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Space-frequency analysis and reduction of potential field ambiguity

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Abstract
Ambiguity of depth estimation of magnetic sources via spectral analysis can be reduced representing its field via a set of space-frequency atoms. This is obtained throughout a continuous wavelet transform using a Morlet analyzing wavelet. In the phase-plane representation even a weak contribution related to deep-seated sources is clearly distinguished with respect a more intense effect of a shallow source, also in the presence of a strong noise. Furthermore, a new concept of local power spectrum allows the depth to both the sources to be correctly interpreted. Neither result can be provided by standard Fourier analysis. Another method is proposed to reduce ambiguity by inversion of potential field data lying along the vertical axis. This method allows a depth resolution to gravity or the magnetic methods and below some conditions helps to reduce their inherent ambiguity. Unlike the case of monopoles, inversion of a vertical profile of gravity data above a cubic source gives correct results for the cube side and density.

Key words *potential field – wavelet – interpretation*

1. Introduction

Interpretation of potential field anomalies has considerably improved in the last ten years. New instruments and survey methods together with powerful data processing, display and modelling methods have consistently enhanced the quality of the interpretation. In modelling methods, a common feature is the attempt to invert data with a reduced amount of ambiguity. Among them, we mention those using the high resolution properties of the derivatives of the gravity and magnetic field for estimating the source boundaries (Cordell and Grauch, 1985); Blakely and Simpson, 1986). It can be shown that when the sources may be reasonably assumed as shallow vertical prisms, boundary detection is very accurate. Since most geological sources consist in contacts and faults, the boundary estimations are therefore generally affected by a small degree of ambiguity.

Other magnetic prospection methods (Nabighian, 1984) are based on the analytic signal, which does not present any anomaly shape ambiguity, since it is only slightly depending on the Earth's magnetic field and magnetization directions. Source boundaries and approximate depths are efficiently computed from the analytic signal, with the only *a priori* assumption that the source is represented by a set of vertical contacts.

Other methods are based on the Euler deconvolution (Thompson, 1982; Marson and Klingelé, 1993), which is again not dependent on the Earth's magnetic field and magnetization directions. Consistent depths and source boundaries are obtained by *a priori* assuming for the source the so-called structural index, *i.e.*, some kind of shape factor.

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Data processing and interpretation has also improved due to the recent developments in the field of non-stationary and fractal signals. It has recently been pointed out that fractal magnetization distributions allow the corresponding field power spectra to have a power law exponent of about −2.9 (Pilkington and Todoeschuck, 1993). But, Fedi *et al.* (1997) also showed that the same exponent arises from an ensemble of magnetized blocks uniformly distributed. Correction for this power law spectral factor will improve the classical method of Spector and Grant allowing more accurate depth estimations.

As regards non-stationary signals, the discrete wavelet transform (Meyer, 1993) has proved more appropriate than Fourier Transform to deal with them and was also superior in residuation of potential field anomalies with respect to long period signals (Fedi and Quarta, 1997).

A different kind of *a priori* information is that the source-space is parametrized in a set of homogeneous blocks (a tomographic scheme). Li and Oldenburg (1996) assumed an empirical depth weighting function to counteract the decay of the kernel function together with positivity susceptibility constraints and a reference model. By this type of inversion, they obtained rather realistic models for the block magnetizations. By also assuming a tomographic scheme, Fedi and Rapolla (1995, 1997) and Fedi *et al.* (1997), were able to show that information on the field along the vertical direction is enough to give a depth resolution to the magnetic and gravity methods. Obviously this is not true for spheres or monopolar sources.

In this paper, we describe two approaches to reduce the interpretative ambiguity of potential fields. The first is related to the inversion of potential field data along a vertical direction. Our aim is to show that the inherent ambiguity of potential fields is not so serious as commonly believed. The second is based on a continuous wavelet transform analysis of magnetic anomalies. We will define the new concept of local spectrum and will propose a local version of the classical Spector and Grant method of depth estimations.

2. Inherent ambiguity and depth resolution of potential fields

The most typical example of inherent ambiguity is that of the fields generated at some fixed level by the infinite number of equal-mass homogeneous spheres centred at some point P. The ambiguity follows from the monopolar form of each potential:

$$\phi(r) = \frac{4}{3}\pi\gamma\frac{\rho a^3}{r} \tag{2.1}$$

since the two source parameters, ρ and the radius a, cannot be determined independently. Note also that external data along the radial direction, as well as along any other direction, do not reduce ambiguity since they invariably give redundant information.

Spheres or monopoles are widely used to illustrate nonuniqueness (*cf.* Menke and Abbott, 1990; Parasnis, 1979; Sharma, 1986). Instead, monopolar ambiguity is rather unrepresentative for most practical cases, as already observed by Al Chalaby (1971). For example, monopolar ambiguity obviously implies that the horizontal extent (diameter) of a sphere cannot be estimated from its field, unless its density is known. Nevertheless, many successful interpretation techniques assume exactly the opposite, *i.e.* the horizontal extent of a source is in principle detectable from its own field, with no *a priori* information about density. See, for instance, the above mentioned boundary analysis (Blakely and Simpson, 1984). Thus, since sources other than monopoles are used in practical applications, it is important to study them with respect to inherent ambiguity.

