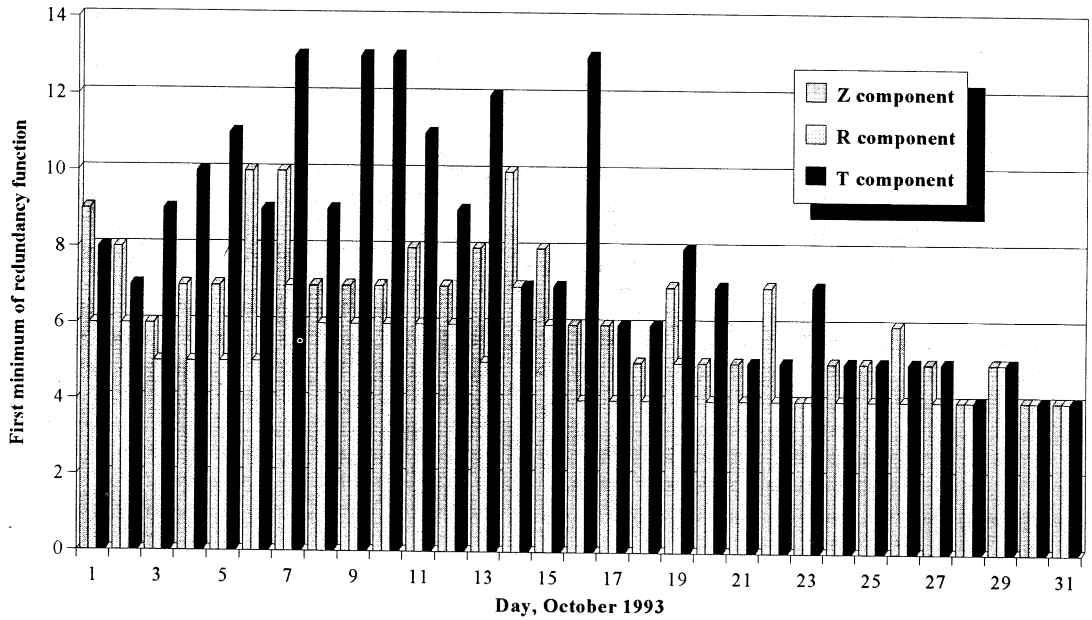

Fig. 2. First minimum of the redundancy function for different tremor samples during October 1993. The vertical scale is expressed in sampling periods of 0.0125 s.

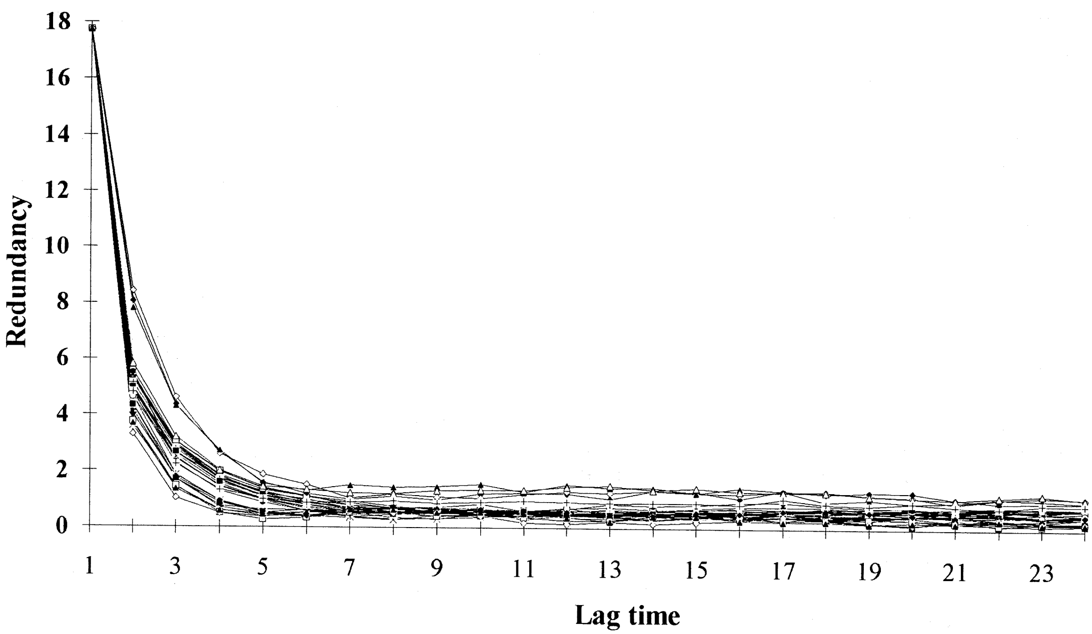

Fig. 3. Behaviour of the redundancy function for vertical component of different tremor samples during October 1993.

