Regional geomagnetic field modelling: the contribution of the Istituto Nazionale di Geofisica

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Abstract
Models of the geomagnetic field are mathematical expressions able to represent the Earth's magnetic field space and time variations. Time variations on the long term basis are represented by the so-called secular variation. This paper describes and reviews recent activities of the Italian group at the Istituto Nazionale di Geofisica in regional magnetic field modelling. The models are introduced starting from the classical technique of Spherical Harmonic Analysis (SHA) undertaken for the first time by Gauss, the polynomial analysis and the regional harmonic analysis, specifically introduced as a regional analogue of SHA. In this last group the recent techniques of Spherical Cap Harmonic Analysis (SCHA), Translated Origin Spherical Cap Analysis (TOSCA) and Adjusted Spherical Cap Harmonic Analysis (ASHA) are also described and discussed.

Key words *regional modelling – spherical cap harmonics – geomagnetic field*

1. Introduction

More than 90% of the magnetic field measured at the surface of the Earth is generated by electric currents flowing in the Earth's outer fluid core. Other minor contributions are the remanent magnetisation (present in a material in zero external field) and core-field induced magnetisation in ferromagnetic minerals of the crust and electromagnetic fields induced in the crust and the upper mantle by external field time variations.

Earth's magnetic field time variations, that range from a few seconds to geological time scales, are caused by internal and external sources. The internal origin time variations observed at the Earth's surface are intrinsically slow (more than 1-2 years characteristic time scale) and generally formed by long spatial wavelengths (longer than 3000-4000 km); they are generated in the core by electric currents. External time variations, on the other hand, are caused by ionosphere-magnetosphere electric currents and concern the higher part of the time variations spectrum (see, for example, Chapman and Bartels, 1940; Parkinson, 1983; Merrill *et al.*, 1996).

As regards the portion of the field given by the Earth's core contribution, the community of scientists that operates in geomagnetism has decided to represent it by means of the so-called reference fields. Geomagnetic reference fields are mathematical models of Earth's magnetic field elements that are computed to represent their space and time variations; time variations generally include only secular variation. Among the scopes of reference models, for a certain region, there is that of a precise calculation of the magnetic components, in particular the magnetic declination, for navigation purposes (see, for example, Haines, 1990).

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Magnetic observatory data, as well as ground, marine, airborne and satellite survey field measurements are the fundamental contribution from which models can be made. For Italy's historical geomagnetic fields measurements date back to the sixteenth century, while a continuous data set from observatory data has been available since 1880 (see, Cafarella *et al.*, 1992a,b). In modern times the whole Italian territory has been very well covered by magnetic repeat stations (benchmarks where measurements are repeated at regular intervals) since 1940 by many institutions there operating, and, in the last thirty years, mainly by the Istituto Nazionale di Geofisica (ING) (Talamo, 1975; Meloni *et al.*; 1988, 1994).

In conjunction with Magnetic Observatory data the availability of a network of repeat stations provides a unique opportunity to monitor the geomagnetic field and its long-term variations on a regional scale. The stations are located in areas free of artificial disturbances, and not characterised by large surface anomalies satisfying the standard requirements indicated by Newitt *et al.* (1996). Absolute measurements, repeated at a given station, furnish a secular variation estimation at that site by finite differences, while repeat station surveys determine spatial variability of the secular variation.

In many countries of Central and Eastern Europe, geomagnetic data were restricted and mainly used for military purposes for many years. If we exclude countries with a long tradition of geomagnetic field measurements, this large portion of Europe has been poorly investigated, while the remaining part, is generally well covered by both observatories and repeat stations (*e.g.*, Schulz and Beblo, 1971; Barraclough, 1992; Gilbert, 1994; Reader and Kerridge, 1995). ING has started to promote cooperations to increase geomagnetic data acquisition and dissemination of results in Eastern Europe. Recent contributions are, for example, the implementations of magnetic repeat station networks in Albania, Hungary and partly Ucraina (see, *e.g.*, Chiappini *et al.*, 1997a; Kovac *et al.*, 1997).