The class of concentric homogeneous cubes is particularly interesting, because cubes are multipolar sources very close to spheres. Kellog (1979) gives the first two terms of the multi-polar expansion of the gravity potential of a homogeneous cube:

$$\phi(r') = \gamma\frac{M}{r'} + \frac{7\gamma Ma}{60}\frac{[3r'^4 - 5(x'^4 + y'^4 + x'^4)]}{r'^9} + R_5 \tag{2.2}$$

where $M = \rho a^3$ and R_5 is the remainder. A reference system with axes parallel to the cube sides and the origin at the (known) mass center are taken. By vertical derivation of Φ and assuming that the truncation error is negligible, the gravity field is approximately:

$$g(r') \simeq -\frac{\gamma M z}{r'^3} + \qquad (2.3)$$

$$-\gamma a M \frac{[140\, r'^2 z'^3 + 105\, r'^4 z' - 315\, z'(x'^4 + y'^4 + z'^4)]}{r'^{11}}.$$

If the coefficients of the above expression were known, the density and the side of the cube could easily be evaluated. They may be computed, within some error induced by truncation, from a set of measured data. For example, let us examine a homogeneous cube, of which only the center is known, having a 1 g/cm^3 density and a 3 km side. Also consider an $N = 100$ data set along the vertical axis, the altitudes ranging from 1.5 to 5.5 km. Using eq. (2.3), the inverse problem is to solve for a and M an overdetermined linear system of equations of the form:

$$g_i = M k_{i1} + M a\, k_{i2}, \qquad i = 1, \ldots, N,$$

where:

$$k_{i1} = -\gamma / z_i^2;$$

$$k_{i2} = 7\, \gamma / 6\, z_i^6.$$

Using the svd method (Menke, 1984) we first note that the approximation (2.2) is acceptable, since the condition number c of the system matrix is fairly low ($c = 10$); then we compute the least square solution giving 0.984 g/cm^3 for density and 2.97 km for a. This example shows that inherent ambiguity for concentric cubes is completely different from that of concentric spheres. In fact, it results that: a) both density and volume can be determined, within some truncation error, from the gravity field at external points, and b) data along the vertical axis do not provide redundant information.

The property (b) is remarkable, since it leads to identification of an inherent depth resolution for the gravity and magnetic methods. To this end, let us consider that for shallow sources the gravity field is normally expected to have most of the energy in the high order multipole terms and the converse for the deepest ones. Consequently, since the high order multipoles decay rapidly with distance, the shallowest sources are better interpreted by near-ground data, while the deepest ones need relatively distant data. We therefore conclude that, unless property (b) does not hold, the best way of interpreting gravity or magnetic data is to jointly invert a set of data lying along all the coordinate axes. In this way, the information relative to data at different altitudes will provide an inherent depth resolution for the gravity and magnetic methods.

3. Space-frequency wavelet analysis

Most geophysical signals consist of transients and local oscillations superimposed on more regular and flat components. The gravity and magnetic anomaly fields may be clearly represented in such a way, due to the presence of intense and local effects of shallow sources and to the weaker and regular effects due to deep and extended sources. Fourier Analysis has provided significant insights into the field structure and the separation and interpretation of both local or regional effects. However, there is an inherent drawback in the Fourier Transform methods: the kernel of Fourier Transform is a sinusoidal function extended on the whole measurement interval, so that it uses global oscillations to analyze local ones. In other words, Fourier analysis has its best performance in analyzing stationary or quasi-stationary signals, *i.e.*, when its probabilistic behaviour is space-invariant up to the second order. Due to its global approach of analysis, Fourier-based spectral descriptions, like the power spectral density, are inherently non-localized in space. Instead, when rapid and unpredictable oscillations appear, the signal can no longer be considered stationary and different representations have to be tried, which may disclose the space variations of the spectral properties. Such techniques, which are specific to the nonstationarity of the signal, are here referred to as space-frequency analysis. From a mathematical point of view such analysis does not use global-space sinusoidal functions but space-frequency localized functions called space-frequency atoms. Wavelets, wavelet packets, local trigonometric functions or matching pursuit waveforms are different examples of such atoms, whose features have been increasingly applied to various physical and numerical fields such as coding speech, music (Mallat and Zhang, 1993), seismology (Chakraborty and Okaya, 1995), fractal signals, sea-floor bathymetry, climate changes, ocean wind waves, nonlinear denoising, signal compression (among others: Goupillaud *et al.*, 1984; Donoho, 1993; Jawerth and Sweldens, 1994). A wavelet analysis may be either continuous or discrete. Differentely from the Fourier analysis, owing to a fixed kernel, different wavelets may be used and the success of the analysis often depends on the appropriate choice of the analyzing wavelet. For example, Fedi and Quarta (1977) used discrete wavelet transform analysis and the triangle bi-orthogonal analyzing wavelet, to effectively separate the regional field from the localized anomalies of the aeromagnetic field of Sicily.

