ANNALI DI GEOFISICA, VOL. 42, N. 3, June 1999

Dynamical tools for the analysis of long term evolution of volcanic tremor at Stromboli

Roberto Carniel and Mauro Di Cecca
Dipartimento di Georisorse e Territorio, Università di Udine, Italy

Abstract
Stromboli is characterised by persistent seismic activity, both in terms of tremor and of discrete events associated with moderate explosions defining the so-called «Strombolian activity». This kind of permanent activity suggests the probable existence of a dynamical system governing the volcano. The purpose of this paper is to extract information on the «Stromboli dynamical system» from the seismic recordings. The analysis is carried out using the theory of non-linear dynamical systems and the delayed coordinates phase-space reconstruction. Several methods are presented and discussed in order to analyse volcanic tremor data in this framework; finally the time evolution of the computed parameters, both in the long term and close in time to paroxysmal phases, is presented and discussed. Evidence for a strong deterministic component in the dynamics of the volcano is shown.

Key words *non linear analysis – volcanic tremor – state space reconstruction – delayed coordinates*

1. Introduction

The activity of Stromboli volcano (Aeolian Islands, Italy) is characterised by a continuous series of moderate explosions that constitute the so-called *Strombolian activity*. Their seismic footprints (Carniel and Iacop, 1996a) are shocks that we will call simply «events», recorded several times per hour. This, together with the persistent presence of the volcanic tremor, *i.e.* the ground movement between these events, leads us to consider the volcano as a dynamical system, *i.e.* a system that evolves with time according to precise rules (Carniel, 1993); these will determine for instance the daily number of events and the intensity of the tremor.

Mailing address: Dr. Roberto Carniel, Dipartimento di Georisorse e Territorio, Università di Udine, Via Cotonificio 114, 33100 Udine, Italy; e-mail: carniel@dgt.uniud.it

In this paper we will try to analyse the dynamics (*i.e.* the time evolution) of the volcanic tremor with the tools offered by the theory of non linear dynamical systems and chaos, in order to extract from the seismic recordings information on the dynamical system that governs the entire volcano.

The data used in this study are sampled by an automatic seismic station installed in 1989 by the Dipartimento di Georisorse e Territorio of the University of Udine and operating since 1992 in its current three-component configuration (Beinat *et al.*, 1994).

More detailed analyses will be performed close in time to some paroxysmal phases. In fact, Stromboli volcano sometimes shows, besides «normal» moderate explosive activity, more energetic phases characterised by powerful explosions or small lava flows.

In the literature, several works have applied results coming from the theory of non linear systems and fractal analysis to the time distributions of earthquakes (Smalley *et al.*, 1987; Turcotte, 1992). In this paper we will go further, as we will study the time behaviour of different dynamical parameters independently computed on many different samples.

2. Non linear analysis

A *dynamical system* is a system that evolves with time. Its state at any time is represented by a vector of *state variables*. Let's now assume that we can record scalar discrete observations of some of these state variables.

We may denote the resulting time series with $\{x_k\} = \{x(k\tau_s)\}$, where N is the length of the series, $k = 0, ..., N$ and τ_s is the sampling interval. We are assuming that the original dynamical system is continuous, *i.e.* the time is running in the real domain. The other typology, that of discrete dynamical systems, assumes that the time domain is discrete (*e.g.*, evolution of populations, rate exchanges etc.). When sampling, we are therefore implicitly passing from the first kind of dynamical system to the second. The relation that links the time series to the original continuous dynamical system equation $\boldsymbol{s}'(t) = f(\boldsymbol{s}(t))$ is given by $x_k = h(\boldsymbol{s}(kt_s))$, where h is a measuring function, and $\boldsymbol{s}$ is the original state variables vector. Regardless of the sampling interval, the function f governing the dynamical system, its dimension N (*i.e.* the number of state variables) and the measuring function h are in general unknown (Kugiumtzis *et al.*, 1994). Another hypothesis we may make is that the dynamics in the long term is confined on a manifold with a (usually fractal) dimension f, smaller than N. This manifold is called an attractor. However, its dimension f is as unknown as the state dimension N. The idea on which all the methods we will describe are based is that we cannot reconstruct the original attractor, but we can try to reconstruct an embedding space where another attractor lies preserving the invariant characteristics of the original one (see fig. 1).

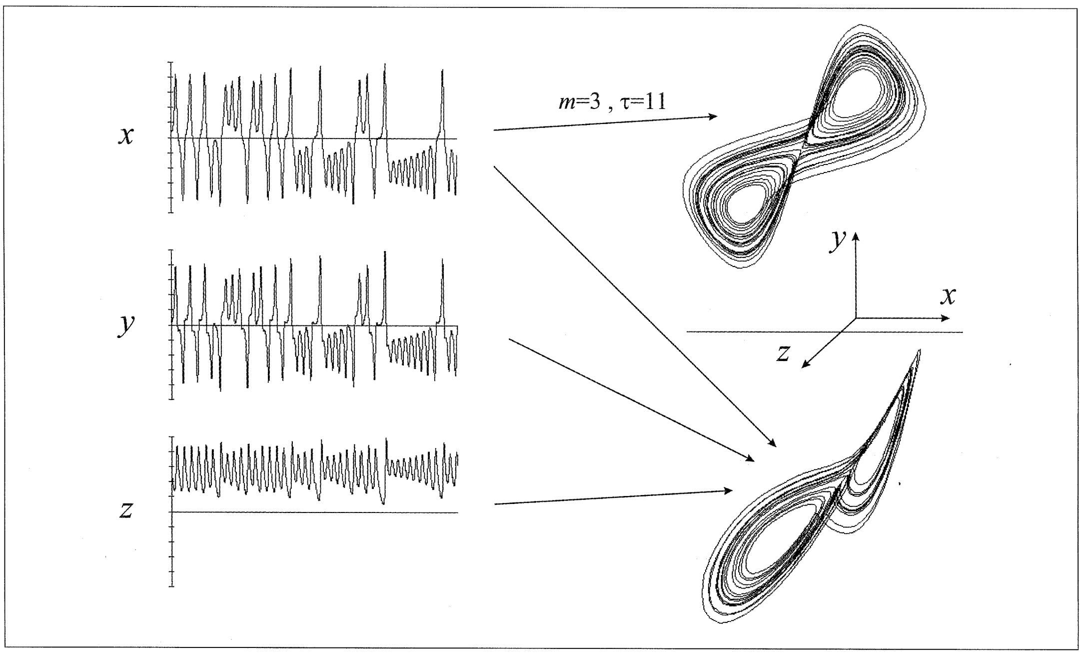

Fig. 1. Reconstruction of Lorenz attractor using delay coordinates method from one data series with a time delay $\tau = 11$ in an embedding dimension $m = 3$. The original attractor is plotted using all three data series (x, y, z).

