ANNALI DI GEOFISICA, VOL. 43, N. 1, February 2000

The use of Monte Carlo simulations for seismic hazard assessment in the U.K.

Roger M.W. Musson
British Geological Survey, Edinburgh, U.K.

Abstract
The input required for a seismic hazard study using conventional Probabilistic Seismic Hazard assessment (PSHA) methods can also be used for probabilistic analysis of hazard using Monte Carlo simulation methods. This technique is very flexible, and seems to be under-represented in the literature. It is very easy to modify the form of the seismicity model used, for example, to introduce non-Poissonian behaviour, without extensive reprogramming. Uncertainty in input parameters can also be modelled very flexibly – for example, by the use of a standard deviation rather than by the discrete branches of a logic tree. In addition (and this advantage is perhaps not as trivial as it may sound) the simplicity of the method means that its principles can be grasped by the layman, which is useful when results have to be explained to people outside the seismological/engineering communities, such as planners and politicians. In this paper, some examples of the Monte Carlo method in action are shown in the context of a low to moderate seismicity area: the United Kingdom.

Key words *seismic hazard – Monte Carlo simulation*

1. Introduction

Of the various methods of seismic hazard analysis in use today, the most widespread is the probabilistic method; probabilistic seismic hazard analysis or PSHA. It derives ultimately from the much-cited work of Cornell (1968) and has been much developed since then. The basis of the method is the «total probability theorem» – that the probability of an acceleration value occurring at site can be calculated by multiplying the conditional probability of obtaining that acceleration from an earthquake of given magnitude at a given distance by the independent probabilities of obtaining that magnitude and at that distance, and integrating over all possible values of distance and magnitude.

The stages of this method, and the input required, are summarised in the well-known and often reproduced fig. 1 (TERA 1980). The elements are as follows. Firstly, the seismicity data must be spatially disaggregated into discrete seismic sources. These are usually represented as line sources (faults) or area sources (faulted areas of more or less uniform seismogenic potential), although in fact these should be considered as existing in three dimensions: the depth distribution of the seismicity is also important and cannot be excluded from the model. The seismic source model should properly be thought of as a set of planes and volumes.

Secondly, for each seismic source, the seismicity has to be characterised with respect to time, that is, the annual rate of occurrence of different magnitude earthquakes. This is most often expressed in terms of a Gutenberg-Richter power law, where the seismicity of the source is written as an activity rate, a frequency magnitude slope (the b value), and a maximum magnitude value at which the curve is truncated. Other distributions (*e.g.*, truncated exponential,

---

Mailing address: Dr. Roger M.W. Musson, British Geological Survey, Murchison House, West Mains Road, Edinburgh EH9 3LA, U.K.; e-mail: R.Musson@bgs.ac.uk

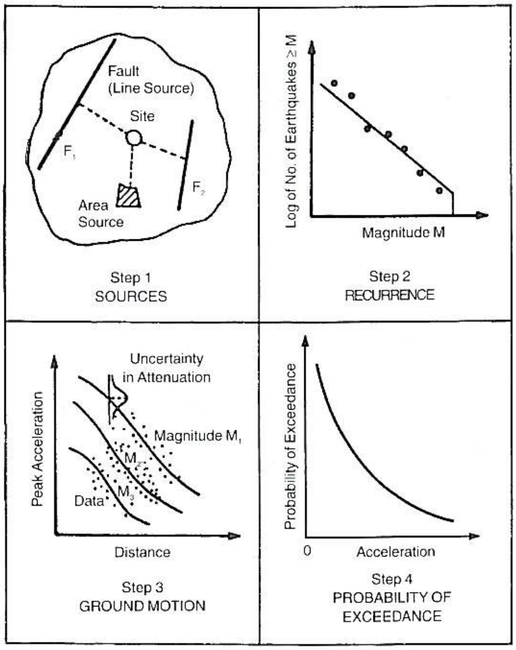

Fig. 1. The elements of a probabilistic seismic hazard study, from TERA (1980).

gamma distribution) can also be used. It is generally assumed that the seismicity of each source is Poissonian in character.

Thirdly, it is necessary to know the rate at which ground motion decays with distance from the epicentre or hypocentre or fault rupture (according to taste) as a function of magnitude. This is the attenuation; the scatter in the attenuation values (sigma) is also an important parameter in the determination of hazard.