4. The continuous wavelet transform

In this paper we utilize a continuous space-frequency analysis of potential fields to define a new method of depth to source estimation, which acts in a local sense.

An admissible wavelet is a $L^2(\boldsymbol{R})$ function, $\psi(x)$ satisfying the following admissibility criteria:

a) zero mean: $\int \psi(x)\,dx = 0$;

b) compact support, or sufficient fast decay. Other additional properties may be required such as integrability of the first moment $x\psi(x)$.

Within such criteria, the continuous wavelet transform of a function $f(x) \in L^2(\boldsymbol{R})$ is defined as

$$\tilde{F}_w(s, b) = \qquad (4.1)$$

$$= \langle f, \psi(s, b) \rangle = \frac{1}{\sqrt{s}} \int_{\boldsymbol{R}} f(x)\, \psi\left(\frac{x-b}{s}\right) dx$$

where b indicates the location, $s \neq 0$ the scale and $\psi(s, b) = \frac{1}{\sqrt{s}}\, \psi\left(\frac{x-b}{s}\right)$.

The (s, b) plane is called the phase plane of the wavelet transform. The phase plane is layered with resolution cells of varying dimensions, but of constant area. In such a way the wavelet transform acts as a mathematical microscope of magnification inversely proportional to the scale. In contrast, for the windowed Fourier transform (Gabor, 1946), the phase plane is layered with resolution cells of fixed dimensions and area, so that it is applicable only when all the signal features have approximately the same scale.

Equation (4.1) indicates that the wavelet transform may be interpreted as a specific band-pass filtering of f through the chosen wavelet at a given scale and location. The reconstruction formula for the wavelet transform is given by

$$f(x) = \frac{1}{C_\psi} \int_{\boldsymbol{R}} \int_{\boldsymbol{R}^+} s^{-2}\, \tilde{F}_w(s, b)\, \psi(s, b)\, db\, ds,$$

where $C_\psi = \int \frac{|\,\tilde{\psi}(\omega)\,|^2}{\omega}\, d\omega < \infty$.

In the Fourier context, the scale s is analogous to the wavelength, while the scale number $p = 1/s$ is analogous to the wave number. If $\tilde{F}_w$ is the wavelet transform of f with the wavelet ψ, an analogue of the Fourier power spectrum can be defined as

$$P_w(s) = \int_{\boldsymbol{R}} |\,\tilde{F}_w(s, b)\,|^2\, db. \qquad (4.2)$$

Note that since the power at a given scale is determined from averaging many squared

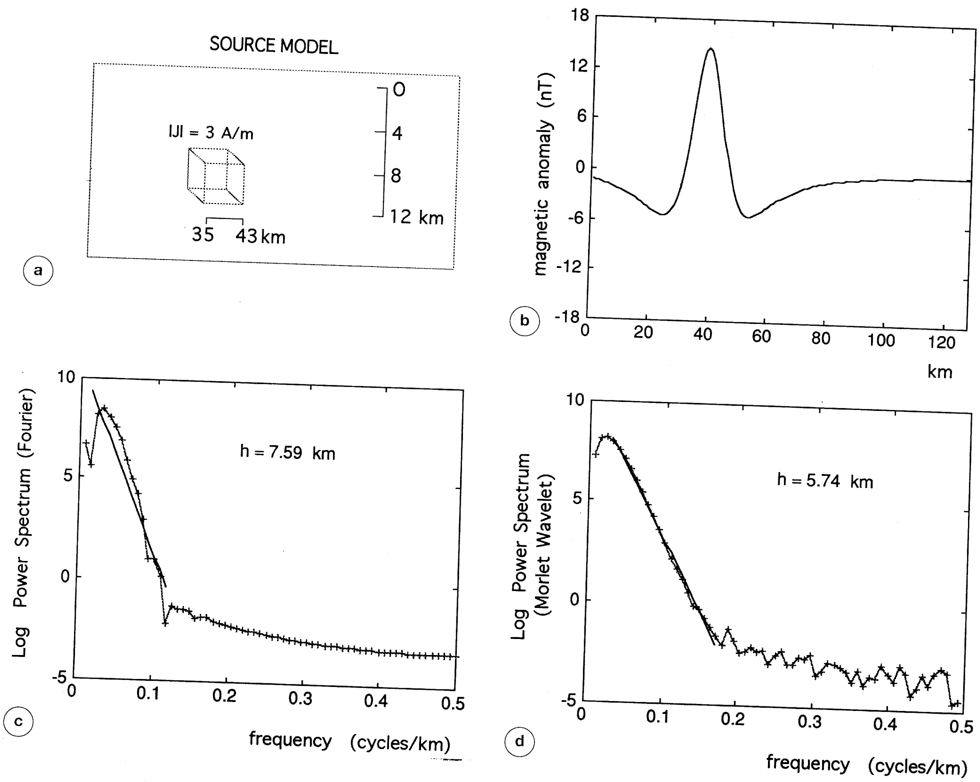

Fig. 1a-d. The wavelet and Fourier power spectra of a shallow magnetic source. The source shown in (a) generates the field in (b). Vertical polarization is assumed. According to the Spector and Grant theory (1970), the logarithm of its Fourier power spectrum (c) has a slope which is proportional to depth to the top of the source, apart from an overestimation error. The wavelet power spectrum (d) also gives a good depth estimate and is a smoothed version of the Fourier one, according to eq. (4.3).