The *embedding dimension m* of the reconstructed state space will in general be different from the unknown dimension $\lfloor d \rfloor + 1$ if d is the dimension of the attractor.

The simplest method (albeit effective!) to derive a state vector $\boldsymbol{x}_k$ from a time series is the one of delay coordinates (Packard *et al.*, 1980)

$$\boldsymbol{x}_k = [x_k, x_{k-\tau}, ..., x_{k-(m-1)\tau}]^T \qquad k = (m-1)\tau, ..., N \tag{2.1}$$

Here τ is a multiple of τ_s, and it is called *delay time*. With this method, the reconstruction problem is simply reduced to the determination of the correct embedding dimension m and of the correct delay time τ.

The most common method for the choice of the delay time makes use of the autocorrelation function. Starting from the definition

$$\rho_{X,Y} = \frac{\sigma_{X,Y}}{\sigma_X \, \sigma_Y} \tag{2.2}$$

where $\sigma_{X,Y}$ represents the covariance and σ_X and σ_Y the standard deviation of the random variables X and Y, we define

$X = \{x_k\}$ for $k = 0, ..., N-\tau$ (original time series)

$Y = \{x_{k+\tau}\}$ for k = 0, ..., N $- \tau$ and $\tau = 0, ... \tau_{max}$ (delayed time series).

Using the variables defined above in eq. (2.2) we obtain the correlation of the original time series with the same series delayed by a time τ. If we now repeat this computation for τ varying from 0 to a given τ_{max}, the choice of the optimal delay time for the reconstruction goes to the first τ for which the value $\rho_{X,Y}$ is negative, *i.e.* the value of τ after the first zero crossing.

The choice of τ with this method induces an independence between the coordinates, which are guaranteed to be linearly uncorrelated. However, considering the particular nature of the time series under study, typically generated by non linear phenomena, we are strongly suggested to look for more sophisticated methods, furnishing a more general independence between the coordinates.

The *mutual information* (Fraser, 1986; Fraser and Swinney, 1986) is one of these criteria. As suggested by Shaw (1984), the value τ for which we find the first minimum of the mutual information will be used for the reconstruction of the pseudo state space.

The mutual information answers the question: «given $\boldsymbol{x}$ measured at time t, which is the amount of information that we have about $\boldsymbol{x}$ measured at time $t + \tau$?» (Shannon, 1948). Iterating the computation on τ going from 0 to τ_{max}, as for the case of the autocorrelation, the first minimum of τ is chosen as the candidate delay time. The choice of this τ guarantees a nonlinear independence between the reconstructed coordinates.

For choosing the correct embedding dimension, the method of *False Nearest Neighbours* (FNN) (Liebert *et al.*, 1991; Kennel *et al.*, 1992) can be used. The procedure identifies the so-called «false neighbours», *i.e.* points that *appear* to be near only because the embedding space is too small (fig. 2), just like two objects which are near in a photograph but well separated in the third dimension in reality.

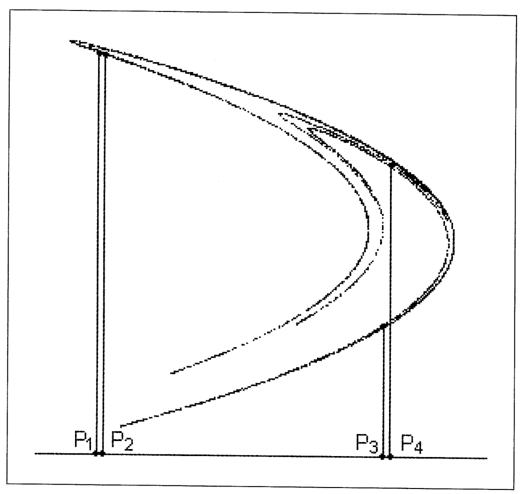

Fig. 2. Henon attractor: P_1 and P_2 are «True» Neighbours, while P_3 and P_4 are «False» Neighbours, *i.e.* appear to be near only because embedded in a too small embedding space.

When, by increasing the embedding dimension, the percentage of FNN goes to zero for a certain value m, the attractor is completely unfolded in that dimension, which is therefore chosen as the optimal one for the reconstruction (Kennel *et al.*, 1992). Of course before applying the procedure one must have already chosen an appropriate delay time τ, *e.g.*, with the first zero crossing of the autocorrelation or with the first minimum of the mutual information.

Another criterion for the choice of the correct embedding dimension makes use of the Singular System Analysis, or Singular Value Decomposition (SVD) (Broomhead and King, 1986).

$$W = \begin{bmatrix} x_1 & x_2 & \dots & x_n \\ x_2 & x_3 & \dots & x_{n+1} \\ \vdots & \vdots & \ddots & \vdots \\ x_N & x_{N+1} & \dots & x_{N+n-1} \end{bmatrix} \tag{2.3}$$

This approach consists in creating a series of $\boldsymbol{n}$-windows which become the rows of a trajectory matrix $\boldsymbol{W}$. The matrix $\boldsymbol{W}$ is then projected to an $\boldsymbol{n}$-dimensional subspace $\boldsymbol{V}$. The basis of $\boldsymbol{V}$ is chosen one vector $\boldsymbol{e}_i$ at a time ($\boldsymbol{i}$ = 1, ..., $\boldsymbol{n}$), in order to maximise its average weight in the construction of a row $\boldsymbol{w}$ of the matrix $\boldsymbol{W}$, with respect to the following vectors. The values $\sigma_i = (<|\boldsymbol{w} \cdot \boldsymbol{e}_i|^2>)^{1/2}$ are called *singular values*. The number $\boldsymbol{m}$ of σ_i above a given noise threshold corresponds to the embedding dimension of the attractor. Note that $\boldsymbol{w} \cdot \boldsymbol{e}_i$ are guaranteed to be only *linearly* independent.