2. Geomagnetic field models

2.1. *Spherical Harmonic Analysis*

The classical technique of Spherical Harmonic Analysis (SHA) introduced by Gauss in 1838, depicts the spatial structure of the geomagnetic field potential in terms of solid spherical harmonics, *i.e.*, powers of radial distance from the centre of a spherical Earth (considered the origin of a spherical reference system), Fourier expansion in longitude and Legendre functions in (cosine of) colatitude. This kind of expansion results from solution of Laplace's equation in electric current-free regions without magnetic sources and appropriate boundary conditions on the sphere. A by-product of this operation is that the resulting basis functions, *i.e.*, the spherical harmonics, are orthogonal over the sphere. This apparent coincidence helps in the practical application of SHA to experimental data. An important aspect of SHA is that it allows the separation of internal and external contributions of the magnetic field, although it cannot give the exact position of the sources which are together ideally grouped at the origin (internal sources) or at infinite (external sources), in the form of so-called multipoles, each characterised by a given spherical harmonic degree n. In mathematical notation, on and over the Earth's surface (supposing an electric current-free atmosphere) the potential V of the magnetic field B satisfies Laplace's equation:

$$\nabla^2 V = 0 \qquad (2.1)$$

with general solution in spherical coordinates (θ = latitude, λ = longitude, r = radial distance from Earth's centre) given by

$$V(\theta, \lambda, r) = a \sum_{n=1}^{N_{max}} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{n+1} \cdot$$

$$\cdot \left(g_n^m \cos m\lambda + h_n^m \sin m\lambda\right) P_n^m(\cos\theta) \qquad (2.2)$$

supposing only internal contributions and degree n and order m up to N_{max}. $P_n^m(\cos\theta)$ are the associated Legendre functions, a is the mean Earth radius (6371.2 km) and g_n^m and h_n^m are the Gauss coefficients. Any global model in ordinary spherical harmonics represents details with minimum wavelengths (Bullard, 1967):

$$\lambda_{\min} = \frac{2\pi r}{N_{\max}} \tag{2.3}$$

The stability of the inversion of any global magnetic data set is inversely related to $N_{\max}$, therefore SHA is more suitable to model the longest-wavelength part of the geomagnetic field. This is also confirmed by the introduction of the IGRF (International Geomagnetic Reference Field; *e.g.*, Barton *et al.*, 1996), which is the accepted global model of the geomagnetic field. It is believed to contain all (or most of) the core field, *i.e.*, the largest part of the field observed at the Earth's surface (for this reason also called the main field). The IGRF is given in the form of sets of spherical harmonic coefficients to degree and order 10, except for the predictive secular-variation model which extends to degree and order 8. The current sixth generation is composed of 10 definitive sets of main-field models, ranging from 1945 to 1990, at 5-year intervals, a provisional main-field set for 1995, and a secular-variation predictive model for the interval 1995 to 2000. Since geomagnetic data do not cover all the sphere uniformly, the most suitable method of finding the spherical harmonic coefficients is that based on a least-square procedure. Because of the irregular geographical distributions of land and of different economic situations of Countries, some regions (*e.g.*, Europe and Northern America) are better represented by the IGRF than others (*e.g.*, the oceans).

2.2. *Polynomial analysis*

According to eq. (2.3), the characteristic minimum wavelength associated with the IGRF is around 4000 km. When the user needs finer details of the geomagnetic field, the IGRF is not the most appropriate to use. This situation is exacerbated when one wishes to know the values for the present (or a very recent) epoch, since after 1990 the IGRF is still considered provisional. Another intrinsic limitation of the IGRF, and typical of present global models, is the fact that to be well defined it needs data covering the entire globe. This implies that its realisation takes a significant time, always longer than the preparation of a regional model. Moreover, when the magnetic field observed only over a limited region is considered, SHA is no longer appropriate apart from some particular and specific cases (*e.g.*, De Santis *et al.*, 1997).

Simple models of the main (and part of the deep crustal) field have been developed in terms of polynomials in latitude and longitude; they are usually based on magnetic survey data at intervals of around 5 years. Despite the fact that these models do not satisfy eq. (2.1) (Laplace's equation), for their simplicity they are quite useful when an approximate knowledge of a reference field is sufficient to deduce anomalies, as for example in the Italian area.

