ANNALI DI GEOFISICA, VOL. 42, N. 3, June 1999

Maximum entropy estimation of *b* values at Mt. Etna: comparison with conventional least squares and maximum likelihood results and correlation with volcanic activity

Concetta Centamore (1), **Giuseppe Patanè** (2) **and Tiziana Tuvè** (3)

(1) *Dipartimento di Geofisica e Vulcanologia, Università di Napoli «Federico II», Napoli, Italy*

(2) *Istituto di Geologia e Geofisica, Università di Catania, Italy*

(3) *Osservatorio Sismologico, Università di Messina, Italy*

Abstract

The variations of the b coefficient in the frequency-magnitude relationship for earthquakes which occurred at Mt. Etna from 01/01/90 to 31/12/92 are analysed; the completeness threshold for our earthquakes catalogue is $M = 2.30$. The b values calculated using the Maximum Entropy Principle (MEP) are compared to those obtained by conventional methods of Least Squares (LS) and Maximum Likelihood (ML). All the differences among the b values computed using these methods, and the reasons for these differences, are discussed and examined. In particular, our results show that the b values obtained by MEP are lower than the others calculated using LS and ML; this implies that, on the average, LS and ML underestimate the seismic hazard at Mt. Etna. Moreover, temporal variations of b_{MEP} are more evident than the corresponding ones of b_{LS} and b_{ML}; indeed, in some cases the trend of b_{MEP} variation is opposite those of b_{LS} and b_{ML}. A significant correlation between temporal variations of the volcanic activity and the b values is evident only if the MEP is used; this means that, if b temporal variations are analysed in order to detect changes in the volcano dynamics and predict the eruptions, the maximum entropy approach should be preferred. Finally, the observed pattern of b_{MEP} temporal variations with regard to the changes in the volcanic activity is consistent with the hypothesis of a compressive stress field acting on Mt. Etna.

Key words *frequency-magnitude distribution – maximum entropy method – Mt. Etna*

1. Introduction

The well known frequency-magnitude relationship of Gutenberg and Richter (1944) can be written as

$$\log N = a - bM \qquad (1.1)$$

where N denotes the number of earthquakes with magnitude greater than or equal to M, a and b are parameters describing regional seismicity. For a certain region the constant a, named «seismic activity» parameter, is related to the total number of earthquakes with non-negative magnitudes; the coefficient b gives information on the ratio between large and small earthquakes: the b value is small when the majority of earthquakes occur at the higher magnitudes, while the b value is high when the magnitude of most earthquakes is low.

The b value depends on the state of stress (Scholz, 1968; Main *et al.*, 1989), on changes in the fracture mechanism (Meredith and Atkinson, 1983), on the heterogeneity of the material

Mailing address: Dr. Concetta Centamore, Via Romano 52, 95125 Catania, Italy; e-mail: concicen@tin.it

(Mogi, 1967) and on the fractal dimension D of fault planes (Aki, 1981; Main, 1987; Hirata, 1989).

A decrease of the b value before a large tectonic earthquake has been widely observed; at Mt. Etna a significant increase in the b_{ML} value (*i.e.* computed using the maximum likelihood method), followed by a sharp decrease, was noted before the 1981 and 1983 eruptions (Gresta and Patanè, 1983a,b). This pattern has been associated with changes in the stress field and to differences of heterogeneity and rigidity among the structures of the volcano.

On the contrary, an analysis of the time evolution of b_{ML} during the period 1981-1987 (Patanè *et al.*, 1992) shows that a significant decrease of the b_{ML} value can rarely be interpreted as a precursor of eruptions.

Similarly, before the explosive eruption which occurred at Mt. St. Helens on 18th May 1980 there is no indication of a decrease in the b_{ML} value, even if an earlier phase of the phreatic eruptions preceding the main eruption is related to a b_{ML} value anomaly (Main, 1987).

Starting from the eighties, the Maximum Entropy Principle (MEP) (Shannon, 1948; Shannon and Weaver, 1949) has been applied to determine the frequency-magnitude distribution and to compute the b parameter of the relation (1.1) (Berrill and Davis, 1980; Shen and Mansinha, 1983; Dong *et al.*, 1984).

The aim of this work is to investigate the implications of using the MEP, instead of the conventional Least Squares (LS) and Maximum Likelihood (ML) (Aki, 1965; Utsu, 1965), to analyse the time variations of b at Mt. Etna volcano. We will discuss all the differences among the results obtained by these methods; the comparison between the observed pattern of b_{MEP} values and the volcanic activity at summit craters will enable us to point out some interesting features of the stress field acting on Mt. Etna.

2. Earthquake catalogue and completeness analysis

The data considered for the present study have been obtained from recordings of the seismic stations ACR (Acireale) and SND (San Leonardello), belonging to the Seismological Observatory of Acireale (fig. 1) and equipped with short period (1 Hz) vertical seismometers.

Our earthquake catalogue includes 1015 events from 01/01/90 to 31/12/92; among them, No. 468 are recorded at both ACR and SND stations, No. 152 only at ACR station and No. 395 only at SND station.

The magnitude of events has been computed using the relations

$$M_{ACR} = 1.64 \log D_{ACR} + 0.03\,(t_S - t_P)_{ACR} - 0.26 \quad (2.1)$$

$$M_{SND} = 1.98 \log D_{SND} + 0.04\,(t_S - t_P)_{SND} - 0.70 \quad (2.2)$$

where D is the coda duration in seconds; $(t_S - t_P)$ is the time interval (in seconds) between S-phase arrival time and P-phase arrival time.

In the relations (2.1) and (2.2) the logarithm to the base 10 is used. For the No. 468 events recorded at both ACR and SND stations the

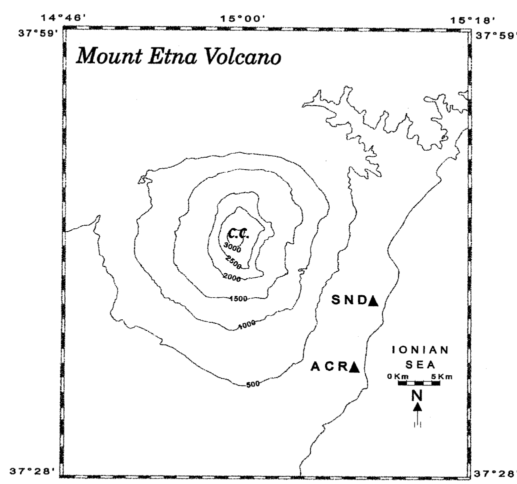

Fig. 1. Map of Mt. Etna. Triangles indicate the location of the seismic stations of Acireale (ACR) and San Leonardello (SND), used in this study.

mean value of the magnitudes computed by (2.1) and (2.2) has been considered; for the events recorded only at ACR or SND the corresponding magnitude has been assigned adopting the relation (2.1) or (2.2), respectively.

In general, the completeness threshold of a catalogue can be estimated using standard procedures such as those developed by Stepp (1972), Caputo (1983), Tinti and Mulargia (1985). These authors analyse catalogues collected during reasonably long time intervals; assuming that the number of events of a given magnitude does not change with time (hypothesis of stationarity), they search for variations in the statistical density distribution of earthquakes for each magnitude interval. The methods of the above-mentioned authors are particularly suitable for catalogues rich in historical events and coming from seismic networks for which the detection capability has been improved with time; but these conditions are certainly not valid for our catalogue.