In order to obtain a correct choice of the size $\boldsymbol{n}$ of the windows we use the bounds (2.4) proposed by Broomhead and King (1986), where τ_s is the sampling period, $\tau_w = n\tau_s$, $\tau^* = 2\pi/\omega^*$ and ω^* is the band frequency limit

$$(2m + 1)\tau_s \le \tau_w \le \tau^*. \tag{2.4}$$

3. The data

The summit station, based on 3 Willmore MKIII/A seismometers, is sited at 800 m a.m.s.l. and at about 300 m from the craters (Carniel and Jacop, 1996a, fig. 2). The receiving station, connected via a radio link, is sited in the village of Stromboli. Here the seismic signal, after an Anti-Alias Filtering at 25 Hz, is digitised with a sampling frequency of 80 Hz at a 12 bit resolution. The resulting files span a 60 s time window each. The discrimination between tremor and events is made by an automatic triggering procedure (Beinat *et al.*, 1994). The tremor is sampled at regular times, usually once per hour. In the following, data will be analysed relative to the volcanic tremor recorded by the seismic station between 11 May 1992 and 30 June 1996. The three components were analysed independently.

The gaps in the graphs are associated with periods during which the acquisition could not take place, mainly due to problems with solar panel efficiency. Due to very different computation times required by the different methods applied in this study, we present the analyses on a subset of the data set in some cases, on the entire data set in other cases.

If not indicated otherwise, the graphs are the result of the application of a moving average procedure to the results of the analyses, with a seven day long window in the long term case, or five day long window in the case of the «zoom graphs» near the paroxysmal phases. This «low pass filter» was needed in order to obtain a better graphical presentation and to highlight the long term evolution of the parameters more than short-lived phases. Arrows refer to single noteworthy eruptive events (vertical arrows) or to periods of intense activity (horizontal arrows). Vertical bars are drawn to indicate the beginning of the different years.

4. Long term analysis of volcanic tremor

4.1. *Intensity and spectra*

The data relative to the tremor intensity come from the reports which are regularly downloaded via modem from Stromboli to Udine and graphically presented on the web site STROMBOLI on-line (Alean and Carniel, 1995, 1996). Every datum represents the daily average of the intensity (integral of the module) as computed on all recorded hourly tremor samples. A moving average with a seven days long window was applied in order to improve the readability.

This first graph (fig. 3) will be used as a reference for the interpretation of successive analyses. We immediately note that the paroxysmal phases (Carniel and Iacop, 1996b) shown in the graph are not predictable on the base of the tremor intensity and that there is no unique behaviour even after such phases: *most of the times* we observe a sudden drop in tremor intensity, but this is not the case of 5 March 1995.

Although the analyses were carried out on all three components, we present in detail only those relative to the tangential one, as we found this the most sensitive with respect to the investigated dynamical parameters. The other components however do not usually show a considerably different behaviour.

The first analysis carried out on the data concerns the spectral content of the volcanic tremor. We do not go into detail, as a much more complete spectral analysis, although with a slightly different procedure, is presented by Carniel and Iacop (1996b). The procedure we applied to each of the three components is the following:

1) The offset of the signal is removed by subtracting the time average from its duration.

2) A Fast Fourier Transform is applied using a 10% Hanning window and a normalisation of the amplitude to the range [0,1].

3) The most interesting section of the spectrum (*i.e.* the range 0.5-10.5 Hz) is subdivided into ten 1 Hz wide bands; the mean value in each band is computed.

4) The daily average of the resulting values is computed.

5) The resulting values are divided by their sum to normalise it to the value 1.

6) The «average frequency» is computed as the weighted average of the 10 band centers.

7) The «dominant frequency band» is also determined.

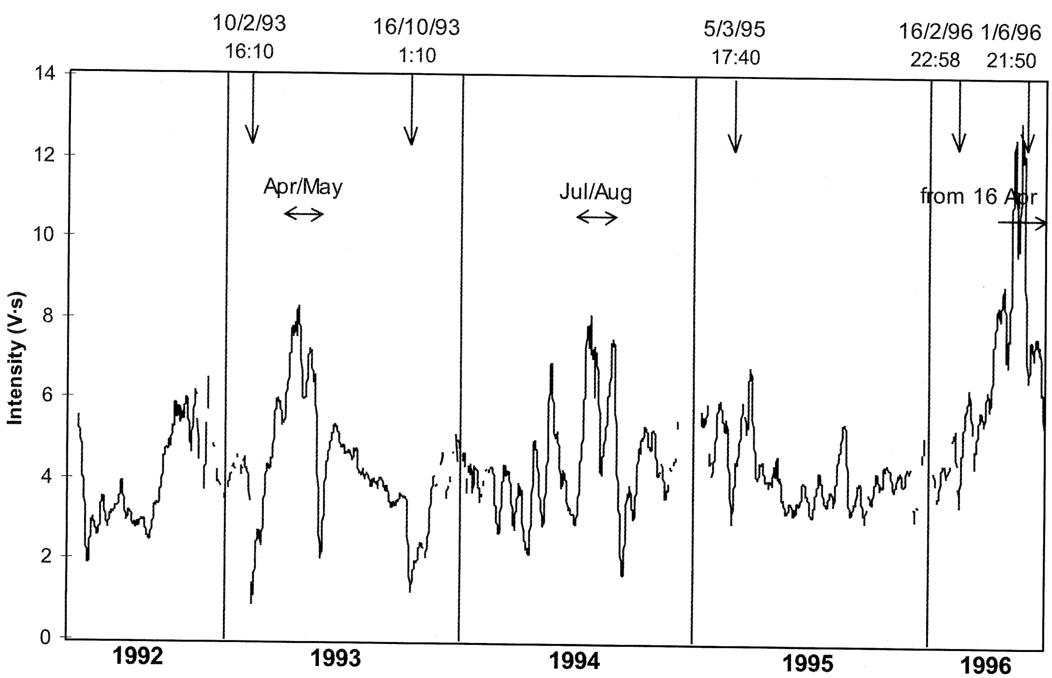

Fig. 3. Evolution of tremor intensity between 11 May 1992 and 30 June 1996.