If $E(\varphi, \lambda)$ is a generic element of the magnetic field, it can be represented, in a region as large as Italy, by a second-order polynomial in latitude φ and longitude λ:

$$E(\varphi, \lambda) = a_0 + a_1\varphi + a_2\lambda + a_3\varphi^2 + a_4\lambda^2 + a_5\varphi\lambda \tag{2.4}$$

The coefficients a_i are the unknowns to be found by means of a least squares fit over the observational values taken for example during a national magnetic survey. An empirical rule given by Bullard (the so-called Bullard's rule; Bullard, 1967) relates the number of coefficients of a global SH model with that of a regional model, and, in turn, the details represented by the regional model itself. De Santis and Torta (1997) explain this rule as a conservation of the physical information. According to Bullard's rule such a model will represent minimum wavelengths of 800 km circa.

Such models suppose independent magnetic elements that, in turn, can imply the physical

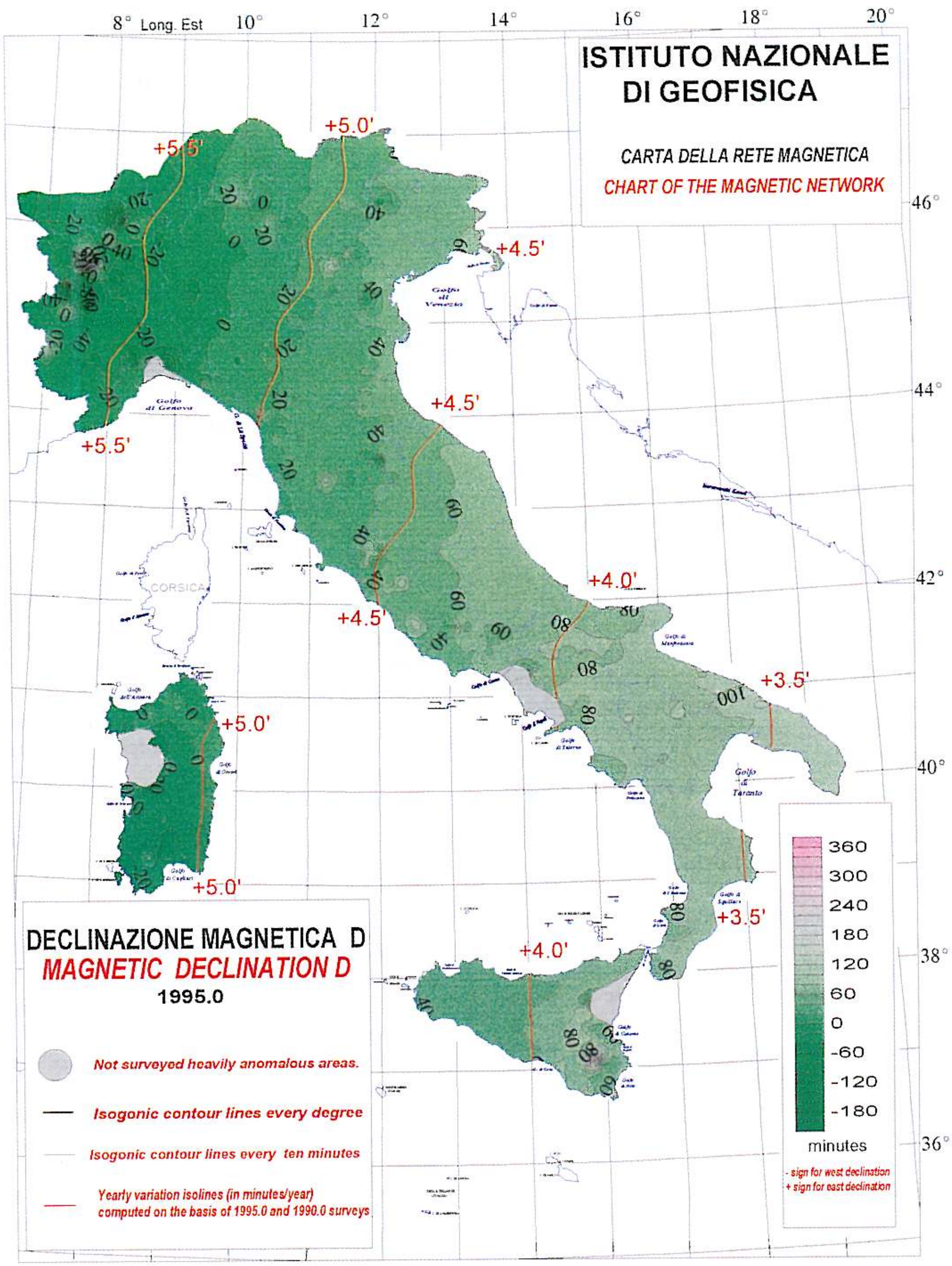

Fig. 1. Geomagnetic field map of Italy for Declination at 1995.0, and secular variation isolines values are in minutes.

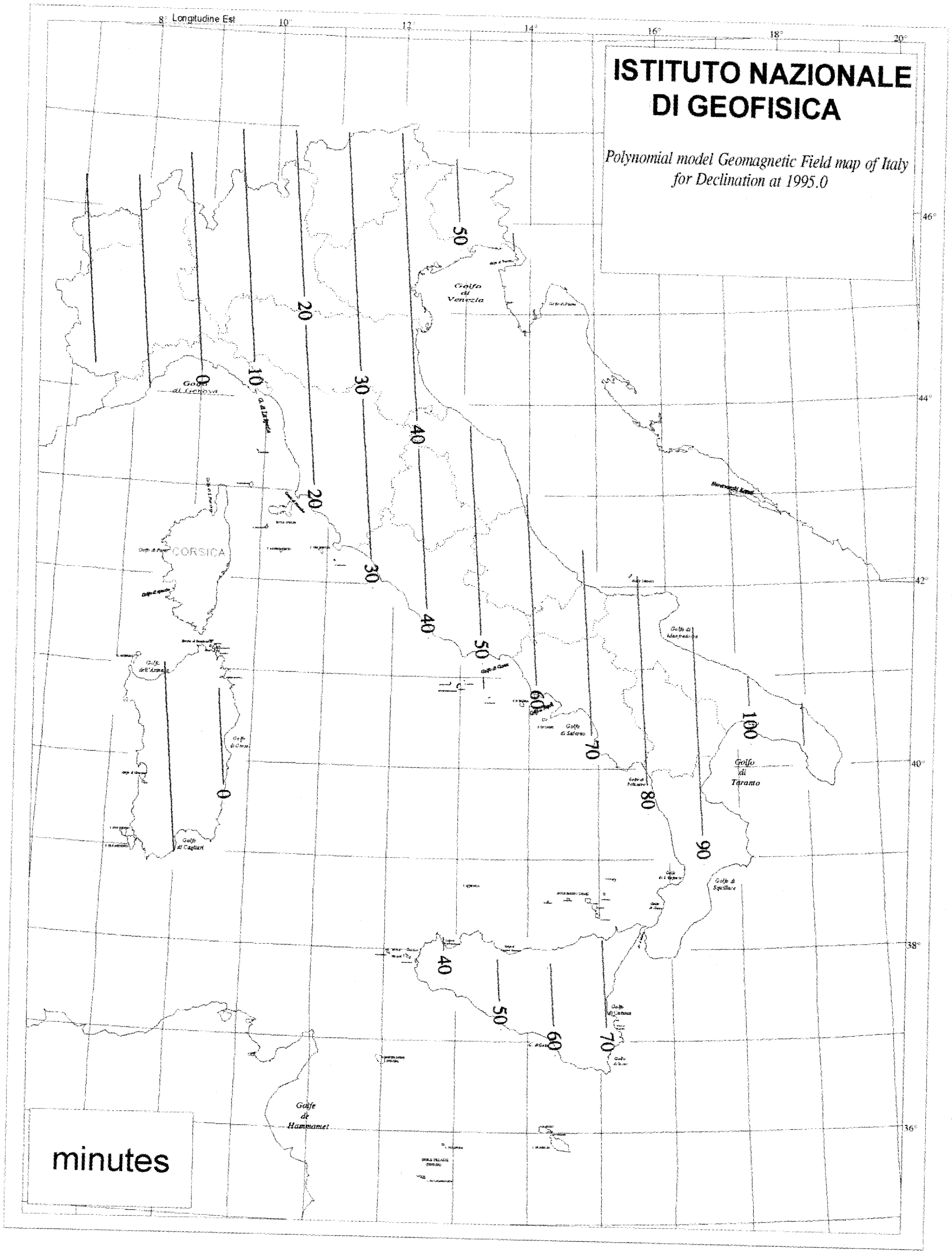

Fig. 2. Polynomial model geomagnetic field map of Italy for Declination at 1995.0, values are in minutes.