Many studies performed on analogous data sets simply assume that the catalogue is incomplete for any magnitude for which Log*N* of relation (1.1) lies below the best-fit line determined from the higher magnitudes (*e.g.*, Caputo, 1983; De Natale *et al.*, 1987; Molchan *et al.*, 1996). This assumption corresponds to attribute any non-linearity of the frequency-magnitude plot, that is any deviation in self-similarity, exclusively to the incompleteness of the catalogue. Aki (1987) reports observations inconsistent with this hypothesis: analysing a set of events recorded at a borehole seismograph station operating in the middle of the Newport-Inglewood fault, he finds a clear departure from linearity for $M < 3$, even if the catalogue is complete at least down to $M = 1$.

The analysis of the spectral parameters of microearthquakes at Mt. Etna (Centamore *et al.*, 1997) clearly demonstrates that self-similarity does not hold for magnitudes lower than 2; for our catalogue this value corresponds to the magnitude threshold below which the Log*N* *versus* *M* plot becomes non-linear (fig. 2).

In such a situation we have decided to apply the completeness test proposed by Rydelek and Sacks (Rydelek and Sacks, 1989; Taylor *et al.*, 1990), which has been developed in order to

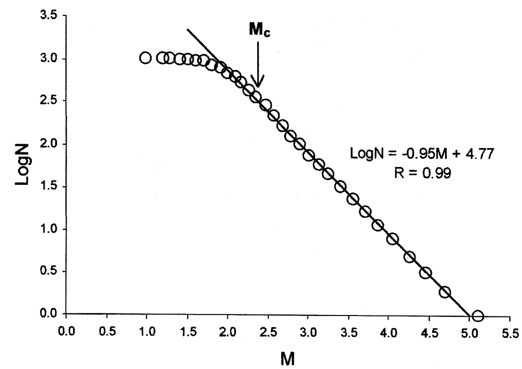

Fig. 2. Log*N* *versus* *M* plot for our catalogue; the regression line for *M* 2.0 is superimposed. M_c indicates the completeness threshold, computed by means of the test proposed by Rydelek and Sacks (1989). See text for details.

identify the effective completeness magnitude M_c of a catalogue when, at the same time, deviation in self-similarity is present. Assuming that the detection of low-magnitude earthquakes is more probable at night, when the noise is reduced, and that the earthquake occurrence is not correlated with time of day, we examined the preference of earthquakes to be detected as a function of the time of day. For this purpose, we assigned to each event a phasor whose direction indicates the time of day on a 24-h clock. The phasors for all earthquakes in a given magnitude interval were then added end to end. If the catalogue is complete within a given magnitude interval, then the phasor sum wanders randomly about the origin. If, however, the catalogue is partially incomplete with a preference for events with low magnitude to be detected at night, then the phasor sum of lower magnitude events wanders away from the origin towards a «night time» direction. The length of the resultant phasor sum was compared with that expected for a random (Poisson) process, which is $1.73\sqrt{N}$ at the 95% confidence level (Rydelek and Sacks, 1989); N is the total number of events of the catalogue within a given magnitude interval. If the phasor sum does not span the chosen confidence level, the catalogue is considered complete in that magnitude interval.

Figure 3 shows the Rydelek and Sacks's test applied for the magnitude intervals 0.80 ÷ 1.70, 1.7 ÷ 2.00, 2.00 ÷ 2.30, 2.30 ÷ 2.60, 2.60 ÷ 2.90. To beware of swarms distorting the results, we ignore all shocks occurring more frequently than twice the average hourly earthquake rate for the catalogue (again within a given magnitude interval). The completeness threshold for the catalogue is clearly M_c = 2.30; therefore, in the range $2.00 \leq M \leq 2.30$ our catalogue does not show significant departures from linearity even if it fails the completeness test.

This result, very similar to that obtained by Rydelek and Sacks (1989) analysing the catalogue of events occurred in the Phlegraean Fields (Italy) during 1983 and 1984, can be explained considering that the completeness test reaches maximum sensitivity when the number of events peaks, whereas in the same situation the Log*N* *versus M* plot has minimum sensitivity (Rydelek and Sacks, 1989).

3. Analysis of *b* values and estimation of errors

At first we scanned the whole sequence of earthquakes by means of 32 groups with a fixed earthquake number N = 100. Each group corresponds, on the average, to a thirty days' interval; the step between two consecutive groups is ΔN = 30 events. For the computation of the *b*-values only the events with magnitudes greater than M_c = 2.30 were considered within each group; this means that the analysis of the *b* parameter was performed, for every group, us-

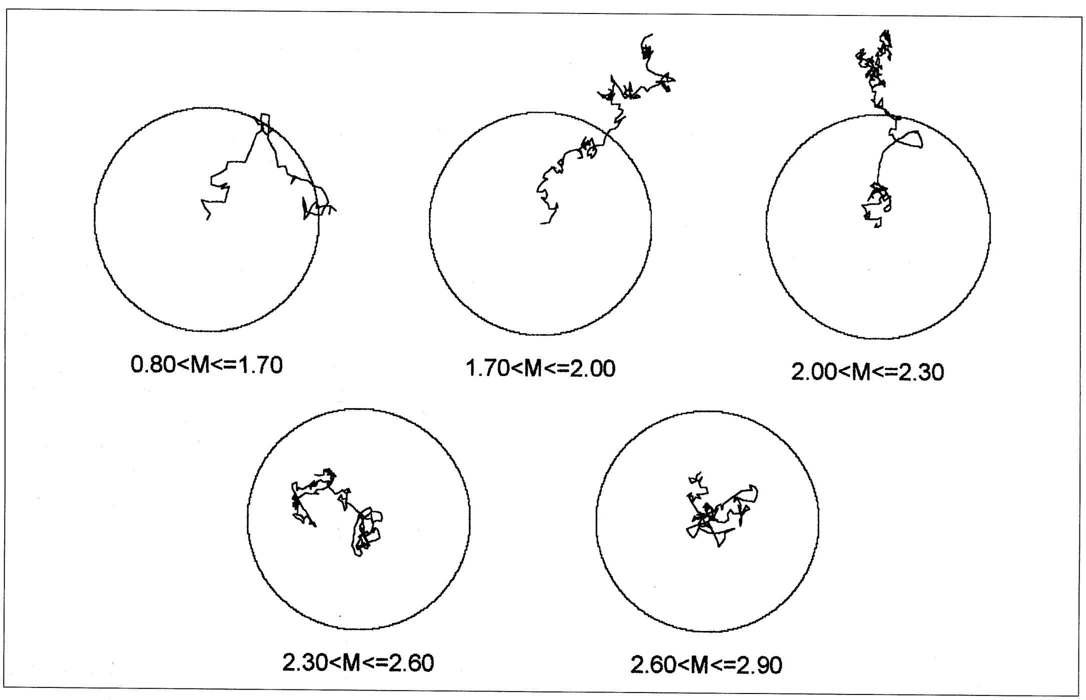

Fig. 3. Rydelek and Sacks's (1989) test for our data set. The magnitude range is included in each plot. The circles represent a 24-h clock with the top corresponding to local midnight and 6.00 a.m. to the right. The radius of each circle corresponds to the 95% confidence level for an equivalent random walk (Brownian motion). The completeness threshold for our catalogue is M = 2.30.

ing a number of events varying between 47 (group No. 32) and 72 (group No. 25). In any case, the number of events for each subset is such as to permit a reliable estimation of the b-value using the LS, ML and MEP methods; in fact at least 25 earthquakes are required to obtain an estimate of b with a standard deviation as low as $\sigma_b = 0.25\, b$ (Bender, 1983). Moreover, the subdivision of the catalogue in 32 subsets with a different number of events allows to justify the assumption of a stationary b inside each group: only in these conditions, in fact, each subset corresponds to a relatively small temporal window (always less than three months).

