

Antipersistent dynamics in short time scale variability of self-potential signals

Vincenzo Cuomo⁽¹⁾⁽²⁾⁽⁴⁾, Maria Lanfredi⁽²⁾⁽⁴⁾, Vincenzo Lapenna⁽²⁾, Maria Macchiato⁽³⁾⁽⁴⁾,
Maria Ragosta⁽¹⁾⁽⁴⁾ and Luciano Telesca⁽²⁾

⁽¹⁾ Dipartimento di Ingegneria e Fisica dell'Ambiente, Università della Basilicata, Potenza, Italy

⁽²⁾ Istituto di Metodologie Avanzate di Analisi Ambientale, CNR, Tito Scalo (PZ), Italy

⁽³⁾ Dipartimento di Scienze Fisiche, Università «Federico II», Napoli, Italy

⁽⁴⁾ Istituto Nazionale per la Fisica della Materia, INFN, Unità di Napoli, Italy

Abstract

Time scale properties of self-potential signals are investigated through the analysis of the second order structure function (variogram), a powerful tool to investigate the spatial and temporal variability of observational data. In this work we analyse two sequences of self-potential values measured by means of a geophysical monitoring array located in a seismically active area of Southern Italy. The range of scales investigated goes from a few minutes to several days. It is shown that signal fluctuations are characterised by two time scale ranges in which self-potential variability appears to follow slightly different dynamical behaviours. Results point to the presence of fractal, non stationary features expressing a long term correlation with scaling coefficients which are the clue of stabilising mechanisms. In the scale ranges in which the series show scale invariant behaviour, self-potentials evolve like fractional Brownian motions with anticorrelated increments typical of processes regulated by negative feedback mechanisms (antipersistence). On scales below about 6 h the strength of such an antipersistence appears to be slightly greater than that observed on larger time scales where the fluctuations are less efficiently stabilised.

Key words *fractal properties – variogram analysis – antipersistence – self-potential signals – earthquake prediction*

1. Introduction

In the context of the scientific research devoted to earthquake dynamics, interest is increasing in a variety of geophysical and geochemical parameters which could provide indirect information on the dynamics underlying the tectonic processes (*e.g.*, Rikitake, 1988).

Among these, geoelectrical parameters might be very useful to monitor and understand a collection of seemingly complex phenomena related to seismic activity (*e.g.*, Johnston, 1997; Park, 1997). As an example, variations in the stress and fluid flow fields can cause changes in the self-potential field, in resistivity, and in other electrical variables (Scholtz, 1990) so that the study of these induced fluctuations might provide information on the driving mechanisms both in normal conditions and during intense seismic activity.

At present, the use of electrical precursors in earthquake prediction is to a large extent still empirical, due to the many difficulties that still exist in understanding the physics underlying the source mechanisms of geophysical precursory phenomena and to well define objective

Mailing address: Dr. Maria Lanfredi, Istituto di Metodologie Avanzate di Analisi Ambientale, CNR, c.da S. Loja, 85050 Tito Scalo (PZ), Italy; e-mail: lanfredi@imaaa.pz.cnr.it

criteria to evaluate the reliability of the short-term predictions based on this type of precursory signals (*e.g.*, Geller *et al.*, 1997). A typical example is the VAN experiment (Varotsos *et al.*, 1993) in which a significant statistical analysis of claimed geoelectrical anomalies and a discrimination of the cultural noisy sources are completely omitted (*e.g.*, Mulargia and Gasperini, 1992; Kagan and Jackson, 1996; Pham *et al.*, 1998).

In order to assess the use of geoelectrical parameters as earthquakes precursors, the fundamental issue to address is if these parameters are able to pick up characteristics of active tectonics. In other words, we have to understand if there is a significant correlation between seismic sequences and electrical fluctuations statistically distinguishable from the electrical background behaviour (anomalous patterns). Obviously, the actual presence of such a connection can be established only after a general statistical characterisation of the given geoelectrical signals is carried out: this is the fundamental step which allows us to define the «anomalous patterns». In agreement with this approach philosophy, in this work we investigate the dynamical properties of self-potential signals, as they can be detected from observational time series (Cuomo *et al.*, 1996; 1998).