here shown for the vertical component of all tremor samples, does not vary so much and the choice of the minimum is not so critical as long as lag times less than 5 are avoided. Having made these considerations, and for the already mentioned reason that we have to process all the data in exactly the same way, a unique lag time was chosen, its value being set equal to 6 sampling intervals which, at a sampling frequency of 80 Hz, correspond to 0.075 s.

The second choice is that of the embedding dimension. The method of false nearest neighbours was used (Kennel *et al.*, 1992). The idea of the algorithm is based on the successive reconstruction of pseudo-state spaces with larger and larger dimensions. When passing from an embedding dimension d to the next one $d + 1$ it is possible to divide the couples of points constituting the d-dimensional trajectory among *true* and *false* neighbours, these latter being those points which appear to be near just because we are looking at the trajectory in a dimension which is still too small. This is exactly what happens when one sees a couple of points in a photograph which appear to be very near but belong to two distinct objects which are far from each other in reality; the appearance of vicinity is simply due to the fact that a 2-dimensional object such as the photo has not a sufficiently high embedding dimension to represent 3-dimensional real objects. The error would be solved passing to a 3-dimensional embedding such as a hologram but no further advantage would be gained passing to a hypothetical 4-dimensional one.

Starting from a one-dimensional time series and iterating the computation of the percentage of false neighbours in the transition from one embedding to the next, the correct embedding dimension will then simply be the one that presents no more false neighbours, while the percentage of false neighbours still surviving in lower dimensions gives the estimate of the error of a reconstruction in such spaces. The main criterion used by the algorithm to detect false neighbours is straightforward: the increase in distance between two embedded points is large when going from dimension d to dimension $d + 1$. However, this criterion is not sufficient for determining a proper embedding dimension. The problem is that with finite data the nearest neighbour of a point is necessarily *close* to it. In order to deal with the problem of increasing sparseness of the points in the higher dimensional reconstructed spaces a second criterion is added, relating the distance in each reconstructed space to an estimate of the size of the manifold; all the details can be found in (Kennel *et al.*, 1992).

Of course when analyzing real data the percentage of false nearest neighbours does not necessarily go exactly to zero. The correct embedding dimension should then be the one for which such percentage falls below a certain threshold, which as we have seen represents the error expected in that embedding.

Figure 4 shows the values the method just described suggests for a number of tremor samples recorded during the days preceding the explosions of 16 October 1993 in the radial, tangential and vertical component. One can immediately note how the embedding dimension increases considerably if one requires a very low percentage of false neighbours, *i.e.* a very small error in the reconstruction procedure. It is trivial to understand that there is a major drawback in the computing times of any subsequent analysis of reconstructed data if one chooses a high dimension. This is particularly true in the case of the neural network, where the training algorithm has to fix the values of all the weights which, according to our network architecture, are $2 \cdot n \cdot (n - 1)$, if n is the embedding dimension. An embedding dimension of 5 was therefore chosen; this choice, as can be seen from fig. 5, guarantees an acceptable reconstruction error while maintaining a reasonable complexity of the neural network to be trained.

6. Neural network construction

Using the results in terms of optimal lag time and embedding dimension described in the previous section, a neural network was built with 5 inputs, 5 outputs and 4 hidden nodes. The input and output data are reconstructed points, *i.e.* 5-ples of tremor time series