3.1. *Least Squares method*

Applying to relation (1.1) the well known Least Squares method, the b value is given by

$$b_{\mathrm{LS}} = \frac{\sum_{i=1}^{n} M_i \sum_{i=1}^{n} (\log N)_i - n \sum_{i=1}^{n} (\log N)_i}{n \sum_{i=1}^{n} M_i^2 - \left(\sum_{i=1}^{n} M_i \right)^2} \qquad (3.1)$$

where, for each group of 100 events, n is the number of magnitudes greater than 2.30 and N is the number of events with magnitude greater than or equal to magnitude M_i $i = 1, 2, \ldots n$.

It should be noted that, for each group, the mean magnitude $\langle M_{\mathrm{LS}} \rangle$ computed by LS does not coincide with the sample mean $\langle M \rangle$ (Dong *et al.*, 1984); in fact we have

$$\langle M_{\mathrm{LS}} \rangle = \frac{1}{b_{\mathrm{LS}}} \left[a_{\mathrm{LS}} - \frac{\sum_{i=1}^{n} \log N}{n} \right] \qquad (3.2)$$

where a_{LS} is the least squares estimation of the «seismic activity» parameter, while

$$\langle M \rangle = \frac{\sum_{i=1}^{n} n_i M_i}{\sum_{i=1}^{n} n_i} \qquad (3.3)$$

where n_i is the number of events with magnitude M_i $i = 1, 2, \ldots n$. The difference between the sample mean and the calculated mean from LS fit is a fundamental point to understand the anomalies in b_{LS} with respect to b_{ML} and b_{MEP} values.

The standard error in b_{LS} is (Taylor, 1990)

$$\sigma(b_{\mathrm{LS}}) = \left[\sum_{i=1}^{n} [(\log N)_i - a_{\mathrm{LS}} - b_{\mathrm{LS}} M_i]^2 \right]^{1/2} \cdot \left[n \sum_{i=1}^{n} M_i^2 - n \left(\sum_{i=1}^{n} M_i \right)^2 \right]^{-1/2} \qquad (3.4)$$

where n, N, M_i $i = 1, 2, \ldots n$ and a_{LS} are the same of those appearing in (3.1) and (3.2).

3.2. *Maximum Likelihood method*

Given a minimum magnitude M_0 and a maximum magnitude M_u, the Maximum Likelihood formula for fitting b is (Page, 1968)

$$\frac{1}{\beta_{\mathrm{ML}}} + \frac{M_0\, e^{-\beta_{\mathrm{ML}} M_0} - M_u\, e^{-\beta_{\mathrm{ML}} M_u}}{e^{-\beta_{\mathrm{ML}} M_0} - e^{-\beta_{\mathrm{ML}} M_u}} - \langle M \rangle = 0. \qquad (3.5)$$

In this equation, which can be solved numerically, $\beta_{\mathrm{ML}} = b_{\mathrm{ML}}/\log_{10} e \approx 2.3\, b_{\mathrm{ML}}$ and $\langle M \rangle$ is the sample mean. If M_u is more than two units larger than M_0, the limit $M_u \to \infty$ can be used (Page, 1968; Bender, 1983) and the relation (3.5) can be approximated by the simplified formula (Aki, 1965; Utsu, 1965)

$$b_{\mathrm{ML}} = \frac{0.4343}{\langle M \rangle - M_0} \qquad (3.6)$$

which is the relation generally adopted but valid only if the condition

$$M_u - M_0 > 2 \qquad (3.7)$$

is verified.

For the purpose of comparing our results with those obtained in previous studies, in this work we computed the b_{ML} values by means of (3.6) even if the condition (3.7) is not satisfied for the whole data set.

If there is no uncertainty in M, the generally adopted 95% confidence limits for the b_{ML} value calculated from (3.6) are (Aki, 1965; Utsu, 1965)

$$\sigma(b_{ML}) = \frac{1.96\, b_{ML}}{\sqrt{n}} \tag{3.8}$$

where n is the number of events, for each group, whose magnitude is greater than 2.30. This relation is valid only supposing that b is constant in time, *i.e.* assuming that the mean magnitude of the sample does not change in time.

Under the less restrictive hypothesis of slow temporal changes in b, the standard error of the b_{ML} value is (Shi and Bolt, 1982)

$$\sigma(b_{ML}) = 2.30\, b^2\, \sigma(\langle M \rangle) \tag{3.9}$$

where

$$\sigma(\langle M \rangle) = \sqrt{\frac{\sum_{i=1}^{n} (M_i - \langle M \rangle)^2}{n(n-1)}}$$

and where n is the same of relation (3.8). It can be demonstrated (Shi and Bolt, 1982) that the relation (3.8) gives an underestimated standard error when the temporal window is not small enough for b to be constant.

Our groups of data correspond to small time intervals for which the assumption of a constant b is acceptable; for this reason we have adopted the relation (3.8) instead of the more general (3.9).

3.3. *Maximum Entropy Principle*

If we consider entropy as a measure of «missing information», then the Maximum Entropy Principle yields the unique probability density function which is the most unprejudiced if information is not available (Berrill and Davis, 1980).

According to Dong *et al.* (1984), the parameter $\beta_{MEP} \approx 2.3\, b_{MEP}$ can be computed solving numerically for β_{MEP} the equation

$$\frac{1}{\beta_{MEP}} + \frac{M_0\, e^{-\beta_{MEP} M_0} - M_u\, e^{-\beta_{MEP} M_u}}{e^{-\beta_{MEP} M_0} - e^{-\beta_{MEP} M_u}} - \langle M \rangle = 0. \tag{3.10}$$

which is identical to (3.5), *i.e.* for magnitudes the probability distribution obtained by MEP is the same as the *a priori* distribution considered in the ML method (Bender, 1983; Dong *et al.*, 1984). It should be noted that the MEP, unlike the LS method, makes direct use of the sample mean magnitude, which is the only unbiased source of information we have; as a consequence of this, the mean magnitude computed by MEP always coincides with the sample mean (Dong *et al.*, 1984)

Applying the standard theory of errors to (3.10) we have

$$\sigma(b_{MEP}) = \frac{k_1\, b_{MEP}\, \sigma(M)}{2.3\, b_{MEP}\, k_3 - \langle M \rangle (2.3\, b_{MEP}\, k_2 + k_1)} \tag{3.11}$$

where

$$\sigma(M) = \sqrt{\frac{\sum_{i=1}^{n} (M_i - \langle M \rangle)^2}{n-1}}$$

$$k_1 = e^{-2.3\, b_{MEP} M_0} - e^{-2.3\, b_{MEP} M_u}$$

$$k_2 = -(M_0\, e^{-2.3\, b_{MEP} M_0} - M_u\, e^{-2.3\, b_{MEP} M_u})$$

$$k_3 = -(M_0^2\, e^{-2.3\, b_{MEP} M_0} - M_u^2\, e^{-2.3\, b_{MEP} M_u})$$

with n equal to the number of events, for each group, whose magnitude is greater than 2.30.