Self-potential signals are the result of the interaction among very heterogeneous and not well known mechanisms (Scholz, 1990). In particular, there are many ambiguities to indicate a common physical process able to describe the possible generation of electrical signals in seismic active areas. Taking into account the geological and seismological setting of the Irpinia-Basilicata Apennines, the preliminary results of our recent monitoring activity and the quantitative dynamical information extracted from the analysis of geoelectrical time series, a reasonable common physical process could be the dilatancy-diffusion-polarization model (Di Maio and Patella, 1991). Local features can be mixed with the general ones (Patella *et al.*, 1997) so increasing the difficulty of rightly characterising and interpreting the signal time variations. In addition, as occurs for many environmental signals, observational data are made even more erratic by the presence of electrical signals coming

from anthropic sources which make its dynamical characterisation harder (Cuomo *et al.*, 1997; Pham *et al.*, 1998).

In the study of seemingly complex phenomena, like those generating self-potential signals, methodologies able to determine time scale structures in observational time series are particularly useful tools to obtain information on the features and on causes of variation at the different time scales. They can also be very useful to test models proposed to explain spatio-temporal variability in complex systems. In particular, fractal analysis techniques, developed to draw qualitative and quantitative information from time series, have recently been applied to the study of a large variety of irregular, erratic signals and by now have proved very useful to detect deep dynamical features. Often, analysing long samples of data it is possible to disclose trends which appear to vary between successive subsamples. Although one may assume the presence of nonlinear trends or generic non-stationary behaviours, similar features naturally arise, *e.g.*, in fractional (fractal) Brownian motion (Mandelbrot and van Ness, 1968). Observed series like these, show a red spectral density with an accumulation of variance in frequencies which are not much greater than the inverse of the sample length (*e.g.*, see Granger, 1966). For these processes, fractional Brownian motion may provide a good study framework especially to understand the interplay between the seemingly non stationary behaviour and the variance distribution between the various time scales with all its dynamical implications.

As far as the methodologies useful to detect fractal properties in time series is concerned, structure functions have become a universally accepted tool to investigate the presence of scale invariance in time and space (Mandelbrot, 1982) and particularly to analyse fractional Brownian motion. In addition, structure function of second order or «variogram», that is the variance of the signal fluctuations at the different scales, is a well used tool in geostatistical and environmental studies to investigate and model spatial features (*e.g.*, Cressie, 1993; Sen, 1995). In the context of the study of time variability, the structure function analysis allows us to detect any possible non stationary behaviour. In particular,

for fractional Brownian motion, the variogram shows the typical power law behaviour (scaling regime) and the value of the power law coefficient characterises the persistence properties of the fluctuations. In the scale range in which a scaling occurs, the signal changing is the result of the accumulation of scale invariant random fluctuations. By estimating the scaling coefficient we are able to obtain quantitative information on the strength of its persistence features and to gain insight into the kind of mechanisms which may be responsible for its generation.

In this work we analyse two time series of self-potential measurements instrumentally recorded in the Basilicata region, one of the most seismic areas of the Apennine chain (Pantosti and Valensise, 1984). Time scales, from a few minutes to several days, are investigated to characterise the short time scale variability of the observational time series and the results are discussed.

2. Data

Our data consist of two sequences of 15 min averaged self-potential measures recorded from 1 January 1995 to 31 December 1995 in two sites, Tito and Tramutola (Basilicata Region - Italy), located in a seismically active area of the Apennine chain (see fig. 1). Such an area is recognised among the most interesting for the analysis of those geophysical phenomena possibly related to earthquakes (Martinelli and Albarello, 1997). Monitoring stations in fig. 1 measure geolectrical parameters (self-potentials), geochemical (CO_2 , $^{222}\text{Radon}$, etc.) and acoustic emissions, and meteoclimatic variables (air temperature, radiance, humidity, etc). Self-potential signals investigated in this paper are measures of voltage differences between electrodes installed with a spacing of 100 m and inserted in the ground at about 1 m depth. As far as the technical features of the experimental equipment are concerned, we refer the reader to Cuomo *et al.* (1997) and for the results of mono- and multi-parametric preliminary statistical analysis of the monitored variables to Di Bello *et al.* (1998). In this paper we focus our attention on the analysis of time scale properties in

geophysical parameters of electrical nature. The samples of 15 min-averaged observations are shown in fig. 2a,b: strong variability and non stationary features are detectable in the plots at a naked eye view.