wavelet coefficients, the wavelet power spectrum is expected to be a smoothed version of the Fourier power spectrum. In fact, it can be shown (Hudgins *et al.*, 1993) that the following relationship exists between wavelet ($P_{w,f}$) and and Fourier ($P_{F,f}$) power spectra of f:

$$P_{w,f}(s) = \int_R P_{F,f}(\omega)\, P_{F,\psi_s}(\omega)\, d\omega, \qquad (4.3)$$

where $P_{F,\psi_s}(\omega)$ is the spectrum of the wavelet at the scale s. In other words, the wavelet power spectrum at a given scale is the Fourier power spectrum averaged by the power spectrum of the specific analyzing wavelet at the same scale.

The smoothing effect may easily be shown by comparing Fourier and wavelet power spectra. We considered the magnetic anomalies due to two different prisms, measuring $8\times 8\times 3$ km^3 (fig. 1a,b) and $8\times 8\times 5$ km^3 (fig. 2a,b) respectively, with a 1 km step and an $N = 128$ data number. Their respective depths to the top were 2 and 6 km, so that we expect to see different slopes in the logarithm of their Fourier spectra. In fact, according to the Spector and Grant theory (Spector and Grant, 1970) the slope of the logarithm of the magnetic anomaly power spectrum is essentially proportional to twice the depth to the source top. This behaviour may be observed in figs. 1c and 2c,

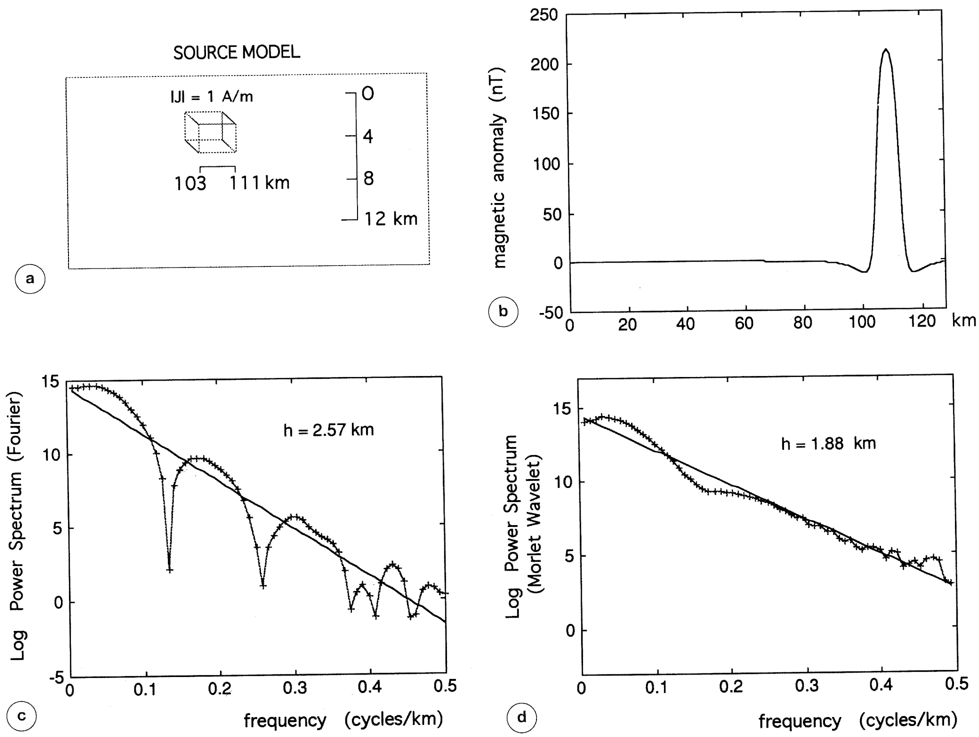

Fig. 2a-d. The wavelet and Fourier power spectra of a deep magnetic source. The source shown in (a) generates the field in (b). Vertical polarization is assumed. Similar to the case of fig. 1a-d, the logarithm of the Fourier power spectrum (c) has a slope which is proportional to depth to the top of the source, apart from an overestimation error. The wavelet power spectrum (d) again gives a good depth estimate and is a smoothed version of the Fourier one, according to eq. (4.3).

where the above depths are well estimated, apart from an overestimation error which is inherent to the method (Fedi *et al.*, 1997). The wavelet spectra may be computed by discretization of eq. (4.2) so that $b = \{1, \ldots, 128\}$; the wave number p was chosen according to the equispaced frequency set of the Fourier analysis, that is: $p = \frac{\{1, \ldots, N/2\}}{N}$. The computed wavelet spectra are shown in figs. 1d and 2d, respectively, and effectively they result as smoothed versions of the Fourier spectra. The slopes of the wavelet spectra may also be utilized to estimate the source depths correctly, with only a slight underestimation error due to the smoothing process.