Once all these data are assembled, the total probability theorem is invoked to determine the probability of a design level of ground shaking being exceeded in a certain period, by a process which involves determining the probability of every possible ground shaking value, from every possible magnitude, at every possible distance, from every possible source. The result can be expressed as a hazard curve giving the annual probability of any level of ground shaking being exceeded at the site of interest.

However, using the same data, one can arrive at the same result by a completely different route. This is the Monte Carlo simulation method, also known as stochastic modelling. This is not a new technique, and has been used in a number of seismic hazard studies in different parts of the world, as in studies by Rosenhauer (1983), Shapira (1983), Johnson and Koyanagi (1988) and Ahorner and Rosenhauer (1993). However, the technique seems under-utilised and relatively little known considering the advantages it offers. For example, it does not feature at all in the text book on seismic hazard by Reiter (1990), although there is a passing reference to using Monte Carlo methods for sampling different models as an alternative to the use of a logic-tree approach to representing uncertainty (Coppersmith and Youngs 1986).

The essence of the Monte Carlo approach is very straightforward. Since the seismic source model describes as completely as possible, both spatially and temporally, the way in which earthquakes occur in a region, it is a fairly straightforward matter to use the model to generate synthetic earthquake catalogues using a Monte Carlo process (controlled use of random numbers). Each catalogue represents a version of what could occur in the way of earthquakes in that region in the next 50 or 100 years that would be consistent with past behaviour (fig. 2). For each earthquake generated, the ground shaking at site can also be simulated from knowledge of the attenuation and the scatter of the attenuation. From observation of the effects of a very large number of simulations, probabilities can be calculated by merely counting the number of results exceeding a critical value. As a simple example, 100 000 simulations of 100 years of seismicity gives the effect of 10 000 000 years of data. When the highest ground motion value from each of these 10 000 000 years is sorted by size, one can determine the ground shaking value with a 10^{-4} annual probability of being exceeded by just picking the 1001st value in the sorted list.

2. Advantages of the Monte Carlo method

The following advantages of the Monte Carlo approach can be identified.

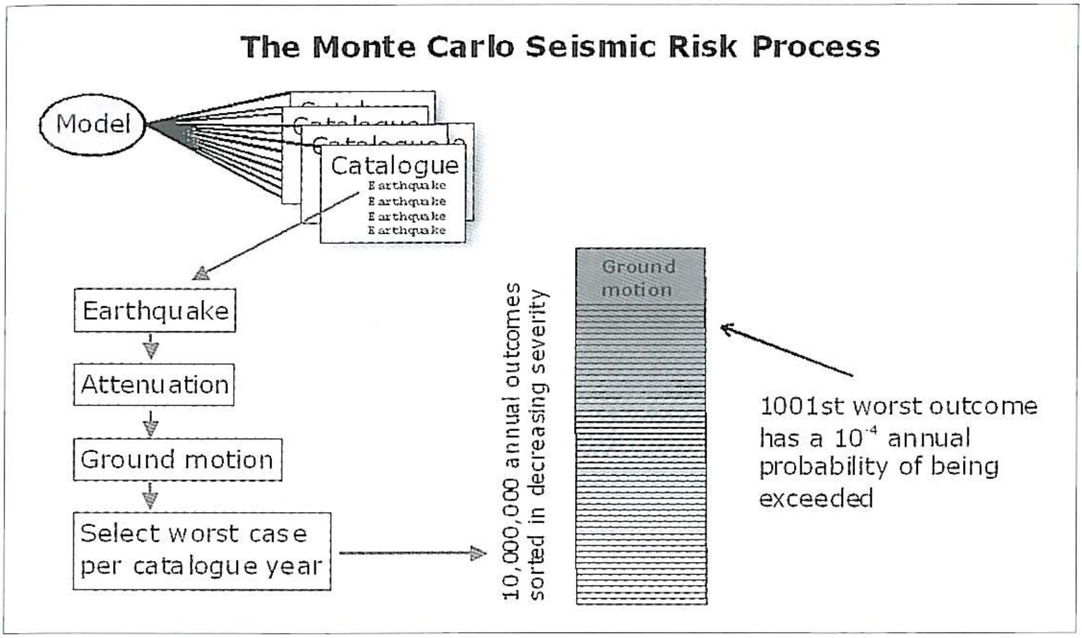

Fig. 2. Diagrammatic representation of the Monte Carlo simulation process in seismic hazard.