3. Basic definitions and methodology

Observed signals, especially those from the natural world, are very often irregular and present details in a wide range of time scales. When plotted, observational time series describe jagged and crinkled curves which may also be highly non stationary. In order to draw dynamical inferences on the properties of the mechanisms generating this irregularity, the concept of fractal dimension or equivalent fractal descriptor can be applied to time series for quantifying the «crinkliness» of the trajectory though phase space which is related to the time correlation of the observed signal (Theiler, 1991).

Various methods for analysing the correlation properties of a time series are available, the variance spectrum being one of the most classical tools. In particular, many techniques have been developed to detect and quantify fractal features in experimental and observational data and many empirical studies have been carried out to validate them (see *e.g.*, Beran, 1994; Taqqu and Teverosky, 1995 and references therein). Among these, the analysis of the structure functions of time series is particularly suitable to qualitatively and quantitatively characterise the persistence properties of the fluctuations in observed signals.

Let $\{x(t)\}$ ($t = 1, \dots, N$) be a time series of length N . The structure function of order p of this time series is defined according to

$$S_p(\tau) = \langle \|x(t + \tau) - x(t)\|^p \rangle$$

with the angular brackets denoting the expected value. For any given time scale τ , $S_p(\tau)$ represents the p -th moment of the magnitude of the signal fluctuations at such time scale.

For fractional Brownian motion, which is the fractal extension of the classical Brownian motion (*e.g.*, Mandelbrot and van Ness, 1968),

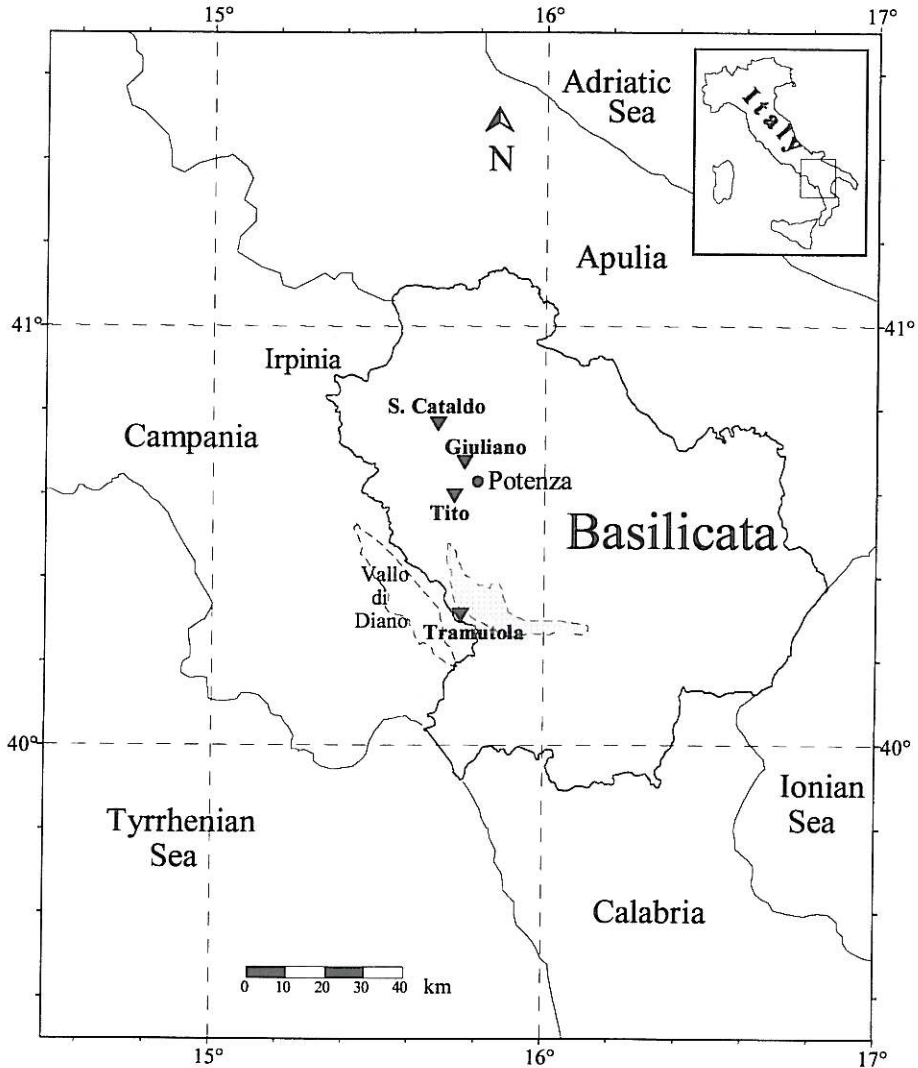


Fig. 1. Monitoring network. Data analysed in this work come from Tito and Tramutola.