inconsistency that rot $\boldsymbol{B} \neq 0$. A partial adjustment is to constrain the total absence of vertical electric currents in the atmosphere (Chapman and Bartels, 1940) through a relation between the two partial derivatives of the horizontal components:

$$(\nabla \times \boldsymbol{B})_z = \frac{\partial X}{\partial \varphi} + \frac{\partial Y \sin\theta}{\partial \theta} = 0 \qquad (2.5)$$

In polynomial models the above equation implies an explicit relationship among the polynomial coefficients of the horizontal components. This condition has the further advantage to decrease the number of coefficients characterising a second-order three-component polynomial model from 18 to 12 coefficients (*e.g.*, Tsubokawa, 1952).

For Italy the very recent accomplishment of the 1995 repeat station network (paper in preparation) has led to the new magnetic maps of Italy; fig. 1 shows the magnetic map of Italy for Declination at 1995.0, while fig. 2 shows the corresponding polynomial reference field.

2.3. *Regional Harmonic Analysis: SCHA and TOSCA*

A technique, which can be used as the regional analogue of SHA, has been specifically introduced for this purpose by Haines (1985, 1990): Spherical Cap Harmonic Analysis (SCHA). It is based on a potential expansion in spherical harmonics with integer order but generally non-integer degree, the latter depending on the order of the expansion and is subject to new boundary conditions in the cap-like region.

This idea consists in choosing adequate basis functions, solutions of Laplace's equation, to have an orthogonal basis within the sub space defined by the spherical cap. In particular these functions form two sets of orthogonal functions within the interval $[0, \theta_0]$ chosen in order to satisfy either

$$P_n^m(\cos\theta_0) = 0 \qquad (2.6)$$

or

$$dP_n^m(\cos\theta_0)/d\theta = 0 \qquad (2.7)$$

Some variations of SCHA have been proposed in order to simplify both its realisation and its application in specific cases, when generally SCHA shows some inherent pitfalls. An example has been the introduction of the Translated Origin Spherical Cap harmonic Analysis (TOSCA) which consists of an SCHA after an appropriate vertical translation of the origin, in order to improve the fit of the crustal field being studied (De Santis, 1991). In this context, SCHA can be regarded as a special case of TOSCA, when no vertical translation is performed. TOSCA takes advantage of the invariance of Laplace equation from coordinates translations. This technique is useful for modelling regional anomalies with evident central peaks in the region of interest.

2.4. *Another kind of Regional Harmonic Analysis: ASHA*

With the same objective, another regional technique has been proposed. It is mainly based on conventional SHA properly adjusted to the spherical cap, after an artificial enlargement of the colatitudes of data to cover the hemisphere. The method is called, for this reason, Adjusted Spherical Harmonic Analysis (ASHA) (De Santis, 1992). ASHA was introduced to solve most of the problems of using ordinary spherical harmonics. In Fourier analysis, the interval containing data is usually normalised to 2π radians; analogously, considering a small spherical cap of half-angle θ_0 (for which $\sin\theta \approx \theta$), we could still apply SHA, after the colatitude interval $[0, \theta_0]$ is scaled to the hemisphere, *i.e.* $[0, \pi/2]$. The corresponding transformation operations from the cap reference system (r, θ, λ) to the new one (r', θ', λ') are

$$r' = r; \quad \lambda' = \lambda; \quad \theta' = s \cdot \theta \qquad (2.8)$$

where: $s = \pi/2\theta_0$.