This procedure is a good compromise between the need to get the long term evolution of the tremor by using the greatest number of samples and the impossibility to maintain the complete spectra of the single samples, as this would lead to an intractable amount of data.

At first sight we note how (fig. 4a) the «average» and «dominant» frequencies show abrupt changes at the time of the explosions. These changes however do not share the same «direction». Another observation is that higher frequencies are generally shown during periods with intense tremor .

During the period July/August 1995 an increase in the average frequencies is observed. This increase cannot be related to significant variations in tremor intensity or external eruptive activity.

If we now examine the «shape» of the graphs relative to the «average» frequency and to the «dominant» frequency (fig. 4a) we can highlight the differences between these two parameters:

1) The «average» frequency shows a smoother aspect, and it describes the general evolution of the spectrum. In the presence of a bimodal spectrum, for instance, the average will assume a value somewhere between the two peaks.

2) The «dominant» frequency band is much more irregular, as it describes the position of the dominant sector of the spectrum. It can assume only integer values, and therefore it «jumps» when it changes value. In the previous example of a bimodal spectrum the parameter would indicate always just one of the two peaks, possibly switching abruptly back and forth from one to the other if these present comparable amplitudes.

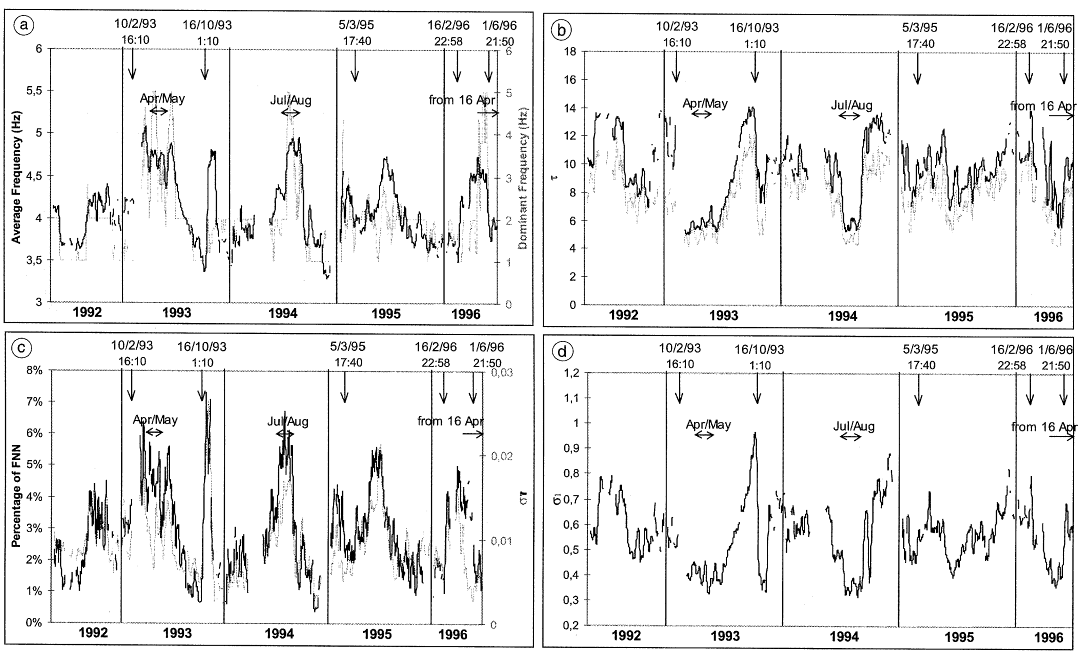

Fig. 4a-d. Long term analysis. a) Evolution of «average» frequency (bold line) and «dominant» frequency (thin line); b) evolution of delay time using first zero crossing of *Autocorrelation* (bold line) and first minimum of *Mutual Information* (thin line); c) percentage of FNN for $m = 7$ (bold line) and 7th singular value (eigenvalue) using SVD method (thin line); d) first singular value using the SVD method.

4.2. *Delay time*

If we now go on to the parameters more linked to the theory of non-linear dynamical systems, the first step towards a proper reconstruction of a pseudo state space is that of determining the correct delay time. As said above, our purpose is that of extracting as much information as possible starting from a single time series of the volcano dynamical system, *i.e.* the volcanic tremor. Actually we have three series available, one per component; these will therefore be analysed separately, by applying exactly the same procedure in order to obtain comparable time evolutions.

For the determination of the delay time we used both methods described in Section 2: the first zero crossing of the autocorrelation function and the first minimum of the mutual information.

For the first method we applied the following procedure to the three components of all available tremor data (26 057 samples):

1) The offset of the signal is removed by subtracting the time average from its duration.

2) The autocorrelation function is computed for every τ between 1 and 20.

3) The first zero-crossing is determined.

4) A daily average is computed of the results of points 2 and 3.

In order to compute the delay time with the method of mutual information we did not use all the available data, but only the files recorded at 12 GMT of each day. This choice was suggested by the relevant computation time required by this method. In this way 1179 files were processed with the following procedure:

1) The offset of the signal is removed by subtracting the time average from its duration.

2) The mutual information is computed for every τ between 1 and 20.

3) The first minimum is determined.