However, note that using either the wavelet or the Fourier power spectrum leads to a cancellation of any local information. This is not what is really desired when space-frequency analysis is performed instead of Fourier analysis. Therefore, something like a local power spectrum has to be defined, which may allow us to fully take advantage of the local frequency-space properties of the wavelet transform.

5. Local power spectrum

The simplest way of performing local spectral estimations is to compute the discrete form of eq. (4.2) in some prefixed subinterval of the

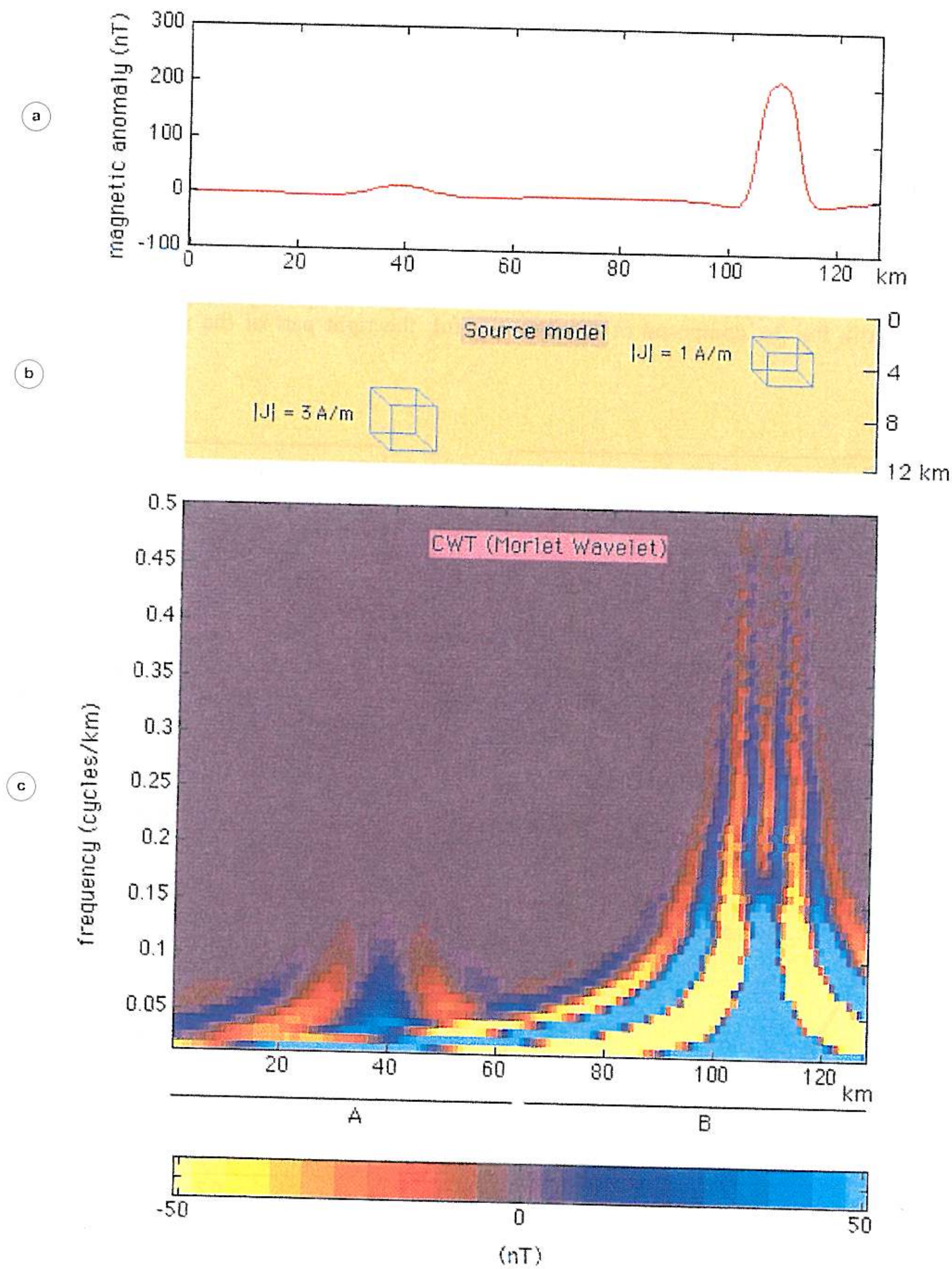

Fig. 3a-c. The phase plane of the two-source case. Considering the source in (b) and the signal in (a), the continuous wavelet transform (c) points out well the presence of the two effects, but unlike Fourier analysis they lie in two well separated parts of the phase-plane, with respect to both the plane axes. In particular, it may be split into two parts, with respect the x-axis (indicated by A and B respectively).

space variable. This is not a rigorous definition of local spectrum, but appears as the wavelet analogue of the traditional Fourier frequency filtering technique. In other words, the representation of f in terms of space-frequency atoms (eq. (4.1)) allows not only filtering in a frequency sense, *i.e.*, suppressing the coefficients related to some set of scales, but even filtering in a space sense, *i.e.* suppressing the coefficients relative to some space subintervals.