In this new «hemispheric» system, we have an approximate version of Legendre equation in θ', with $P_k^m(\cos\theta')$ as new eigen-functions. These adjusted Legendre functions are composed of two orthogonal sets, and each set satisfies one of the following boundary conditions:

$$P_k^m(\cos\theta_0') = 0; \qquad \frac{dP_k^m(\cos\theta_0')}{d\theta'} = 0 \quad (2.9)$$

In the scaled reference system the magnetic

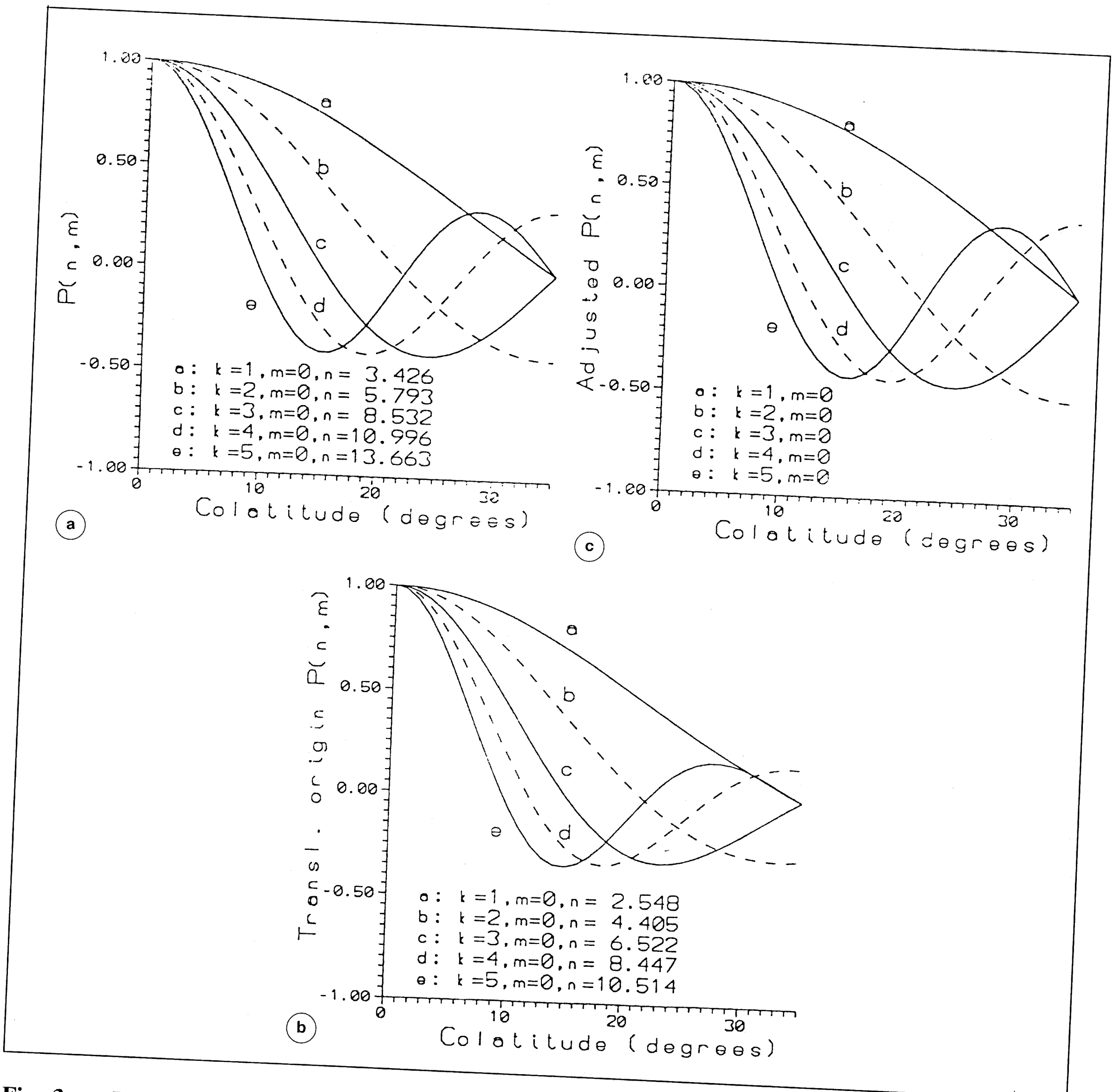

Fig. 3a-c. Basis functions of (a) SCHA, (b) TOSCA, and (c) ASHA, with similar characteristics; n, m are the spatial degree and order of the regional harmonic expansions; k is the integer index that enumerates in increasing orders the corresponding roots in n satisfying (2.6)-(2.7) (SCHA and TOSCA) or (2.9) of the expansions (adapted from De Santis and Falcone, 1995).