Of course no daily average is needed in this case. The first result is the high correlation between the evolution of the values suggested by the autocorrelation function and of the ones coming from the mutual information (fig. 4b). This is an encouraging result, as we may say that regardless of the method chosen for the determination of the delay time, we obtain in general comparable results.

Nevertheless, an anomalous behaviour is shown during the period November 1992-February 1993. A significant increase in the delay time suggested by the autocorrelation is evident in fig. 4b, while no noteworthy variations can be seen in the evolution of the mutual information delay time. This behaviour can be related to the duration of the «pre-crisis» configuration observed before the 10 February 1993 explosion (Carniel and Iacop, 1996b).

In the graph we can also note a discontinuity at the time of the most energetic explosions, and significant variations of the parameters during the periods of the lowest and highest tremor intensity.

Far from the explosions, the tremor intensity and the delay time usually present opposite trends. The simplest interpretation of this result is that the Stromboli dynamical system is simply showing different dynamics as the tremor intensity grows. The second interpretation is more speculative. We can think that when the tremor intensity is high, a greater contribution due to site and path effects is recorded, effects which need a higher intensity to be triggered. These affect the recorded signal significantly, and this perturbation increases its non deterministic component. This in turn is reflected in a decrease of the delay time, as even the nearest time-delayed series possess a low mutual correlation/information. On the contrary, when the tremor is low, the site and path effects decrease, therefore showing an increased delay time, which should be more linked to the real dynamics of the system.

Nevertheless, behaviours which do not fit this scheme are observed close in time to the paroxysmal phases of the volcano. Immediately after these phases, as already pointed out, the tremor intensity generally shows a rapid drop (fig. 3), but after the 1993 explosive events, the decrease of the tremor intensity is accompanied by a similar decrease of the delay time (fig. 4b). This result is additional evidence that the explosions are associated with significant changes in the dynamics of the system.

If we now compare the graphs relative to the delay time (fig. 4b) and the dominant frequencies (fig. 4a) we can note how the trend is gen-

Table I. Mean and standard deviation for the values of the delay time found with the autocorrelation and with the mutual information.

	Autocorrelation			Mutual information		
	Z	R	T	Z	R	T
Mean	6.75	5.31	9.49	5.97	4.75	7.79
Standard deviation	1.52	1.10	3.26	1.45	0.94	2.33

erally opposite: when the dominant frequency increases, the suggested delay time decreases and *vice versa*.

Finally, from table I we can see how the results of the two methods, besides having a highly correlated time evolution, also show comparable absolute values.

4.3. *Embedding dimension*

For the determination of the embedding dimension we used the method of False Nearest Neighbours in the version proposed by Kennel *et al.* (1992) (FNN K.B.A.) and the method of Singular Value Decomposition (SVD) (Broomhead and King, 1986).

For an efficient computation of the correct embedding dimension with FNN-K.B.A. a preliminary analysis was carried out to determine a suitable range of dimensions on which to operate the method. We observed that, even for high dimensions, the percentage of FNN did not go down to zero. This result was expected, as we are dealing with experimental time series, and therefore data subject to noise. We proceeded by excluding both very small dimensions, for which the percentages of FNN for any considered series would have been very high, and too high ones, for which the considerable additional computation time was not justified by significant achievements.

Of course, before applying the method we have to choose the delay time. For every analysed tremor, we applied the following four options:

1) The first zero-crossing of the autocorrelation as computed on the single sample.

2) The first minimum of the mutual information as computed on the single sample.

3) The mean (over the whole time evolution) of the first zero-crossing of the autocorrelation.

4) The mean (over the whole time evolution) of the first minimum of the mutual information.

The mean values in (3) and (4) are actually the nearest integers to the values indicated in table I

The FNN percentage in a given dimension may be interpreted as the error we would commit if we used that dimension for the embedding. It is interesting to note that the results of the computation of FNN percentages coming from the different choices of the delay time described above did not show significant variations (less than 0.1%). This result is particularly important, as it states that the estimate of the embedding dimension is not very influenced by the particular choice of the method of determination of the delay time, *i.e.* that the two estimations are in some sense independent. Obviously this is true if the choice of the delay time is made according to one of the rules (1)-(4) but *not* if its choice is made in a completely arbitrary way, as by simply choosing $\tau = 1$; in this case the dynamics remains confined in an embedding space of dimension 4 with an error smaller than 1% for all three components. In this example a wrong choice for τ leads to a completely wrong estimate of the embedding dimension!

We now present the procedure followed in the computation. Also in this case, due to the considerable computing time, we chose to process one tremor sample per day, *i.e.* only the data recorded at 12 GMT. The steps applied to each component are the following:

1) The offset of the signal is removed by subtracting the time average over its duration.

2) The percentage of FNN is computed for every m between 5 and 9.

The mean value of the dimension at which the FNN percentage goes below 5% is 6.26. On the basis of this value, we analyse the behaviour of FNN in an embedding dimension equal to 7 (fig. 4c). We may recall that this is the error we have when reconstructing a 7-dimensional pseudo state space from the tremor time series.

It is interesting to note how the graph of the evolution of FNN percentage in dimension 7 (fig. 4c) is very similar to the one relative to the «average» frequency (fig. 4a). As a greater error in a given dimension suggests a greater optimal embedding dimension, this seems to increase with an increasing «average» frequency. We can therefore say that the system presents a more complex behaviour when the signal shows higher frequency components. A possible explanation could be that the higher frequency component of the dynamics is linked to the less deterministic features of the system.

If we now proceed to the second method (SVD), the first problem is that of choosing the size of the n-window. The application of eq. (2.4) gives a value of 20 as the upper bound and, if we assume a «first guess» of 7 for the embedding dimension m, a lower bound equal to 15.

If we apply the SVD method to the original data, we obtain a high correlation of the results with the behaviour of the tremor intensity. This is explained with an intrinsic dependence of the results on the amplitude of the input data. In order to avoid this problem, we applied a normalisation of the data to the interval [– 1,1]. The complete procedure applied to each component is therefore the following:

1) The offset of the signal is removed by subtracting the time average over its duration.