Figure 4a-c shows the b_{LS}, b_{ML} and b_{MEP} temporal variations, calculated by (3.1), (3.6) and (3.10) respectively; error bars, computed using the relations (3.4) in fig. 4a, (3.8) in fig. 4b and (3.11) in fig. 4c, are superimposed. Figure 5 represents the cumulative graph comparing the trends of b_{LS}, b_{ML} and b_{MEP}.

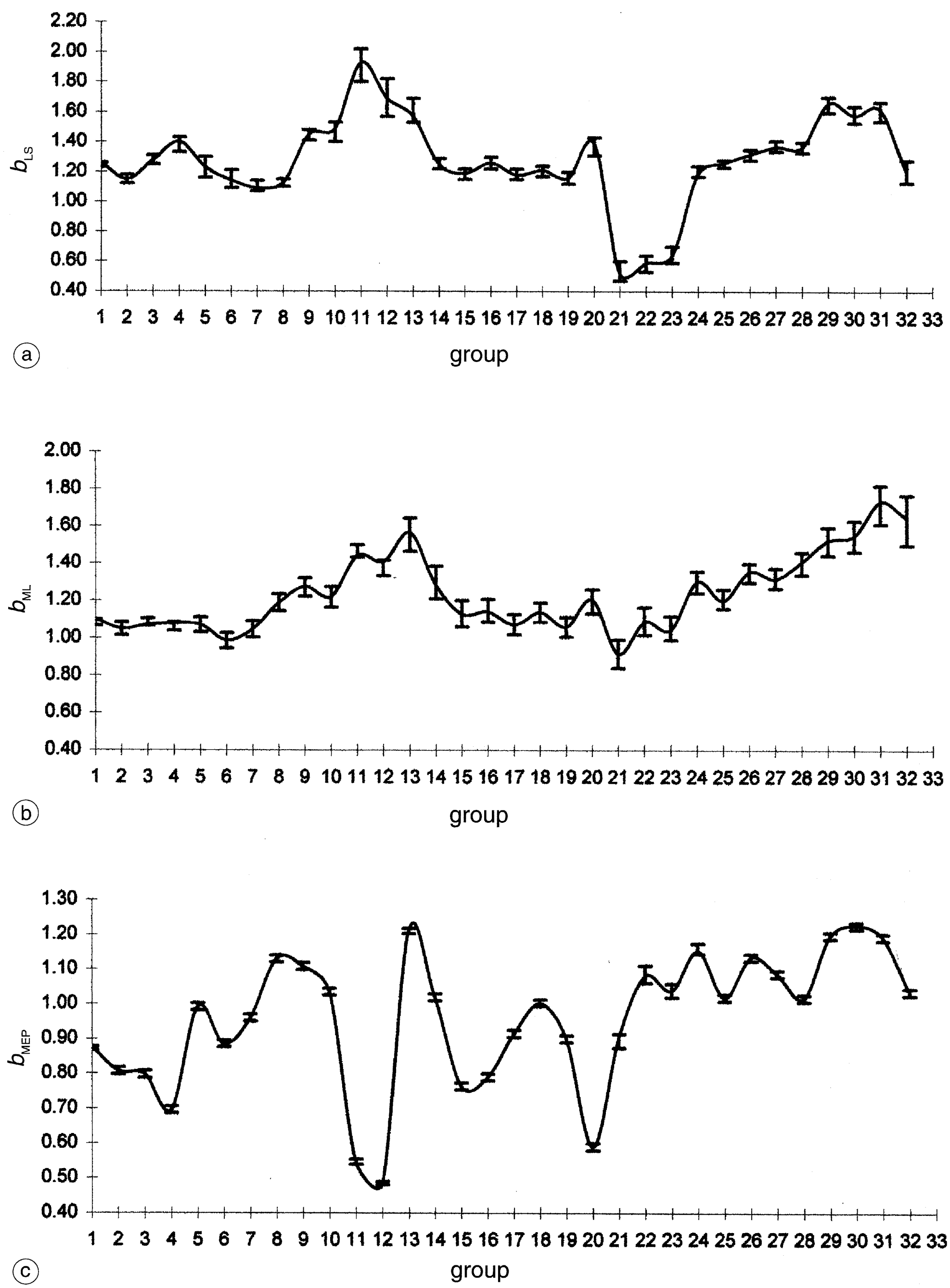

Fig. 4a-c. Time variations of b_{LS} (a), b_{ML} (b) and b_{MEP} (c). Standard error bars, computed using relations (3.4) in fig. 3a, (3.8) in fig. 3b and (3.11) in fig. 3c are superimposed.

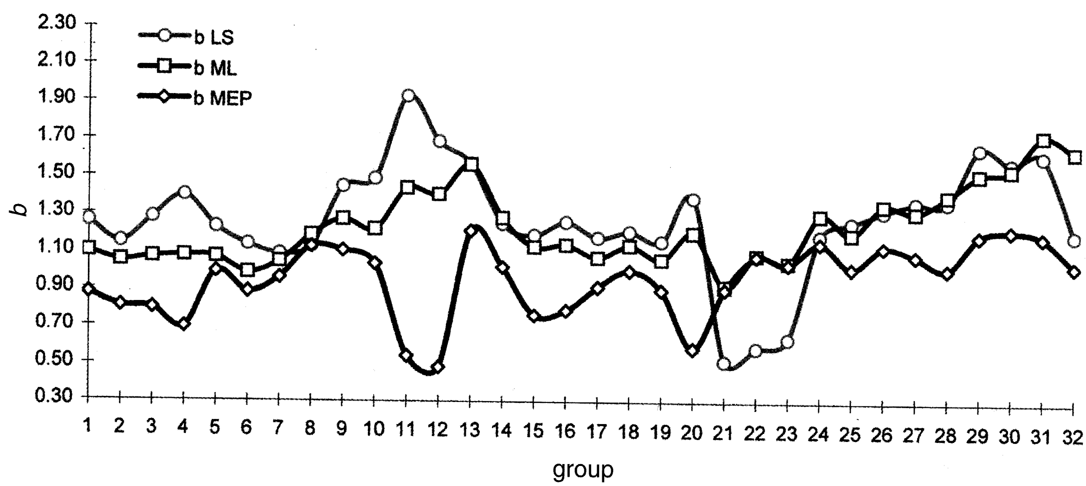

Fig. 5. Graph comparing the temporal changes in b_{LS}, b_{ML} and b_{MEP}.

4. Intensity of volcanic activity at summit craters from 01/01/90 to 31/12/92

The intensity of volcanic phenomena occurring at summit craters during the time interval analysed in this study was evaluated day by day using the empirical scale (Patanè *et al.*, 1991) reported in table I and based on continuous observations of the summit craters. Six different levels of degassing rate have been defined (table I) by a visual estimation of the amount of matter and ash which is thrown out of the four summit chasms. To each level an arbitrary numerical value, or degree of intensity, has been attributed in a similar way to that of the representation of the intensity of an earthquake determined by empirical observations (for more details see Patanè *et al.*, 1991).