potential can be expressed as

$$V(\theta', \lambda, r) = a \sum_{k=0}^{K_{max}} \sum_{m=0}^{n} \left(\frac{a}{r}\right)^{q+1} \cdot$$

$$\cdot (g_k^m \cos m\lambda + h_k^m \sin m\lambda) P_k^m (\cos\theta') \quad (2.10)$$

where q is roughly $s(k+0.5)$. The magnetic field components are defined as

$$X' = \frac{X}{s}; \quad Y' = \frac{Y \sin\theta}{\sin\theta'}; \quad Z' = Z \quad (2.11)$$

The presence of the $k = 0$ term in (2.10) is due to the fact that the surface integral of the vertical component over the cap might not be zero (while it is zero for SHA).

Conditions (2.9) are equivalent to those given by Haines (1985) for his basis functions, *i.e.*, the spherical cap harmonics, that differ from the adjusted spherical harmonics mainly for the use of Legendre functions of non-integer degree.

The value of K_{max} sets up the number N_c of coefficients to be included in the series:

$$N_c = (K_{max} + 1)^2 \quad (2.12)$$

ASHA models can represent minimum wavelengths λ_{min} much shorter than those obtained by SHA by using the same number of coefficients, because (De Santis, 1992)

$$\lambda_{min} = \frac{4\theta_0 r}{K_{max}} \quad (2.13)$$

since K_{max} is roughly the maximum number of wavelengths contained in the spherical cap. For instance, to represent a wavelength of 1000 km in a 20° cap, we need an expansion with K_{max} = 9, *i.e.*, N_c = 100 coefficients. By using SHA, to get the same detail all over the globe we would need N_{max} = 40, *i.e.*, 1680 (= $N_{max} \cdot (N_{max}+2)$) coefficients, *i.e.*, many more coefficients than those involved by ASHA.

The basis functions of SCHA, TOSCA and ASHA possess many similarities, especially the former with the latter; fig. 3a-c, shows the behaviour of some functions with similar characteristics (the figures have been adapted from De Santis and Falcone, 1995).

3. Conclusions

Regional modelling of the geomagnetic field allows the use of data taken in a restricted region as effectively as possible. Italian data, although very limited in space, have covered a long span of time, providing also historical sets of observatory and repeat station data (Cafarella *et al.*, 1992a,b). The geomagnetic group of the Istituto Nazionale di Geofisica has used simple techniques, like polynomial reference models (*e.g.*, Molina and De Santis 1987, Meloni *et al.*, 1988, 1994), together with newer methods such as SCHA to study and represent the magnetic field in the European region, with particular emphasis on the Italian area (*e.g.*, De Santis *et al.*, 1996), but also for larger areas. For instance, a European vector model, based on Magsat satellite data, of the crustal magnetic field at an altitude of 400 km was proposed (De Santis *et al.*, 1989). A geomagnetic reference field for Italy at 1985 was also developed (De Santis *et al.*, 1990), including combined data taken from magnetic survey, observatory and satellite data.

The geomagnetism group of ING has also worked with the corresponding national foreign groups to propose a TOSCA-based Geomagnetic Reference Field for Spain with its secular variation (Torta *et al.*, 1993) and normal reference fields for Hungary (Kovac *et al.*, 1997) and Albania (Chiappini *et al.*, 1997a). One result of the collaboration with Spanish colleagues has been the discovery of some problems with SCHA when it is used to represent long wavelength fields, such as secular variation and the suggestion of some ways to avoid the occurrence of these problems. Other difficulties concerned with the internal-external separation have been described in a separate paper (Torta and De Santis, 1996).

ASHA has also been applied for modelling 2D regional ionospheric parameters in Europe (De Santis *et al.*, 1991, 1994). Many other references of the use of SCHA on geomagnetism and other fields can be found in De Santis and Torta (1997), which also exploits the use of SCHA for gravity field representation.

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