2) The data are normalised to the interval [– 1,1].

3) The first 20 singular values are computed (20-window)

4) A daily average of the results is computed.

The similarity between the resulting graph and the time evolution of FNN for $m = 7$ is noteworthy (fig. 4c). This is another encouraging result, as we once again obtain similar estimations for the same parameter (in this case the embedding dimension) from two different methods (FNN and SVD); this increases our confidence in the results and in their time evolution.

It is also interesting to observe the behaviour of the eigenvalues corresponding to the first singular eigenvector (fig. 4d). This graph is very similar to the ones representing the evolution of the delay time (fig. 4b) suggested by the autocorrelation (the correlation between the two graphs is 0.85) and by the mutual information (correlation 0.89). This similarity is found, although less pronounced, also for the second eigenvalue, while the third one already shows a behaviour more similar to the dimension parameters.

5. Analysis of paroxysmal phase

In the following we will analyse the periods surrounding two paroxysmal phases, *i.e.* the powerful explosions of 16 October 1993 and 1 June 1996. The choice was made on the basis of the completeness of the data set in the days before and after the events. Also in these cases, we will present the results relative to the tangential component. The procedure used for the spectral analysis, the autocorrelation and the SVD is the same as for the long term analysis except for the averages, which were taken over 6 h windows. In order to limit the computing times, only the files recorded at 1 GMT, 7 GMT, 13 GMT and 19 GMT of each day were used for the mutual information and FNN. This choice leads to the same 6 h data lag as for the other methods.

The graphs relative to the tremor intensity (figs. 5 and 6) show the sudden drop immediately after the explosions.

From the spectral point of view, while the 16 October 1993 case shows the «typical» 1 Hz pre-crisis configuration before the explosion (fig. 7a) and a rapid shift towards higher frequencies afterwards (Carniel and Iacop, 1996b), the behaviour (fig. 8a) is opposite for the 1 June 1996 case.

Examining the delay time evolution (figs. 7b and 8b), we can note in both cases significant variations near the crises. In the 16 October 1993 case the major variation is shown about four days after the explosion (fig. 7b). Moreo-

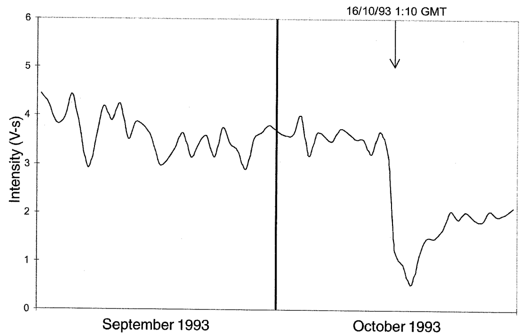

Fig. 5. Evolution of tremor intensity during September/October 1993. Explosion on 16 October 1993.

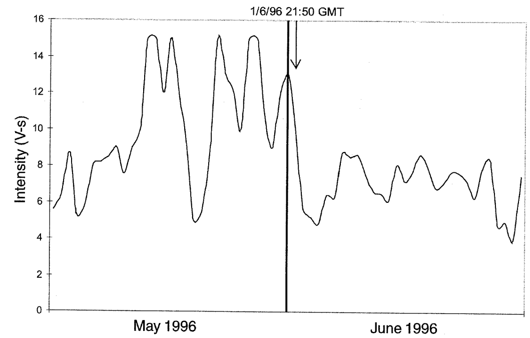

Fig. 6. Evolution of tremor intensity during May/June 1996. Explosion on 1 June 1996.

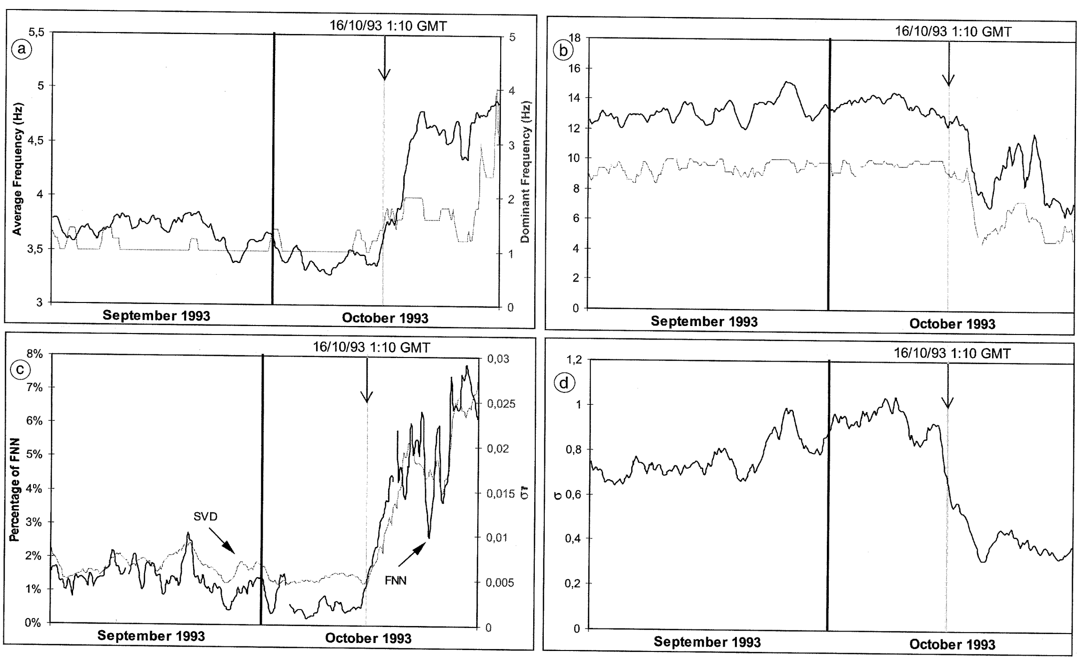

Fig. 7a-d. Zoom analysis near the 16 October 1993 explosion. a) Evolution of «average» frequency (bold line) and «dominant» frequency (thin line); b) evolution of delay time using first zero crossing of *Autocorrelation* (bold line) and first minimum of *Mutual Information* (thin line); c) percentage of FNN for $m = 7$ (bold line) and 7th singular value (eigenvalue) using SVD method (thin line); d) first singular value using the SVD method.

ver, we note how the periods with higher values of the delay time – before 16 October 1993 and after 1 June 1996 – can be associated to the dominance of lower frequencies (1 Hz in fig. 7a; 2 Hz in fig. 8a). This is in agreement with the general trend (fig. 4a,b) and striking if we look at the long period analysis of 1993 where the frequency evolution and the delay time evolution are almost monotonous from May to the 16 October explosion.