Table I. Empirical scale adopted for the estimation of intensity of volcanic activity at summit craters (from Patanè *et al.*, 1991).

Type of volcanic activity	Intensity (arbitrary units)
Lack of degassing or fumarolic activity	0
Very weak	1
Weak	2
Moderate	4
Strong	6
Very strong	9

Every day a mean volcanic intensity was obtained averaging the intensities observed at the SE crater, NE crater, central chasm and W chasm (fig. 6a,b).

In order to make a comparison among the *b* trends and the observed temporal changes in volcanic activity, the whole sequence of smoothed daily intensities shown in fig. 6b was divided into 32 groups, each of which starts and ends on the same days as the corresponding group of seismic events; for every group of volcanic intensities, a mean intensity of the summit crater activity was computed. In the resulting set of 32 mean intensities (fig. 7), the values relative to groups No. 1-12 are coherent with the observations, reported by Patanè *et al.* (1991), of summit craters activity at Mt. Etna during 1990; in particular, the higher intensities of groups No. 6-9 and the lower intensity of group No. 11 correspond, respectively, to the explosive activity which occurred from early June to the second half of September and to the drop in the degassing rate observed in the second half of

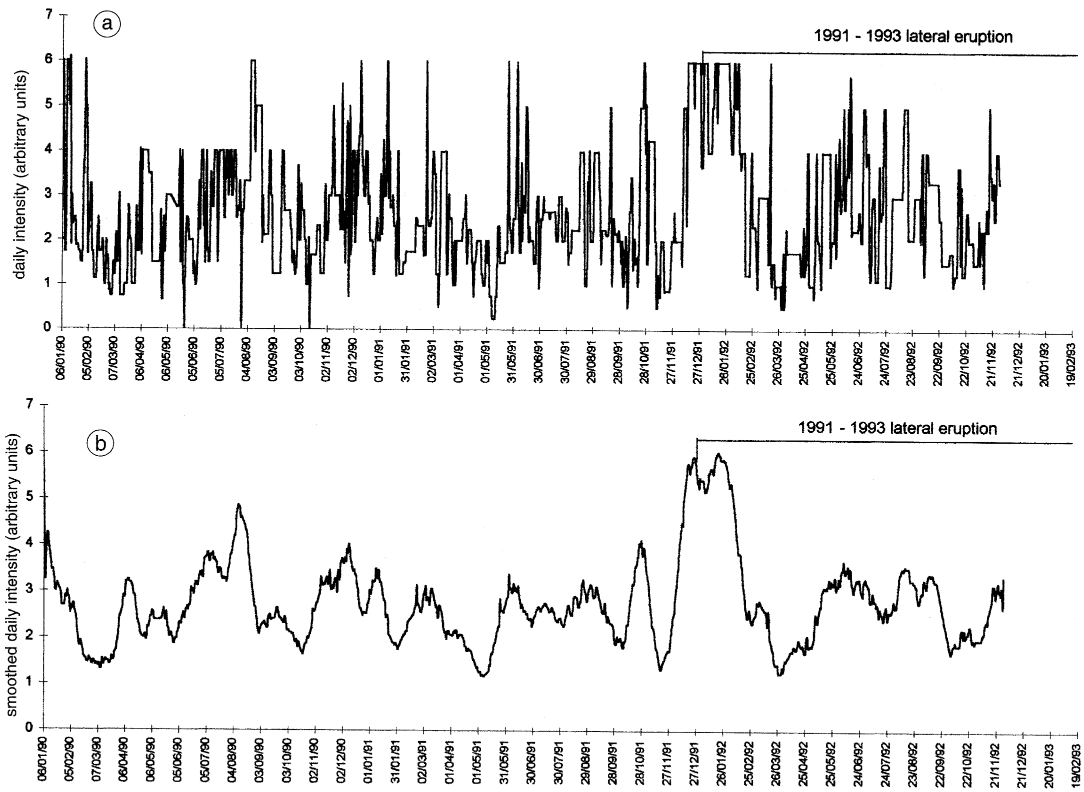

Fig. 6a,b. a) Daily intensity at summit craters from 01/01/90 to 31/12/92; b) the same smoothed by means of a temporal window 20 days wide. Days relative to the 1991-1993 lateral eruption are marked.

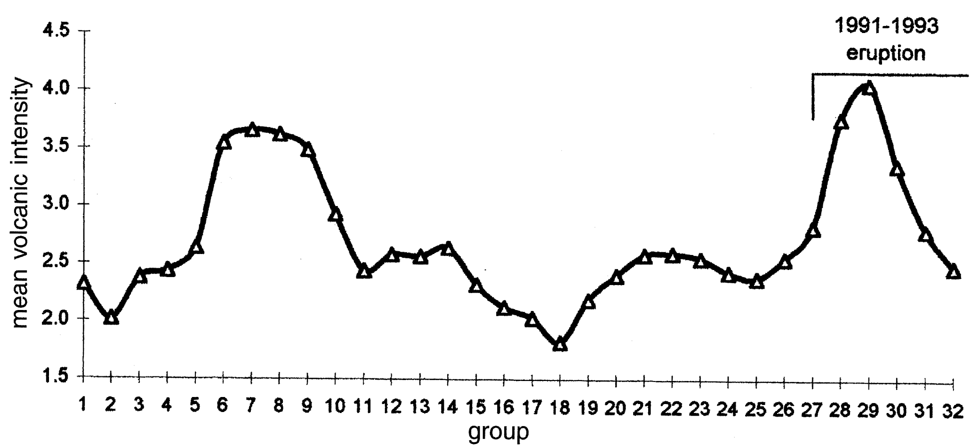

Fig. 7. Mean intensity of summit craters activity for the 32 groups analysed. Intensities corresponding to the 1991-1993 eruption are marked.

November. The values of mean intensity relative to groups No. 12-24 are consistent with the fluctuations in activity from December, 1990 to the first half of November, 1991; the highest intensities of groups No. 28-30 refer to the explosive activity at summit craters connected with the 1991-1993 lateral eruption.

5. Discussion of the results

Figure 5 suggests at least two important considerations. The b_{MEP} values are systematically lower than the corresponding b_{LS} and b_{ML} values: this means that the LS and ML methods, on the average, underestimate the seismic hazard on Mt. Etna. Moreover, for groups No. 20 and 21 (about six months before the 1991-1993 eruption) the significant variations of b_{MEP} seem to precede the corresponding changes calculated by LS and ML; this implies that it would be better to apply MEP if the b variations are used as seismic or volcanic precursors.