In this section we apply this concept to the signal shown in fig. 3a, composed of superposition of both the effects shown in figs. 1a-d and 2a-d, whose sources are sketched in fig. 3b. The continuous wavelet transform, computed with respect to the already mentioned discrete sets of p and b, is shown in fig. 3c. The transform was computed using the Morlet wavelet:

$$\psi(x) = \pi^{-1/4}\,(e^{-i\omega_0 x}\, e^{-x^2/2}).$$

The space-frequency analysis is very meaningful: the right part of the phase plane is domi-

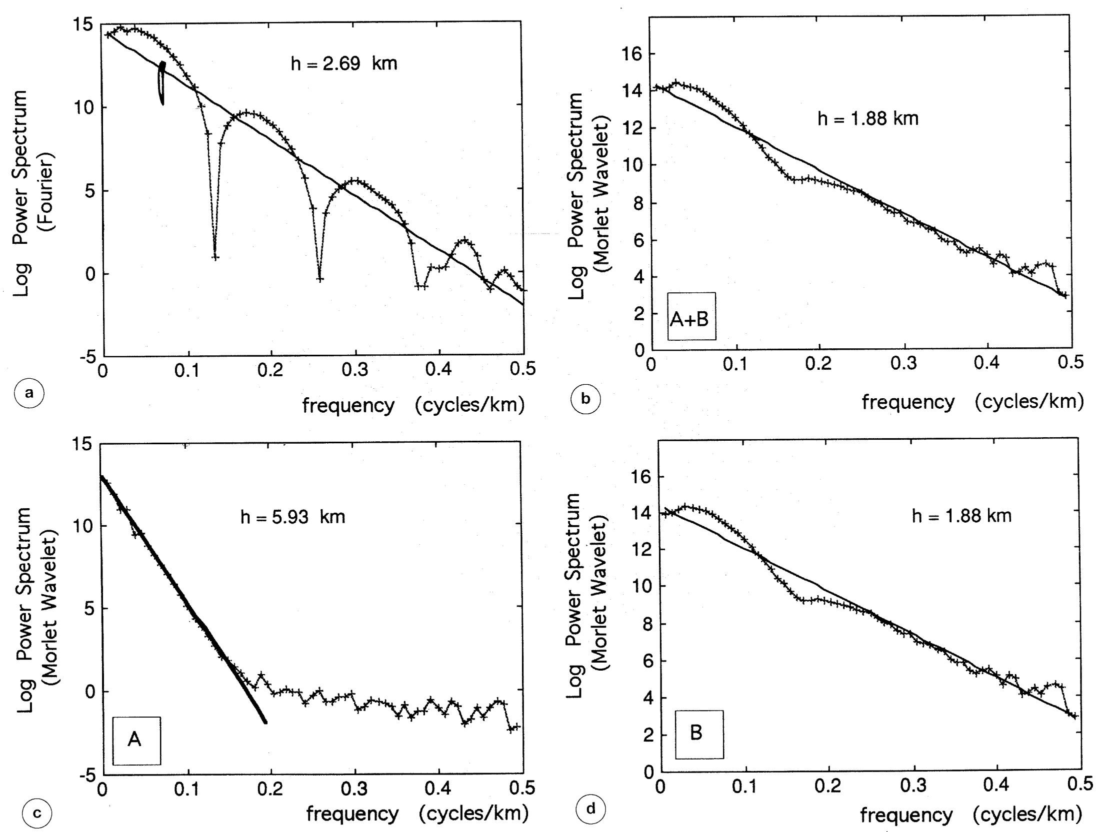

Fig. 4a-d. Local wavelet power spectral analysis. The Fourier power specrum (a) and the wavelet power spectrum (b) of the signal in fig. 3a, are mainly affected by the stronger contribution of the shallow source, so giving a depth corresponding to the case in fig. 2a-d. However, local wavelet spectral analysis allows to operate independently in the A and B zones of the phase plane (fig. 3c) and their respective power spectra to be computed. The depth results (c and d) are very meaningful for both A and B zones.