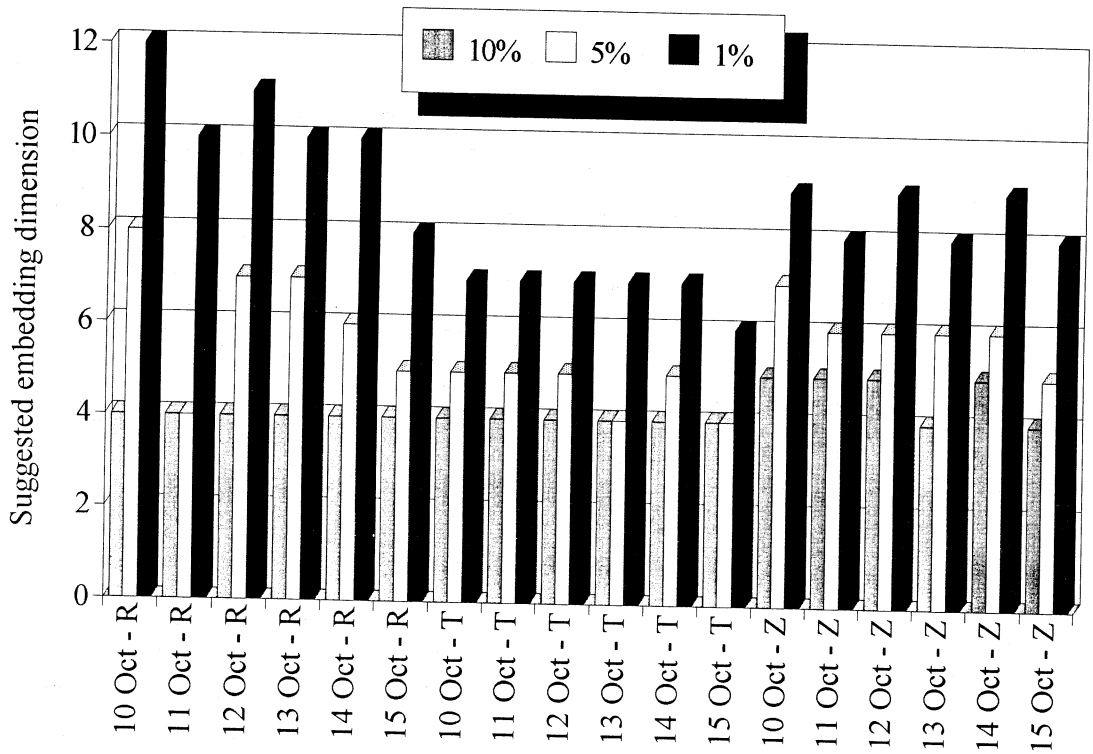

Fig. 4. Suggested embedding dimension by false nearest neighbours method for different October 1993 tremor samples and different thresholds.

samples separated by a lag time equal to 0.075 s.

Tremor records used to feed the network are 60 s long, *i.e.* 4800 points sampled at 80 Hz. For each record the following procedure is applied:

– the weights of the synapses are set equal to the ones computed by training the network with the previous tremor record (we will call them *old weights*);

– the current record is processed by the network and the mean value of the error function, *i.e.* the modulus of the difference between the output data and the real input data which the network is supposed to mimic, is computed;

– initialized with the old weights, the network is now trained again by the back-propagation scheme, using as input data the current tremor record. The values of the weights are stored at the end of this training phase; we will call them *new weights*;

– the neural network is now used again to process the current tremor record, this time with the new weights; the mean value of the error function is computed once again.

Changes in the dynamics, in the framework of the volcano dynamical system, may be of two kinds: internal changes and external ones.

Supposing that the dynamics is developing on a manifold which can be called a strange attractor, an internal change may consist simply in a shift of the dynamics towards a different region of the same attractor. It is well known that an attractor is a completely invariant set, *i.e.* the dynamics cannot lead us from a point of the attractor to another one which does not belong to it any more; moreover, it is topologi-