In order to understand the differences between the b_{LS} and b_{MEP} trends, fig. 8 shows their time variations together with the changes in the quantities $\langle M_{LS}\rangle - \langle M\rangle$ and $M_u - M_0$. It is evident that the difference between the LS mean magnitude and the sample mean magnitude is always greater than zero: this means that the LS method is greatly influenced by those earthquakes in the sample which are relatively larger than the others; in other words, small and large seismic events, in the LS fit, do not have the same weight. In particular, for groups No. 21-23 the presence of just one event of moderate size (M = 5.18) causes an overestimation of the $\langle M_{LS}\rangle$ and, as a consequence, an underestimation of b_{LS} with respect to the b_{MEP} value. For groups No. 4, 11, 12 and 20, instead, the Least Squares method gives a b_{LS} much greater than the corresponding b_{MEP}, because of the absence of events significantly larger than the others (as confirmed by the small $M_u - M_0$). On the contrary, the b_{MEP} values, unlike the b_{LS} corresponding ones, are not directly related to the $M_u - M_0$ variations: for example, when

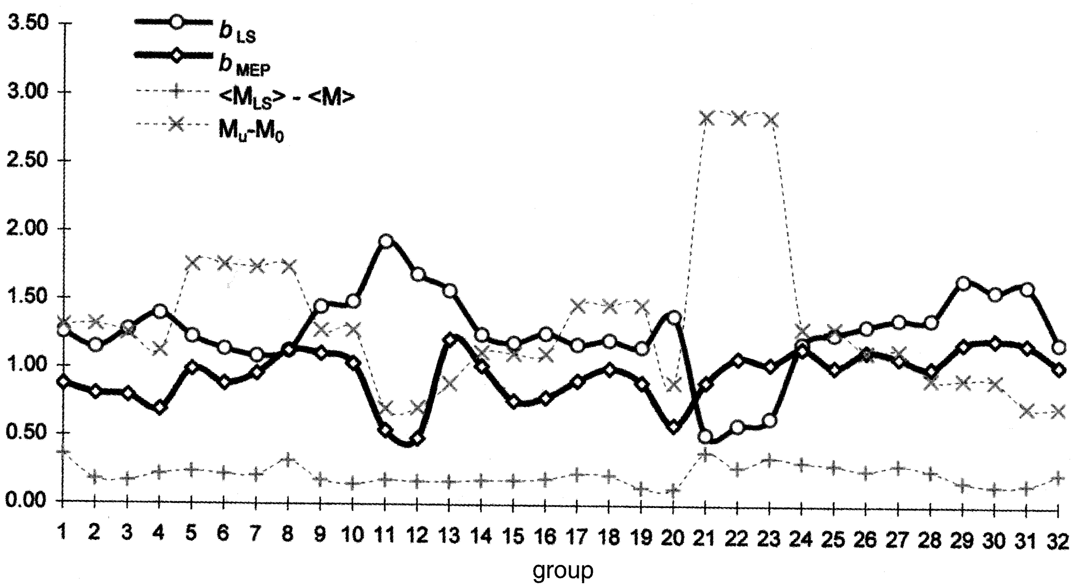

Fig. 8. Time variations of b_{LS}, b_{MEP}, $< M_{LS} > - < M >$ and $M_u - M_0$. $< M_{LS} >$ is the mean magnitude obtained by the LS method, $< M >$ is the sample mean magnitude, M_u and M_0 are the upper and the lower magnitudes of each group, respectively.

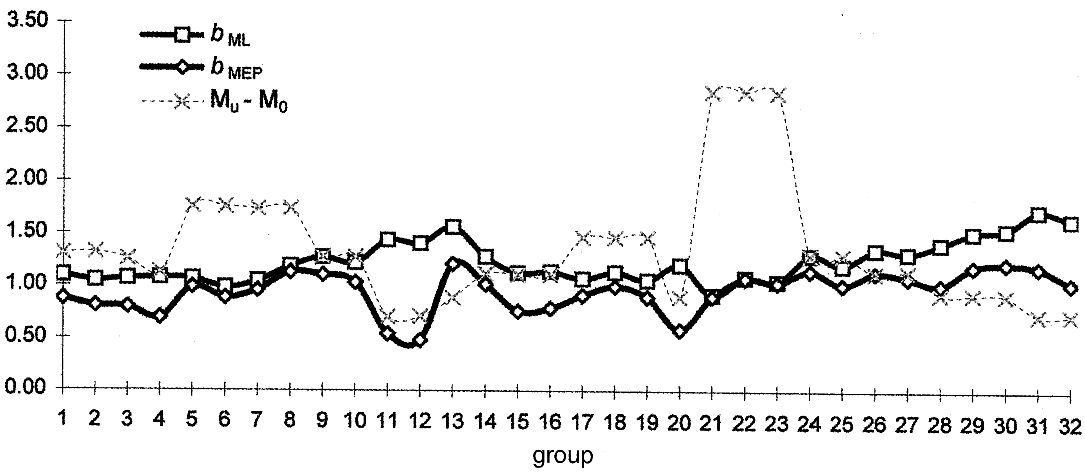

Fig. 9. Time changes in b_{ML}, b_{MEP}, and $M_u - M_0$. M_u and M_0 are the same of fig. 7.

the $M_u - M_0$ value increases the b_{MEP} parameter can increase (groups No. 5, 13, 17, 21) or decrease (groups No. 2, 10, 14, 25, 27).

The deviations of b_{ML} with respect to the b_{MEP} temporal changes can be explained simply in terms of the $M_u - M_0$ values (fig. 9): if $M_u - M_0 << 2$ (groups No. 4, 11-13, 20, 25-32) the b_{ML} are always greater than the b_{MEP} values; when $M_u - M_0 \approx 2$ (groups No. 5-8) the b_{ML} and the b_{MEP} are approximately coincident; if the condition (3.7) is fully verified (groups No. 21-23) the b_{ML} and the b_{MEP} agree exactly. Moreover, it should be remarked that for $M_u - M_0 << 2$ the b_{ML} and b_{MEP} temporal variations show an opposite trend, as confirmed by the analysis of groups No. 4, 11, 12 and 20. This observation suggests that, if the temporal changes in b are used as seismic or volcanic precursors, they should be computed only by means of the Maximum Entropy Principle, which does not depend on the restrictive condition (3.7); on the contrary, the results obtained by the ML method are reliable only if the relation (3.7) is fully satisfied.

On the basis of the results shown in figs. 8 and 9, we discuss the correlation between volcanic and seismic activity referring only to the time variations of b_{MEP}. Figure 10a shows the time variations of mean volcanic intensity superimposed on the variations of b_{MEP}; the result of their cross-correlation analysis is represented in fig. 10b. The cross-correlation function $C(\delta)$ has been computed as follows (Levitch, 1971):

$$C(\delta) = \left\langle \left(mvi(g) - \overline{mvi} \right) \cdot \left(b_{MEP}(g+\delta) - \overline{b_{MEP}} \right) \right\rangle \tag{5.1}$$

In (5.1) mvi indicates the mean volcanic intensity, g = 1, 2, ... 32 is the group index, $\delta = \pm\, 0, 1, \ldots 31$ is the group lag, $\overline{mvi} = \frac{1}{32} \sum_g mvi(g)$ and $\overline{b_{MEP}} = \frac{1}{32} \sum_g b_{MEP}(g)$. The angular parentheses correspond to average all values with fixed δ.