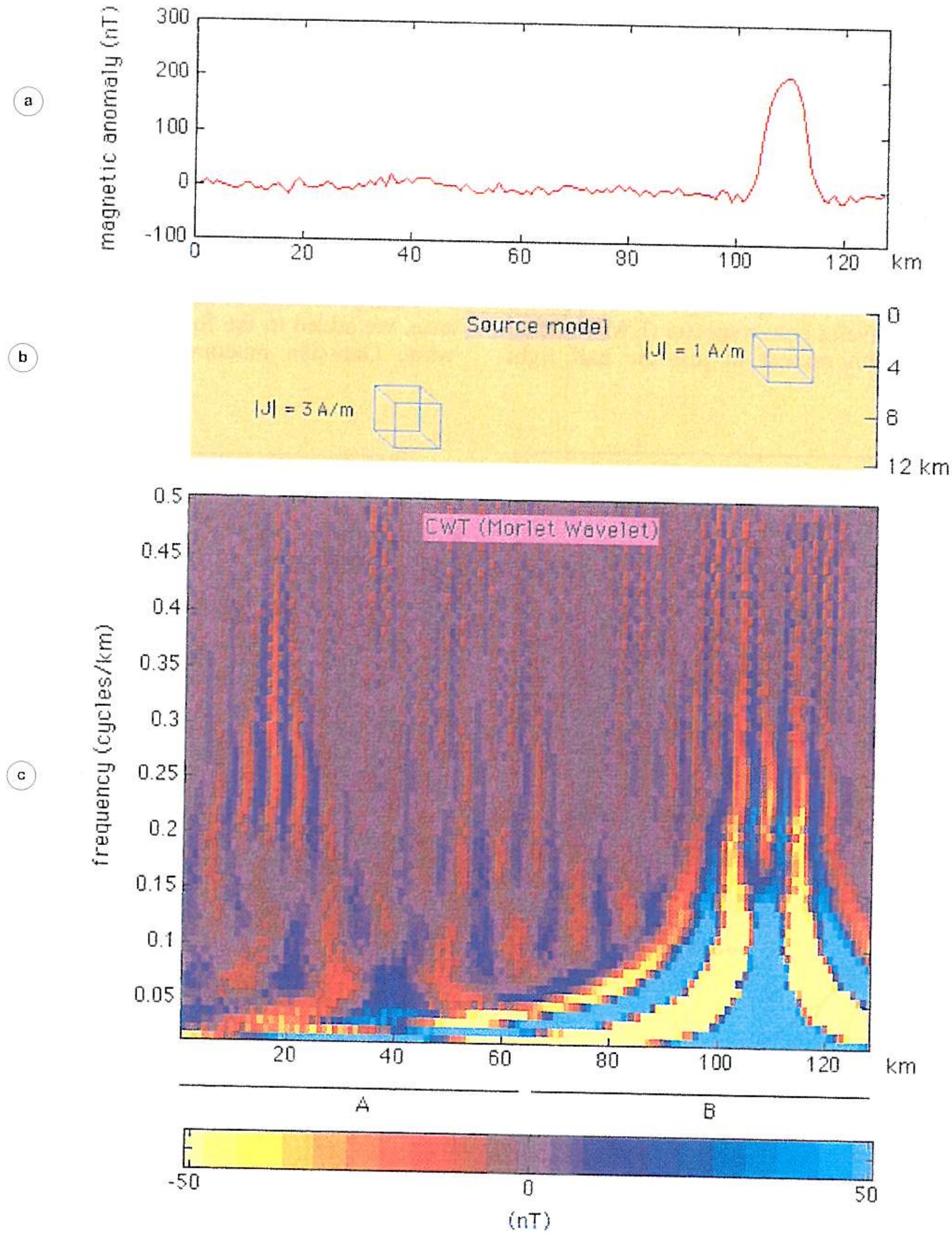

Fig. 5a-c. The phase plane of the two-source case adding a Gaussian random white noise. We now consider the same sources (b) as in fig. 3b, but corrupt the signal (a) throughout an intense white Gaussian random noise, which tends to hide the weak deep-seated source anomaly. The continuous wavelet transform (c) now shows a number of oscillations all over the plane (due to the noise), but the two source effects are again evident. Note that the presence of the deepest source anomaly is clearer in the phase plane than in the anomaly profile.

nated by the strong effect of the shallower source, which takes place along most of the considered scales. Instead, the weaker anomaly of the deepest source is limited to the left part of the phase plane and, as expected, regards just the low frequencies. Since the right effect is considerably more intense than the other, both Fourier and wavelet spectra (fig. 4a,b) result very similar to the spectra of the shallow source by itself (figs. 1c,d). In order to point out the two effects separately, we considered two local wavelet power spectra (LWPS) using eq. (4.2) with respect to just the half right (called A in fig. 3c) part of the phase plane and then considering only the left part (called B in fig. 3c). In both cases, no frequency filtering was performed. The corresponding wavelet power spectra are respectively shown in figs. 4c,d. It is evident that both the LWPS refer now correctly to the two considered sources. In particular, the very weak effect of the deepest source is well recovered by LWPS and the depth to the source is correctly estimated.