2.1. *Adaptability to different seismicity models*

In conventional PSHA methods, the use of an assumption of Poissonian behaviour is convenient, as the integration of the probability of earthquake occurrence is made technically simple. While moving to non-Poissonian models is by no means impossible, the mathematical and computational problems increase with the complexity and non-uniformity of the seismicity model required. With the Monte Carlo approach, characteristic, time-dependent, non-Poissonian, Markovian or other models can be adopted with relative ease, as it is only necessary to be able to describe the model for the simulation process, not to be able to integrate it.

2.2. *Powerful/flexible handling of uncertainty*

The Monte Carlo method allows uncertainties in the input parameters to be dealt with in a very powerful way – parameters can be entered as distribution functions with observed means and standard deviations. A different value can be sampled from the distribution for each simulation. This has attractions over the use of a logic-tree, where the choice of weights for each branch in the tree tends to be subjective.

2.3. *Adaptable to risk analysis*

The PSHA method is rarely, if ever, carried through directly into analysis of actual risk to buildings. To do so would require the integration of vulnerability functions so as to be able to calculate the conditional probability of damage given a certain level of ground shaking. With the Monte Carlo method this is not necessary, and as long as one has vulnerability functions, it is a straightforward matter to carry the simulation process one step further down the line and simulate loss to buildings from each synthetic earthquake catalogue. This is clearly of interest to insurers, reinsurers, planners and politicians.

Commercial software operating in this way is available to the reinsurance industry (see, for example http://www.gsrg.nmh.ac.uk/hazard/monica.html), but such programs tend not to be documented in the scientific literature, hence the under-representation of this technique in published papers on seismic hazard.

2.4. *Identifies design earthquakes*

An effect of using conditional probabilities is that actual earthquakes disappear from the system, making it hard to characterise the design earthquakes that actually would produce a given ground motion value (Scott *et al.*, 1998). With the Monte Carlo method, the synthetic earthquakes can be interrogated to discover the most significant magnitude/distance combinations from a design point of view. This seems not to have been previously realised, and is discussed more fully in a separate paper (Musson, 1999).

2.5. *Conceptually straightforward*

This may seem a trivial point; one cannot demand of science that it be simple. However, when seismologists are working with planners and politicians on the implementation of earthquake disaster plans, it is potentially an advantage that seismic risk values obtained from the Monte Carlo method can be explained in a way that is clearly understandable by the layman, who may therefore have greater faith in the values provided by the seismologist which he then has to implement.

3. Disadvantages of the Monte Carlo method

3.1. *Results non-unique*

Because probabilities are not calculated analytically, very low probability events may not be represented to the same extent in the synthetic catalogues sampled, depending on the random numbers used. As a result, subsequent runs of the same calculation can produce slightly different results. This can be corrected for by increasing the number of iterations performed, as will be demonstrated later in this paper.

3.2. *Computationally slow*

If a large number of simulations are required, with a complex model in a high seismicity area, the time taken for running the computations can be large, especially if a hazard map is required. This is also true of complex logic-tree models, though, and the rapidly increasing availability of cheaper, more powerful computers makes this problem less and less significant. Calculation times are acceptable on a 200 MHz PC, where formerly a high-end workstation would have been obligatory.

4. Illustrative example

As an example of the procedure in action, a hazard analysis for an arbitrary site in the U.K. (Keele University, Staffordshire) is undertaken. Since the object of this paper is to discuss the hazard analysis technique, the details of the seismic source model can be passed over relatively quickly.

Seismicity – The U.K. earthquake catalogue of Musson (1994) is used, with updates for the most recent years taken from the annual bulletins of the British Geological Survey. The catalogue was purged of aftershocks and foreshocks prior to analysis.