In fig. 10a it is evident that an increase of the mean volcanic intensity (groups No. 6, 14, 27) is preceded by an increase in the b_{MEP} value (groups No. 5, 13, 21); the increases in the two quantities are shifted by one group (groups No. 5 for b_{MEP} and No. 6 for the mean intensity, No. 13 for b_{MEP} and No. 14 for the mean intensity) or five groups (groups No. 21 for b_{MEP} and No. 27 for the mean intensity). Likewise, a decrease of the mean volcanic intensity (groups No. 10, 18)

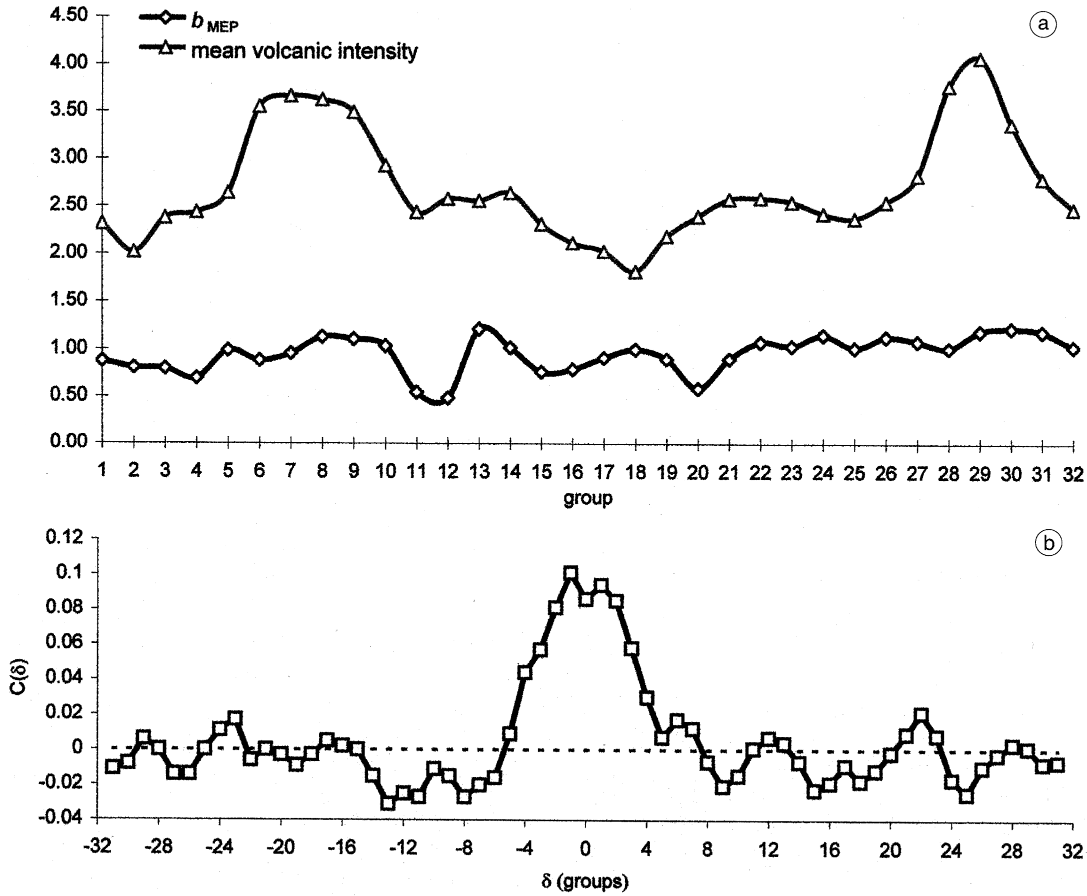

Fig. 10a,b. a) Temporal variations of mean volcanic intensity superimposed on the time changes in b_{MEP}; b) result of the cross-correlation analysis between these time series. δ is the lag (in groups) appearing in the cross-correlation function (5.1).

is followed by a decrease of the b_{MEP} value (groups No. 11-12, 19-20) and, over again, the decreases are shifted by one group (groups No. 10-11 for the mean intensity and No. 11-12 for b_{MEP}, No. 19 for the mean intensity and No. 19-20 for b_{MEP}).

These qualitative interpretations are confirmed by the cross-correlation function (fig. 10b), which shows a clear maximum for $-4 \leq \delta \leq 4$ and oscillates around zero elsewhere. Then, the b_{MEP} pattern actually observed must be regarded as mainly linked to the volcanic activity rather than to the random fluctuations.

Considering that the b value increases with decreasing effective stress (Mogi, 1967; Scholz, 1968; Main *et al.*, 1989), and that the effective stress is given by superposition of the confining stress and the pore pressure, our results seem to confirm that the confining (tectonic) stress field at Mt. Etna is prevalently compressive: only in this case, in fact, the increase in the pore pressure due to the magma acting on the volcanic structures can be related to the decrease in the effective stress, that is to the increasing b value actually observed before the eruptive events.

Further proof of this assertion is given by the *b* temporal variations estimated by Main (1987) before the eruption of Mt. St. Helens on May 18th, 1980 (fig. 11). The growth of the magma chamber produced a tensile stress, as confirmed by the graben areas formed on the mountain before the explosive eruption. During the phreatic eruption from March 27th to April 13th, the tensile stress and the high pore pressure overlapped, causing an increase in the effective stress and a decrease of the *b* value.

At Mt. Etna, moreover, when the eruption is lateral (as in 1991-1993), even if the concomitant summit crater activity reduces itself (groups No. 30-32) the b_{MEP} value does not change significantly: this means that the *b* values are related to the effective stress acting not only on pipes of the summit craters, but on a wide portion of the crust under the volcanic structure.

The decrease of the effective stress observed before the increase in the activity at summit craters is consistent with the hypothesis of a magma infiltration in a large storage system formed by a complex network of veins of melt, at a depth of about 20 km below Mt. Etna (Sharp *et al.*, 1980; Cosentino *et al.*, 1982); similarly, the increase in the effective stress after the end of eruptive phenomena could be related to the depletion of this storage system.

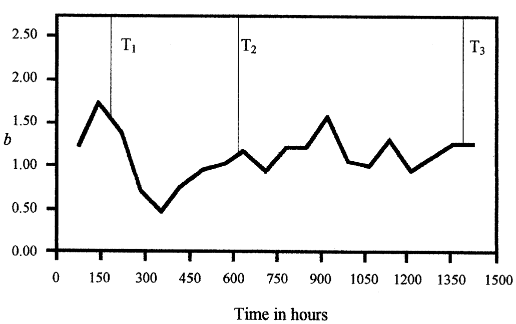

Fig. 11. Variations of the *b* value with time before the eruption of Mt. St. Helens on 18 May 1980. The starting time on the *x*-axis is $T = 0$ h on 20 March 1980; a phreatic eruption begins at $T_1 = 180$ h and ends at $T_2 = 580$ h; the main explosive eruption starts at $T_3 = 1432$ h (from Main, 1987; modified).

6. Conclusions

The present analysis enables us to draw some methodological considerations and to point out some important features of Mt. Etna dynamics.

In order to study the magnitude-frequency relationship of earthquakes and the *b* value, the application of the usual Least Squares method leads to an uncorrected statistical distribution, because the LS fit is strongly influenced by the events in the sample with magnitude relatively larger; this fact implies an overestimation or, alternatively, an underestimation of the *b* value depending on the presence or on the absence of these larger earthquakes. Similarly the standard Maximum Likelihood formula, if used without satisfying the condition (3.7), overestimates the *b* parameter; besides, the resulting trend of b_{ML} temporal variation is opposite to the corresponding one determined by MEP.