$S_p(\tau)$ scales as $\tau^{c(p)}$

$$S_p(\tau) \propto \tau^{c(p)}$$

In particular, the scaling coefficient of the second-order structure function (variogram) is usually denoted by $2H$ with the constrain $0 < H < 1$ in order to ensure that the structure function

does not diverge (Panchev, 1971)

$$S_2(\tau) = \langle [x(t + \tau) - x(t)]^2 \rangle \propto \tau^{2H}$$

The coefficient H describes the power law «diffusion» rate. For small times τ , if $0 < H < 1$ the motion is necessarily crinkly whereas smooth motion leads to $H = 1$. In the large τ limit, a

linear drift gives $H = 1$ whereas bounded motion implies $H = 0$. As a consequence, in the proper fractional Brownian range $0 < H < 1$, the motion is crinkly and unbounded.

According to Mandelbrot and van Ness (1968), for $H = 1/2$ we deal with classical Brownian motion (ordinary diffusion), which is characterised by a zero correlation between the process increments. To see this, we define the increments $\Delta_H(\tau) = x(\tau + 1) - x(\tau)$ ($\tau \geq 1$) and set $\Delta_H(0) = 0$. The covariance between the past increment $-\Delta_H(-\tau)$ and the future increment $\Delta_H(\tau)$ is

$$\begin{aligned} & \langle -\Delta_H(-\tau)\Delta_H(\tau) \rangle = \\ & = 2^{-1} \{ \langle [\Delta_H(\tau) - \Delta_H(-\tau)]^2 \rangle - 2 \langle [\Delta_H(\tau)]^2 \rangle \} = \\ & = 2^{-1} (2\tau)^{2H} - \tau^{2H}. \end{aligned}$$

Dividing by $\langle \Delta_H(\tau)^2 \rangle = \tau^{2H}$, we obtain the increments correlation

$$\rho(H) = 2^{2H-1} - 1$$

which is independent of τ and vanishes for $H = 1/2$.

Such a correlation is instead positive if $1/2 < H < 1$, that is increasing (decreasing) fluctuations in the past imply on the average increasing (decreasing) fluctuations in the future. This feature leads to persistence; the signal appears rather smooth and shows seemingly periodic patterns. Persistent fractional Brownian motion is useful to describe processes generated by positive feedback mechanisms. Examples of persistent behaviour can be found in climatic processes (*e.g.*, Mandelbrot and Wallis, 1969) or in the statistic of ocean waves (see Feder, 1988) where it is possible to observe clear trends in wave-height with relatively little noise. The upper boundary $H = 1$ indicates deterministic behaviour. Conversely, if $0 < H < 1/2$, increasing (decreasing) fluctuations in the past imply decreasing (increasing) fluctuations in the future and the signal appear very noisy. Such kind of fractional Brownian motion is suitable to describe antipersistent signals governed by stabilising mechanisms (negative feedback). Stochastic components with antipersistent features are detectable

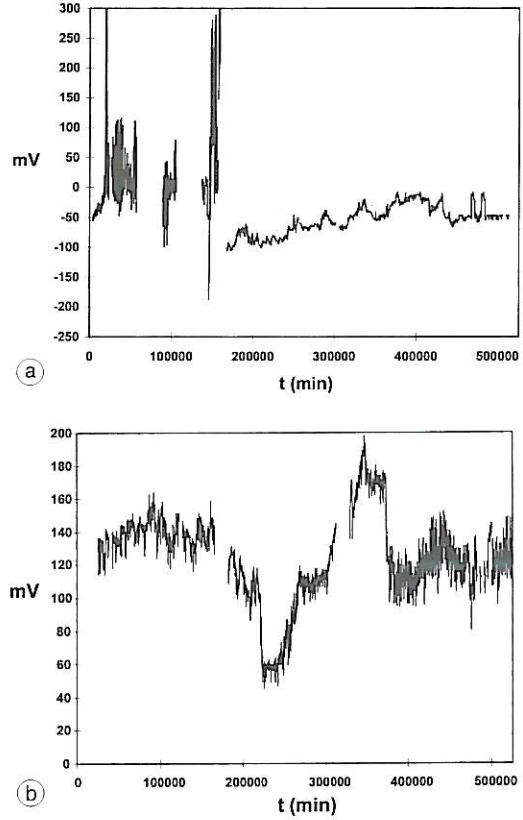


Fig. 2a,b. 15-min averaged self-potential data recorded from 1 January 1995 to 31 December 1995 (1 year = 525 600 min): a) measures recorded at Tito; b) measures recorded at Tramutola.

e.g., in time series of air pollution concentration (Lanfredi *et al.*, 1998) where atmospheric dispersion mechanisms and chemistry act as stabilising factors. The lower bound, $H = 0$, expresses perfect stabilisation (stationary fluctuations).