The evolution of the embedding dimension parameter estimated by the FNN percentage in dimension 7 and by the 7-th singular value (figs. 7c and 8c) show in general a good agreement. However, it is interesting to note how the evolution of the two graphs (figs. 4c and 8c) diverges close in time to the 1 June 1996 explosion. The FNN percentage shows a stable mean value of about 3% during the weeks before the crisis, and a decrease only after the explosion; on the other hand, the SVD parameter starts its decreasing trend about one month before the explosion, during the period characterised by high tremor intensities and high frequencies. One hypothesis to explain this behaviour is the following: during this period, notwithstanding a more or less stable embedding dimension, the dynamics of the volcano changes to a trajectory branch which makes a privileged exploration of different directions from the one represented by the eigenvector associated with the plotted SVD parameter, which therefore loses importance (*i.e.* the absolute value of the corresponding eigenvalue decreases). The accordance between the two estimates of the embedding dimension grows again after 1 June.

Again, analysing the graphs relative to the first singular value (figs. 7d and 8d) we note a similarity with the evolution of the delay time (figs. 7b and 8b). The accordance is more evi-

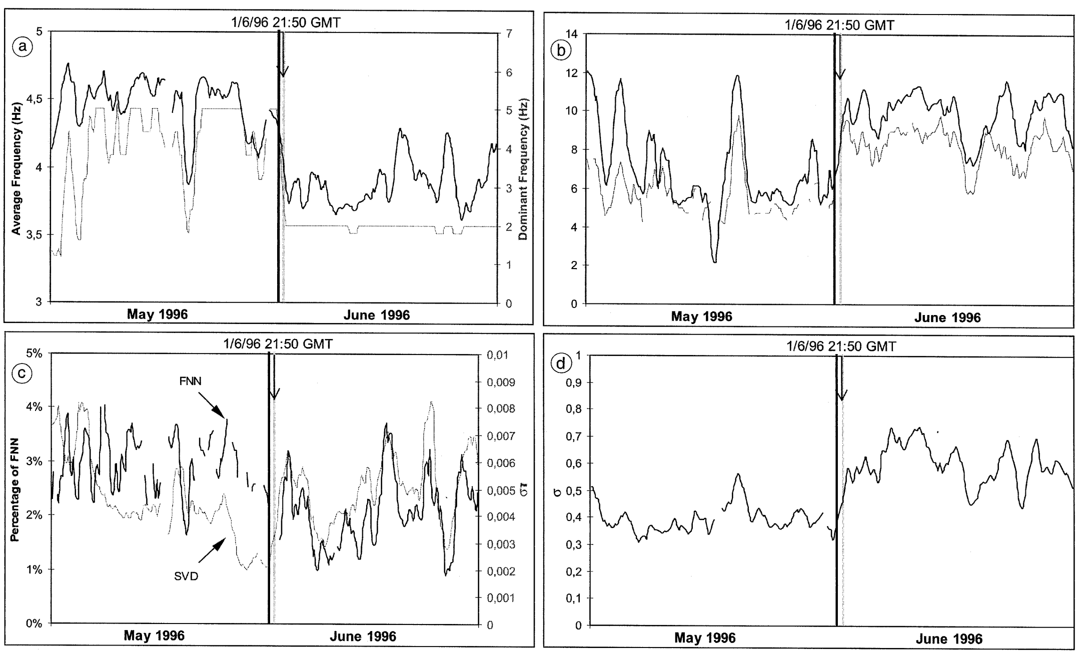

Fig. 8a-d. Zoom analysis near the 1 June 1996 explosion. a) Evolution of «average» frequency (bold line) and «dominant» frequency (thin line); b) evolution of delay time using first zero crossing of *Autocorrelation* (bold line) and first minimum of *Mutual Information* (thin line); c) percentage of FNN for $m = 7$ (bold line) and 7th singular value (eigenvalue) using the SVD method (thin line); d) first singular value using the SVD method.

dent during the periods when the dominant frequency shows stable low values. An interesting difference is that, while the delay time shows a drop only four days after the 16 October 1993 explosion (fig. 7b), the first singular value decreases immediately after the paroxysmal event (fig. 7d).

6. Conclusions

Studying a volcano like Stromboli requires very long time series, as the paroxysmal (eruptive and effusive) phenomena are spread over years of «normal» Strombolian activity, which in turn shows considerable time variations. The fact that persistent behaviour (the Strombolian activity) is shown at the volcano for so many years points towards the assumption that a dynamical system is governing the volcano. But is it deterministic? Any time series (*e.g.*, volcanic tremor) recorded at the volcano has to be interpreted as a (function of) a state variable of the volcano dynamical system. The fact that abrupt, often paroxysmal, changes appear in the form of «jumps» in the time evolution of any recorded time series suggests that the underlying deterministic system (if any) must be non linear. In this paper we applied different kinds of analyses to extract information on the Stromboli dynamical system, by using the method of time delayed coordinates for the reconstruction of a trajectory in an n-dimensional pseudo-state space. We estimated parameters such as the delay time and the embedding dimension of such reconstruction using different methods. We then analysed the time evolution of these estimates for more than four years (11 May 1992 - 30 June 1996). This is a completely new approach to the analysis of volcanic activity and doubts may arise: is this approach appropriate? Is there really an underlying deterministic dynamical system? In our opinion there is evidence for an affirmative answer to these questions.