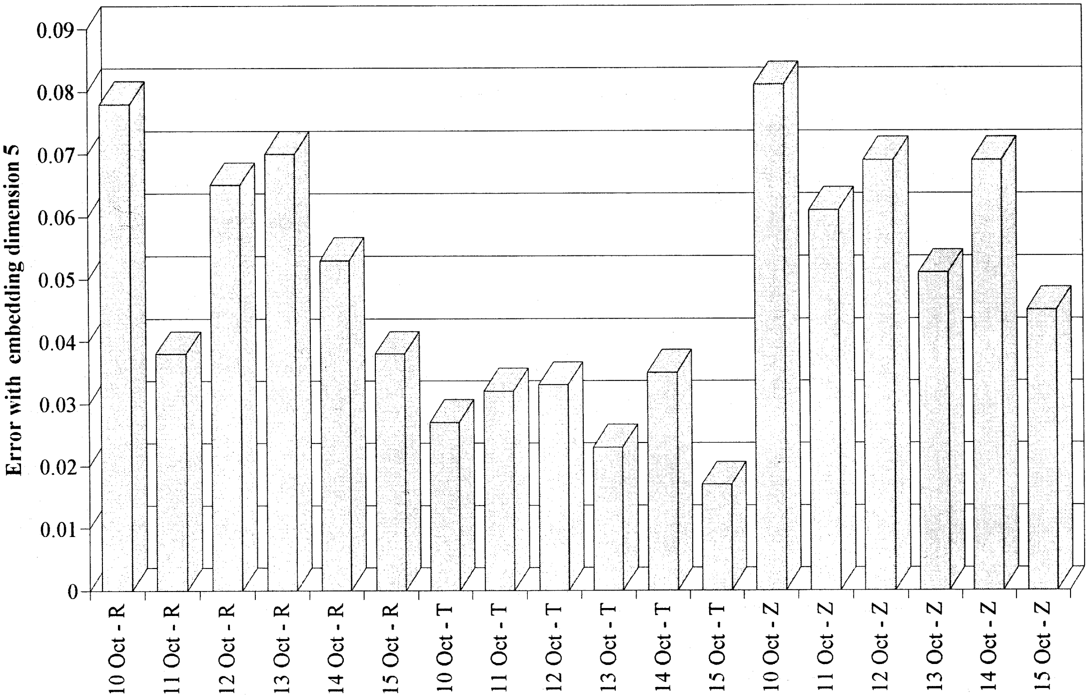

Fig. 5. Error estimate by false nearest neighbours method for different October 1993 tremor samples and embedding dimension 5.

cally transitive, *i.e.* it cannot be decomposed into two subsets which are invariant themselves. This means that starting from any point of the attractor, the dynamics will sooner or later enter a neighbourhood of any other point of the attractor itself. However, nothing is known about the frequency (or probability) with which each region of the attractor is visited. If we think of a particular region being associated to a paroxysmal phase, an internal change may be just an indication of the beginning of a paroxysmal phase.

A dynamical system may possess two or more different attractors for the same values of all its parameters; each of them being completely invariant, no jump is allowed between two of them unless an external intervention forces the system state to change abruptly. A dynamical system may also possess different attractors, perhaps in the same region of the state space, for different values of its parameters. Once again, an external intervention is needed in order to trigger the change of the parameter values. These are examples of external changes and, as a different attractor may be associated to a different external behaviour of the volcano, these also can be forecasters of a paroxysmal phase.

The problem is now that of finding a good parameter to monitor in order to signal possible changes in the dynamics, both internal and external.

The first parameter one can observe is the value of the error function, both in the first run, *i.e.* with the old weights, and in the second, where the new weights are used. Both these values suffer from the dependence from the tremor intensity: the error function usually increases when the tremor ground motion shows higher velocity values. On the other hand, a normalized error function does not seem to offer good results. A different parameter is therefore introduced, *i.e.* the ratio between the error function computed using the new weights and the one computed using the old synapses configuration.

This parameter has the advantage of being independent from the mean absolute value of the ground velocity and of being able to «catch» just what we are looking for, *i.e.* changes in the dynamics of the tremor and – hopefully – of the whole volcano system.

In fact, if such a change appears, the old weights, being computed in order to optimize the recognition of the dynamical features of the previous tremor sample, are not supposed to fit well the distribution of the reconstructed points of the new tremor record. On the other hand, new weights, being tuned according to the characteristics of the new tremor sample, are expected to offer good results, *i.e.* a low mean value of the error function. Therefore, the ratio between the mean error function computed with the old weights and the one observed using the new ones should be quite high.

In the «normal case», when analyzing two similar consecutive tremor records, the precision of the approximation should not be so dependent on the choice between the two sets of synapse weights; therefore, a value of the ratio close to 1 should be expected.

A difficult choice is that of the time separating two consecutive analyzed tremor records; in fact, a separation which is too long may result in a very complex evolution of the «old to new weights» ratio, as the old weights are not likely to work too well with a tremor record which is very distant in time. On the other hand, a phenomenon which causes a gradual change in the dynamics of the system will probably not be detected if the time lag between analyzed tremor records is too short, because the weights would be modified too slowly to cause a significant increase in the monitored parameter.

As regards the analysis of Stromboli tremor data, in this phase two kind of analyses were investigated: in the first case (daily analysis) a tremor sample per day was used, always recorded around noon (GMT), in the second (hourly analysis) all hourly tremor records recorded by our seismic station were used.