Finally, in order to consider a more realistic case, we added to the former signal an intense white Gaussian random noise (fig. 5a) and

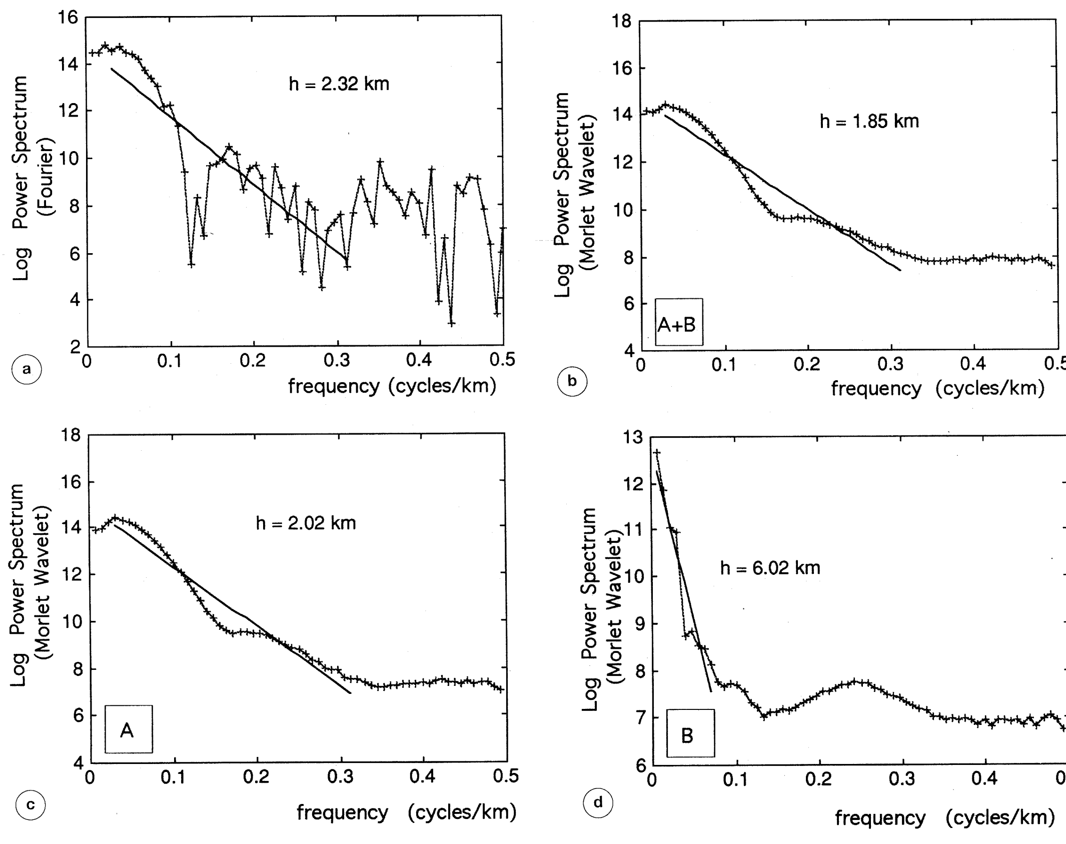

Fig. 6a-d. Local wavelet power spectral analysis with a low signal-to-noise ratio. The Fourier power spectrum (a) and the wavelet power spectrum (b) of the signal in fig. 5a, are now affected again by the contribution of the shallow source, but also by the intense noise at higher frequencies. As in the case of fig. 4a-d, local wavelet spectral analysis allows independent operation in the A and B zones of the phase plane (fig. 3c) and meaningful computation of depth results (c and d) from the local power spectra from A and B zones.

computed again the corresponding continuous wavelet transform. The phase plane (fig. 5c) is nevertheless meaningful: the space-frequency atoms related to the two sources are clearly discernible apart of some distortion related to the noise. Note that the intense noise tends to hide the deep source effect in the anomaly field, while the same effect appears sufficiently clear in the space-plane. Fourier and wavelet power spectra (figs. 6a,b) are now characterized by the shallow source at just the low frequencies, while a high frequency noise-related effect is also evident. Passing to the LWPS spectra, the shallow and deep source effects are well separated and the depths are again correctly estimated.

6. Conclusions

Ambiguity in interpretation of potential fields may be reduced using new techniques of data analysis and interpretation. Use of potential field data along the vertical direction completes the field information related to the sources and may help to solve some ambiguous cases. The considered case of a prismatic source indicates that the ambiguity of monopolar sources cannot be assumed as a general rule for the potential field methods. Theoretical papers (Brodsky, 1986) confirm our reasoning. On the other hand, new techniques of space-frequency wavelet analysis may help to point out the effects of weak sources, practically hidden by noise, and to isolate their spectral content via a local spectral analysis. Local spectral analysis is a natural extension of the classical Fourier frequency filtering, but decomposition in space-frequency atoms now allows to suppress also spatially-related spectral contributions. Obviously, such filtering has to be performed with caution, due to possible overlap effects with respect to both the frequency and space variables of the phase-plane. In other words, the effective separation of the phase plane in two or more parts is somewhat arbitrary; nevertheless our results show that even if the considered separation is affected by some overlapping effects (check fig. 5c around the $x = 64$ axis) of the two sources, the spectrum-based depth estimations are well recovered.

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