First of all, we obtained a comparable time evolution of the estimates of the different parameters obtained from completely independent methods (autocorrelation and mutual information for the time delay, FNN and SVD for the embedding dimension). The correlation between the different estimates of the same parameters is maximum when the spectral content is more constant. Secondly, the time evolution of both the time delay and the embedding dimension shows precise trends, *i.e.* the parameters do not show «random-like» evolution. Moreover, this evolution may be related to corresponding variations in more «classical» parameters such as the tremor intensity or its spectral content. Finally, there are well defined trends in the evolution of one or more parameters (*e.g.*, before the 16 October 1993 explosion) that lead to paroxysmal phases.

At the end of this analysis we may conclude that at least 90-95 % of the dynamics of Stromboli volcanic tremor (and, as a consequence of the assumption that the volcanic tremor is just a state variable, of the whole volcano) may be explained by the trajectory in a 7-dimensional state space of a deterministic dynamical system. The percentage is given by the residual fraction of FNN present in a 7-dimensional embedding.

There is a high correlation between the evolution of the embedding dimension parameters and that of the «average» frequency of the tremor. This observation suggests that higher frequencies are related to a greater complexity of the system, which shows up as a greater embedding dimension. However, this could simply be related to a more consistent weight of the random component (noise) with respect to the deterministic one. Going further, this random component could be associated with path and site effects.

We can underline the importance of the SVD method. As the first singular values are strongly related to the delay time evolution, while the following ones are obviously related to the embedding dimension, this method alone could furnish most of the dynamical information derived in this study.

Another important observation is that at the time of the paroxysmal phases, our analysis showed a significant change of the dimension parameters. We also note a variation of delay time estimates but, considering the low influence of the choice of the exact delay time used for the computation of FNN (of course in the appropriate range!) the first change cannot simply be a consequence of the second, *i.e.* there are two independent changes. These changes, together with the observation that the behaviour of the delay time with respect to the tremor intensity becomes anomalous near the explosions, strongly suggest that these explosions are real turn points that substantially modify the dynamics of the volcanic dynamical system. This is in fact reflected in the change of most analysed «classical» and dynamical parameters. Unfortunately, the interpretation of these changes is not straightforward as even within this limited statistics they show different behaviours. This still prevents the definition of «precursor scenarios».

This study proves that the deterministic dynamical system hypothesis fits the volcanic tremor data recorded at Stromboli, and that the time evolution of the features of this system may reveal new information on the volcano. Of course this is just a step in a new direction. New methods can be applied and new time series can be analysed with this approach in order to better understand both the Strombolian activity and its paroxysmal phases.

REFERENCES

ALEAN, J. and R. CARNIEL (1995): Stromboli on-line: recent and ongoing activity of Stromboli and other volcanoes: *http://stromboli.net*

ALEAN, J. and R. CARNIEL (1996): Stromboli on-line: ein Vulkan auf dem World Wide Web, *Switch Journal - Teleinformatikdienste für Lehre und Forschung*, **2/96**, 36-40.

BEINAT, A., R. CARNIEL and F. IACOP (1994): Seismic station of Stromboli: 3-component data acquisition system, *Acta Vulcanol.*, **5**, 221-222.

BROOMHEAD, D.S. and G.P. KING (1986): Extracting qualitative dynamics from experimental data, *Physica D*, **20**, 217-236.

CARNIEL, R. (1993): Embedding seismic time series of Stromboli dynamical system, in *Proceedings of the International Conference «Applications of Time Series Analysis in Astronomy and Meteorology», Padova, Italy, 6-10 September 1993*, 79-82.

CARNIEL, R. and F. IACOP (1996a): On the persistency of crater assignment criteria for Stromboli explosion-quakes, *Ann. Geofis.*, **39** (2), 347-359.

CARNIEL, R. and F. IACOP (1996b): Spectral precursors of paroxysmal phases of Stromboli, *Ann. Geofis.*, **39** (2), 327-345.

FRASER, A.M. (1986): Using mutual information to estimate metric entropy, in *Dimensions and Entropies in Chaotic Systems* (Springer Verlag), 82-91.

FRASER, A.M. and H.L. SWINNEY (1986): Independent coordinates for strange attractors from mutual information, *Phys. Rev. A*, **33** (2), 1134-1140.

KENNEL, M.B., R. BROWN and H.D.I. ABARBANEL (1992): Determining embedding dimension for phase-space reconstruction using a geometrical construction, *Phys. Rev. A*, **45** (6), 3403-3411.

KUGIUMTZIS, D., B. LILLEKJENDLIE and N. CHRISTOPHERSEN (1994): Chaotic time series, part I: estimation of some invariant properties in state space, *Modeling, Identification and Control,* **15** (4), 205-224.

LIEBERT, W., K. PAWELZIK and H.G. SCHUSTER (1991): Optimal embeddings of chaotic attractors from topological considerations, *Europhys. Lett.*, **14** (6), 521-526.

PACKARD, N.H., J.P. CRUTCHFIELD, J.D. FARMER and R.S. SHAW (1980): Geometry from a time series, *Phys. Rev. Lett.*, **45** (9), 712-716.

SHANNON, C. (1948): A mathematical theory of communications, *Bell System Tech. J.,* **27**, 379-423; 623-656.

SHAW, R.S. (1984): *The Dripping Faucet as a Model Chaotic Systems* (Aerial Press, Santa Cruz, CA), pp. 114.

SMALLEY, R.F. JR., J.L. CHATELAIN, D.L. TURCOTTE and R.A. PREVOT (1987): A fractal approach to the clustering of earthquakes: applications to the seismicity of the New Hebrides, *Bull. Seismol. Soc. Am.*, **77** (4), 1368-1381.

TURCOTTE, D.L. (1992): *Fractals and Chaos in Geology and Geophysics* (Cambridge University Press), pp. 221.