For a theoretical fractional Brownian motion these properties apply for any possible t -value, differently from other random functions (*e.g.*, Markov processes) having the properties that at sufficiently distant times, functions samples are independent.

The coefficient H is related to the exponent in the spectral power law $1/f^\beta$ (see Mandelbrot van Ness, 1968) by the formula $\beta = 2H + 1$. As

noted by Theiler (1991), in real cases, pure $1/f^\beta$ power spectra are physically impossible. Indeed, the total power would diverge to infinity: explicit (of dynamical origin) or implicit (induced by the finite length and by the sampling rate of the series) cutoffs have to be expected which makes band-limited fractals behaviours in real time series.

The variogram estimator used in our analysis is the classical one (Cressie, 1993)

$$\hat{S}_2(\tau) = \frac{1}{|N(\tau)|} \sum_{N(\tau)} (x(t+\tau) - x(t))^2$$

where the sum is over the number $N(\tau)$ of the observations located at an τ distance apart in the observational sample. The time series has not to necessarily be sampled at equal time intervals and a lack of information in periods which are short in comparison with the length of the series is not expected to dramatically affect the characterisation of background behaviours.

4. Results and discussion

Figure 3a,b shows the variograms estimated from the observational time series concerning respectively Tito and Tramutola on the scales going from a quarter of an hour to ten days. As it is possible to note, a very different behaviour is detectable in the two plots. In the variogram relative to Tito (fig. 3a) a 24 h cycle is shown which instead is not present in the plot concerning Tramutola (fig. 3b). Since the variogram represents the correlation structure of a given time series in the time domain, such a cycle is the signature of the presence of a daily modulation. This modulation was already noted and its correlation with the ambient temperature was already discussed in previous works (e.g., Di Bello *et al.*, 1994). As at Tramutola the sensors were slightly deeper than the sensors of the Tito station, the temperature variations did not suffer.

In spite of the presence of an additional forcing in the signal measured at Tito, if we investigate the early time scales in log-log plots, very similar behaviours are detectable (see fig. 4a,b). Below about 360τ (6 h), clear power laws are shown which express the occurrence of scale

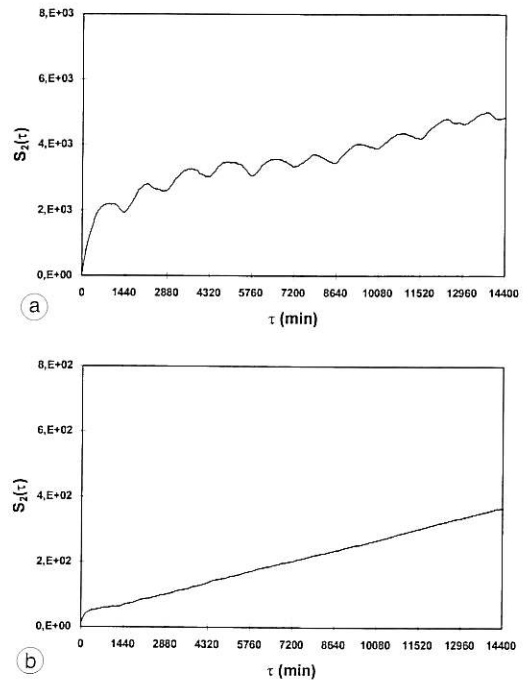


Fig. 3a,b. Second order structure function (variogram) estimated from the samples of fig. 2a,b. The range of time scales covers 10 days (1 day = 1440 min): a) Tito; b) Tramutola.