On the contrary, the Maximum Entropy Principle determines the best statistical distribution of earthquakes, taking correctly into account the mean magnitude and the difference between the maximum and the minimum magnitude of the sample.

At Mt. Etna the analysis of temporal changes in the magnitude-frequency relationship performed by MEP shows that there is an evident correlation between the significant variations of the b_{MEP} value and the sequence of observed eruptive phenomena.

From 1990 to 1992 every increase in volcanic activity has been preceded by an increase in the b_{MEP} value; for the eruptive events at summit craters the time interval between the b_{MEP} increase and the eruption is, on the average, thirty days, whereas for the lateral eruption of 1991-1993 it is six months. On the basis of these results we can therefore conclude that the significant b_{MEP} increases, at Mt. Etna, can be considered reliable eruptive precursors. Moreover, after the end of the eruptive events a decrease of the b_{MEP} was observed; the overall pattern of b_{MEP} temporal changes can be explained only assuming a compressive regional confining stress and taking into account the effect of the superposition of the magma pressure on this compressive stress.

The conclusions so far must be considered only as preliminary indications; further interdisciplinary research is needed to extend our knowledge of a system as complex as the Etna volcano.

Acknowledgements

The authors wish to thank Giuseppe De Natale, Cataldo Godano and the unknown reviewer for their criticism of the manuscript.

The work was supported by MURST 40% and 60% grants.

REFERENCES

AKI, K. (1965): Maximum likelihood estimate of b in the formula log $N = a - bM$ and its confidence limits, *Bull. Earthquake Res. Inst., Tokyo Univ.*, **43**, 237-239.

AKI, K. (1981): A probabilistic synthesis of precursory phenomena, in *Earthquake prediction – an International Review, Maurice Ewing Series 4*, Am. Geophys. Union, 556-574.

AKI, K. (1987): Magnitude-frequency relation for small earthquakes: a clue to the origin of f_{max} of large earthquakes, *J. Geophys. Res.*, **92** (B2), 1349-1355.

BENDER, B. (1983): Maximum likelihood estimation of b values for magnitude grouped data, *Bull. Seismol. Soc. Am.*, **73** (3), 831-851.

BERRILL, J.B. and R.O. DAVIS (1980): Maximum entropy and the magnitude distribution, *Bull. Seismol. Soc. Am.*, **70** (5), 1823-1831.

CAPUTO, M. (1983): The occurrence of large earthquakes in South Italy, *Tectonophysics*, **99**, 73-83.

CENTAMORE, C., A. MONTALTO and G. PATANÈ (1997): Self-similarity and scaling relations for micro-earthquakes at Mt. Etna volcano (Italy), *Phys. Earth Planet. Int.*, **103**, 165-177.

DE NATALE, G., G. IANNACCONE, M. MARTINI and A. ZOLLO (1987): Seismic sources and attenuation properties at the Campi Flegrei Volcanic area, *Pageoph.*, **125**, 883-917.

DONG, W.M., A.B. BAO and H.C. SHAH (1984): Use of maximum entropy principle in earthquake recurrence relationships, *Bull. Seismol. Soc. Am.*, **74** (2), 725-737.

GRESTA, S. and G. PATANÈ (1983a): Variations of b values before the Etnean eruption of March 1981, *Pure Appl. Geophys.*, **121**, 287-295.

GRESTA, S. and G. PATANÈ (1983b): Changes in b values before the Etnean eruption of March-August 1983, *Pure Appl. Geophys.*, **121**, 903-912.

GUTENBERG, B. and C.F. RICHTER (1944): Frequency of earthquakes in California, *Bull. Seismol. Soc. Am.*, **34**, 185-188.

HIRATA, T. (1989): A correlation between the b value and the fractal dimension of earthquakes, *J. Geophys. Res.*, **94** (B6), 7507-7514.

LEVITCH, B.G. (1971): *Theoretical Physics, vol. 2: Statistical Physics* (North Holland Publishing Company, Amsterdam).

MAIN, I.G. (1987): A characteristic earthquake model of the seismicity preceding the eruption of Mt. St. Helens on 18 May 1980, *Phys. Earth Planet. Inter.*, **49**, 283-293.

MAIN, I.G., P.G. MEREDITH and C. JONES (1989): A reinterpretation of the precursory seismic b-value anomaly from fracture mechanics, *Geophys. J.*, **96**, 131-138.

MEREDITH, P.G. and B.K. ATKINSON (1983): Stress corrosion and acoustic emission during tensile crack propagation in Whin Sill dolerite and other basic rocks, *Geophys. J. R. Astron. Soc.*, **75**, 1-21.

MOGI, K. (1967): Earthquakes and fractures, *Tectonophysics*, **5**, 35-55.

MOLCHAN, G., T. KRONROD and G.F. PANZA (1996): Hazard oriented multiscale seismicity model: Italy, *ICTP, IAEA, UNESCO, Int. Rep. IC/96/23*, Trieste, Italy, pp. 50.

PAGE, R. (1968): Aftershocks and microaftershocks of the great Alaska earthquake of 1964, *Bull. Seismol. Soc. Am.*, **58**, 1131-1168.

PATANÈ, D., T. CALTABIANO, E. DEL PEZZO and S. GRESTA (1992): Time variation of b and Q_c at Mt. Etna (1981-1987), *Phys. Earth Planet. Inter.*, **71**, 137-140.

RYDELEK, P.A. and I.S. SACKS (1989): Testing the completeness of earthquake catalogues and the hypothesis of self-similarity, *Nature*, **337**, 251-254.

SCHOLZ, C.H. (1968): The frequency-magnitude relationship of microfracturing and its relation to earthquakes, *Bull. Seismol. Soc. Am.*, **58**, 399-417.

SHANNON, C.E. (1948): A mathematical theory of communication, *Bell Syst. Tech. J.*, **27**, 379-423 and 623-656.

SHANNON, C. E. and W. WEAVER (1949): *The Mathematical Theory of Communication* (University of Illinois Press), pp. 117.

SHEN, P.Y. and L. MANSINHA (1983): On the principle of maximum entropy and the earthquake frequency-magnitude relation, *Geophys. J. R. Astron. Soc.*, **74**, 777-785.

SHI, Y. and B.A. BOLT (1982): The standard error of the magnitude-frequency b value, *Bull. Seismol. Soc. Am.*, **72** (5), 1677-1687.

STEPP, J.C. (1972): Analysis of the completeness of the earthquake hazard sample in the Puget Sound Area and its effect on the statistical estimates of earthquake hazard, in *Proc. Internat. Conf. on Microzonation for Safer Constr. Res. Appl. Seattle, Washington*, **2**, 887-909.

TAYLOR, D.W.A., J.A. SNOKE, I.S. SACKS and T. TAKANAMI (1990): Nonlinear frequency-magnitude relationships for the Hokkaido Corner, Japan, *Bull. Seismol. Soc. Am.*, **80** (2), 340-353.

TINTI, S. and F. MULARGIA (1985): Completeness analysis of a seismic catalog, *Ann. Geophysicae*, **3**, 407-414.

UTSU, T. (1965): A method for determining the value of b in formula log $N = a - bM$ showing magnitude-frequency relation for earthquakes, *Geophys. Bull. Hokkaido Univ.*, **13**, 99-103.