The results are plotted in figs. 6 and 7, respectively. For each case the sum of the «old to new weights» ratios computed independently for the three components of the ground motion (radial, tangential and vertical) is graphed together with the value of the sum of the mean of the error function obtained using the new weights for the same components. This is done in order to better reveal global modifications in the seismic activity. As one can see from both graphs, a change is clearly detected by the ratios between the dynamics before and after the big explosions of 16 October, 1:10 GMT. This is a first suggestion that the method is able to detect such changes.

Other peaks can be noted on the graphs, usually preceded by a value which is less than 3, which is the value we expect for a «stable» case (*i.e.* 1 for each component). This suggests that for the previous tremor sample old weights perform better than new ones in the recognition of reconstructed points. This may seem very strange at first sight, but it can be explained having a look at the tremor samples analyzed. In fact those were deliberately chosen without any further control to study the reaction of the neural network to a real situation in which a tremor sample is recorded every hour after a normal trigger procedure, which in our case is a combination of the application of both amplitude and frequency thresholds and of the well-known STA/LTA ratio.

As a matter of fact, the trigger procedure cannot have a 100% efficiency and parts of small explosion-quakes may be present, during periods of high strombolian activity, also in the tremor samples. The presence of (part of) an event in the very last part of a record is the worst case for the back propagation training method. In fact the new weights, tuned on the first part of the record, are changed by the algorithm when analyzing the very last points in order to try to model the distribution of the explosion-quake. This tuning cannot in general be completed because the number of explosion-quake points is insufficient and the synapse weights are fixed at the end of the training phase to a value which is not optimal

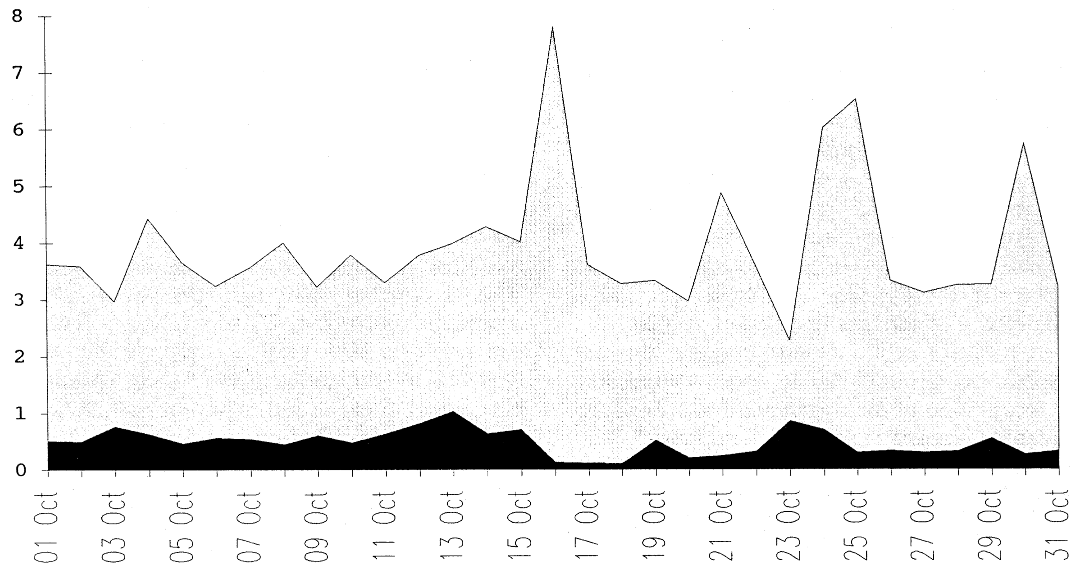

Fig. 6. Daily analysis of October tremor samples; solid area represents neural network error function with new weights, shaded area the ratio of results with old weights and with new ones.