invariance. In both cases the scaling coefficients estimated, H , is less than $1/2$ and gives us an indication of antipersistent behaviour. Since H measures the strength of the stabilising skill of the mechanisms underlying the signals (the value $H = 0$ characterising a perfect stationary behaviour), the series recorded at Tramutola appears rather well stabilised whereas the negative feedback mechanisms ruling the self-potential measured at Tito seem to be less efficient. Also this difference might be explained as an effect of the daily temperature change. Indeed, perfectly stabilised signals are bounded and do not show significant trends. If we suppose that a given observed signal represents the position sequence of a particle motion, about stabilised features express the tendency of the particle to turn back where it came from. On time scales as short as $\tau = 6$ h, which is only a quarter of the

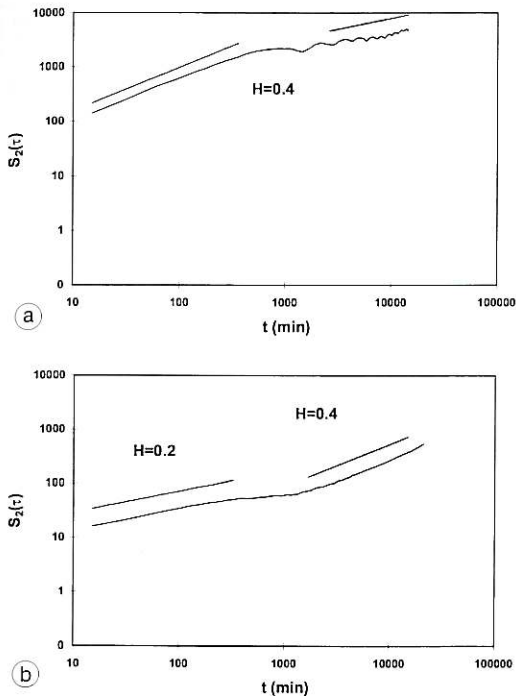


Fig. 4a,b. Log-log variograms on time scales going up to about 10 days (1 day = 1440 min): linear patterns disclose fractal features. a) Tito; b) Tramutola.

daily period, the temperature change is seen on the average as a drift which tends to send away the particle (persistence mechanism) and attenuates the antipersistent effect of stabilising mechanisms. In any case, the differences in the values of H do not express a particularly significant difference of dynamical character.

Above the 1440 τ scale (about 24 h), another very clear linear behaviour is detectable in fig. 4b with a H -value slightly greater than that characterising shorter time scales (lower stabilising effectiveness). Note as in the Tito variogram (fig. 4a) the oscillatory behaviour is also displaced on a near linear trend with about the same slope. If one neglects the diurnal modulation, the two signals appear to have very interesting similar dynamical features.

As still seen in fig. 2a,b, showing self-potential data, on time scales greater than those ana-

lysed above, the signals appear very irregular. At a variogram analysis of our series, these irregularities would trivially reflect in irregularities in the plot of $S_2(\tau)$. In order to investigate also these scales, longer observational time series are needed which can allow us to understand how these fluctuations fit in the dynamics of the signals.

5. Conclusions

The investigation of the time scale properties of two one year time series of self-potential recorded at two sites of a seismically active area located in the Basilicata region, has disclosed very evident fractal features. On time scales going from a few minutes to several days, although one of the two signals is modulated by a daily cycle, the variograms of both the signals show two scaling regimes characterised by fractal exponents typical of fractional Brownian motion with anticorrelated increments. Such kind of motion well describes signals governed by negative feedback (antipersistent) mechanisms. Fractional Brownian motions are not stationary so that our findings show that the investigated fluctuations are not perfectly stabilised. Nevertheless, especially in the signal measured at Tramutola which is not modulated by a daily cycle, the strength of the stabilisation, measured by the distance from 0 of the fractal exponent H (in our case $H \sim 0.3$), is efficient. These mechanisms allow the variance of the signal to explode only on very large times so that, if such a regime occurs only on short time scales, like those detected in our analysis, the fluctuations appear rather stationary.

The irregularity of the time series on time scales larger than those investigated in this paper suggests analysing longer samples of measures in order to assess large scale variation features. Having a more complete time scale characterisation of the persistence properties of the signals we could better understand the background electrical variability and gain insight into the possibility of defining «anomalous» patterns which are not made up of intrinsic fluctuations and might be related to the seismic activity forcing. At the same time, because of

the similar dynamics shown by the two sites, we believe that a similar analysis, carried out on data coming from different sites located in the same zone and from sites belonging to different seismic areas might be very interesting to gain insight into the links among fractal properties, features peculiar to the specific site like the local tectonics and very general properties of the self-potential signals.

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