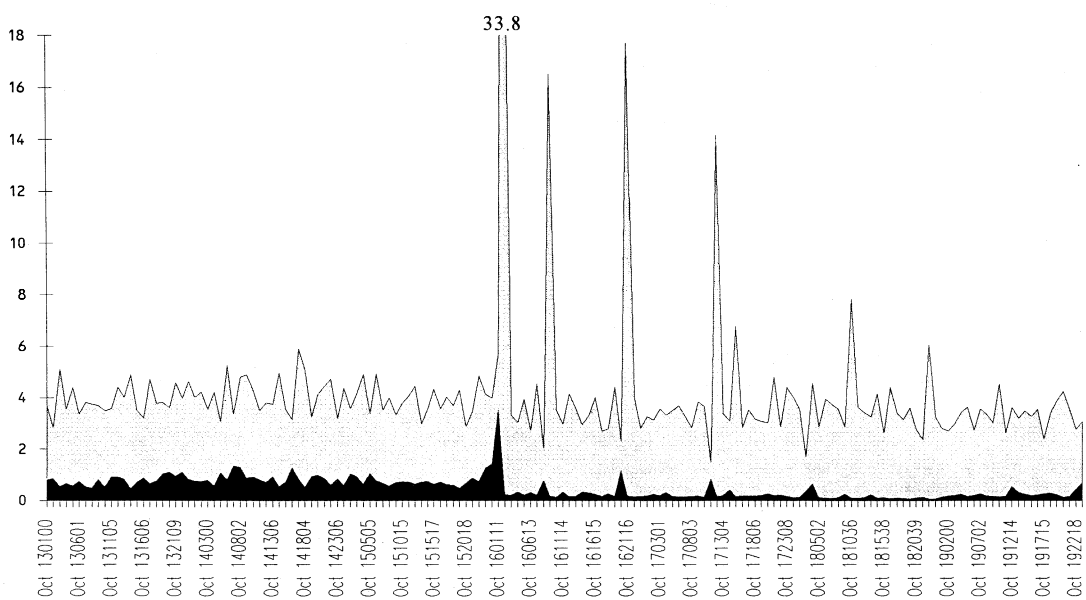

Fig. 7. Hourly analysis of October tremor samples; solid area represents neural network error function with new weights, shaded area the ratio of results with old weights and with new ones. The abscissa labels have the format «Oct ddhhmm»; the strong explosions were felt at Oct 160110.

to reproduce the explosion-quake but certainly not able to reproduce the volcanic tremor correctly. This causes the error function with the new weights to be worse than the one derived with the old ones, resulting in a very low «old to new weights» ratio.

It is now obvious that these new weights will not be able to reproduce the successive record correctly, resulting in a very high value of the «old to new weights ratio» in correspondence of the next tremor sample, signalling a change which is not of the kind we are looking for, *i.e.* the method fails, although with the good «side-effect» of detecting previously undiscovered events. A rule of thumb to identify the occurrence of this problem, a rule which can be automatically checked, consists in examining the value of the ratio just before the peak; if this is too low, the explanation could be the one described above. If, on the contrary, nothing strange is noticed in the previous value of the ratio or in the examination of the tremor records, this could be interpreted as a change in the dynamics of the tremor itself. Hints can be searched for also directly in the evolution of the error function for the new weights, but it is always important to remember that this is strictly linked to the evolution of the mean tremor level, and therefore a high value is not necessarily an indication of the new weights being badly tuned.

7. Conclusions

A volcano can be seen as a dynamical system and any experimental time series, *e.g.* each component of the ground motion derived from volcanic tremor, can be considered one of its state variables. After a pseudo-state space reconstruction with suitably chosen lag time and embedding dimension, a trajectory of N-dimensional points is analyzed by a neural network which first tries to mimic the behaviour with the synapses weights computed for the previous tremor sample, then with new ones tuned on the current record. The ratio of the two error functions is used as a monitoring parameter. This seems to be able to recognize changes in the dynamics of the system.

Unfortunately, undesired peaks are also present in the time evolution of the ratio, often more linked to the characteristics of the previous sample than to the ones of the current record; these «false alarms» can however be easily recognized with a careful analysis.

Some kind of integral value of the «old to new weights» ratio could also be introduced in order to identify slower changes which cannot be detected by a single peak of the ratio as it is. Of course an analysis extended over a long period of time is needed in order to test the method and maybe to find out different parameters to be monitored.

In fact, many minor changes to the algorithm were and can be experimented. The original tremor data can be rescaled to a common maximum value before the analysis, in order to minimize the highlighting of simple amplitude changes. Additionally, the sequence of the reconstructed points can be randomized in order to smooth the evolution of the old to new weights ratio and to reduce the highlighting of simple «post-event» tremor changes. Of course, the final interpretation of the type of change indicated by the neural network is always up to the researcher. The main purpose of the network is that of indicating points which deserve further analysis and this becomes more and more useful as the length of the dataset increases.

Although volcanic tremor ground motion of Stromboli seems to be quite a promising choice for the time series to analyze, the method could be tuned to be applied to any other physical variable almost continuously recorded on a volcano.

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