Source zones – A set of seismic source zones specifically designed for hazard maps of the U.K. has been published in Musson and Winter (1997) and further discussed in Musson (1997). However, a seismic source model is inevitably something that evolves over time (see, for example, Basham *et al.*, 1997) and this model is in the process of being revised to take into account new ideas on the seismotectonic environment in the U.K. (Chadwick *et al.*, 1996). Also, the two-tier system of zones discussed in Musson (1997) is specifically intended for hazard maps and not for site studies. For this study a simplified set of

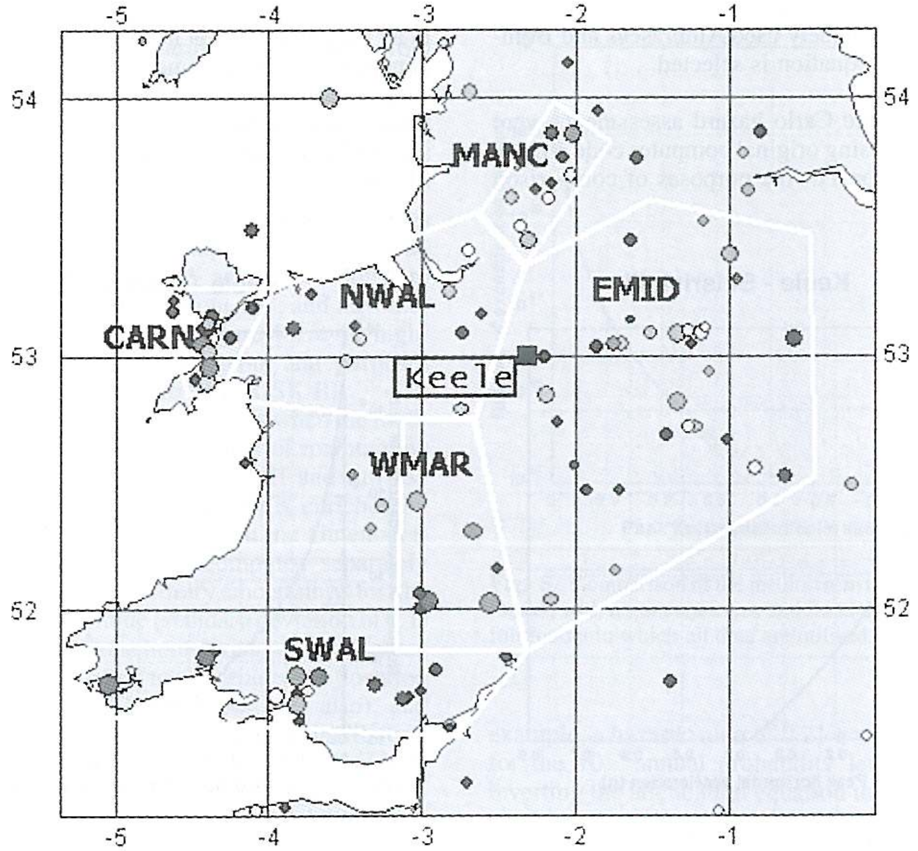

Fig. 3. The source zone model used for this study, with a plot of earthquake locations. The site (Keele) is plotted as a black square.

zones is used; the simplicity is partly in order to be able to make comparisons with the standard program SEISRISK III (Bender and Perkins, 1987) which is less able to cope with complicated source geometry than the Monte Carlo code used. The model, together with earthquakes above magnitude 4.0 M_L, is shown as fig. 3. Each zone is given a four-letter identity code. The seismicity in each zone was analysed separately to determine the magnitude frequency parameters, assuming a linear Gutenberg-Richter law with an abrupt termination at the maximum magnitude value (set, rather conservatively, at 7.0 M_L for all zones). One could argue that the numbers of earthquakes involved are too few to determine separate *b* values in this way; but such discussion is beyond the scope of the present paper. In this study, Poissonian earthquake behaviour is assumed.

Attenuation – The attenuation of peak ground acceleration in the U.K. is a fraught subject. In the absence of adequate recorded data, various imported relationships have been proposed. Musson and Winter (1997) used the attenuation formula of Dahle *et al.* (1990). When compared with other possible formulae, this gives notably non-conservative results, and, while side-stepping the issue of which formula gives the most realistic results, for site studies it is probably advisable to use a more conservative option. In this case the widely used Ambraseys and Bommer (1991) equation is selected.

The Monte Carlo hazard assessments were conducted using original computer code written by the author. For the purposes of comparison with SEISRISK III, the depths of all earthquakes in all the zones was set to 10 km, and all uncertainties in various parameters were removed. A small modification to the SEISRISK III code was made to truncate the scatter in attenuation to three standard deviations, to bring it into line with the Monte Carlo code, which has this limit imposed.

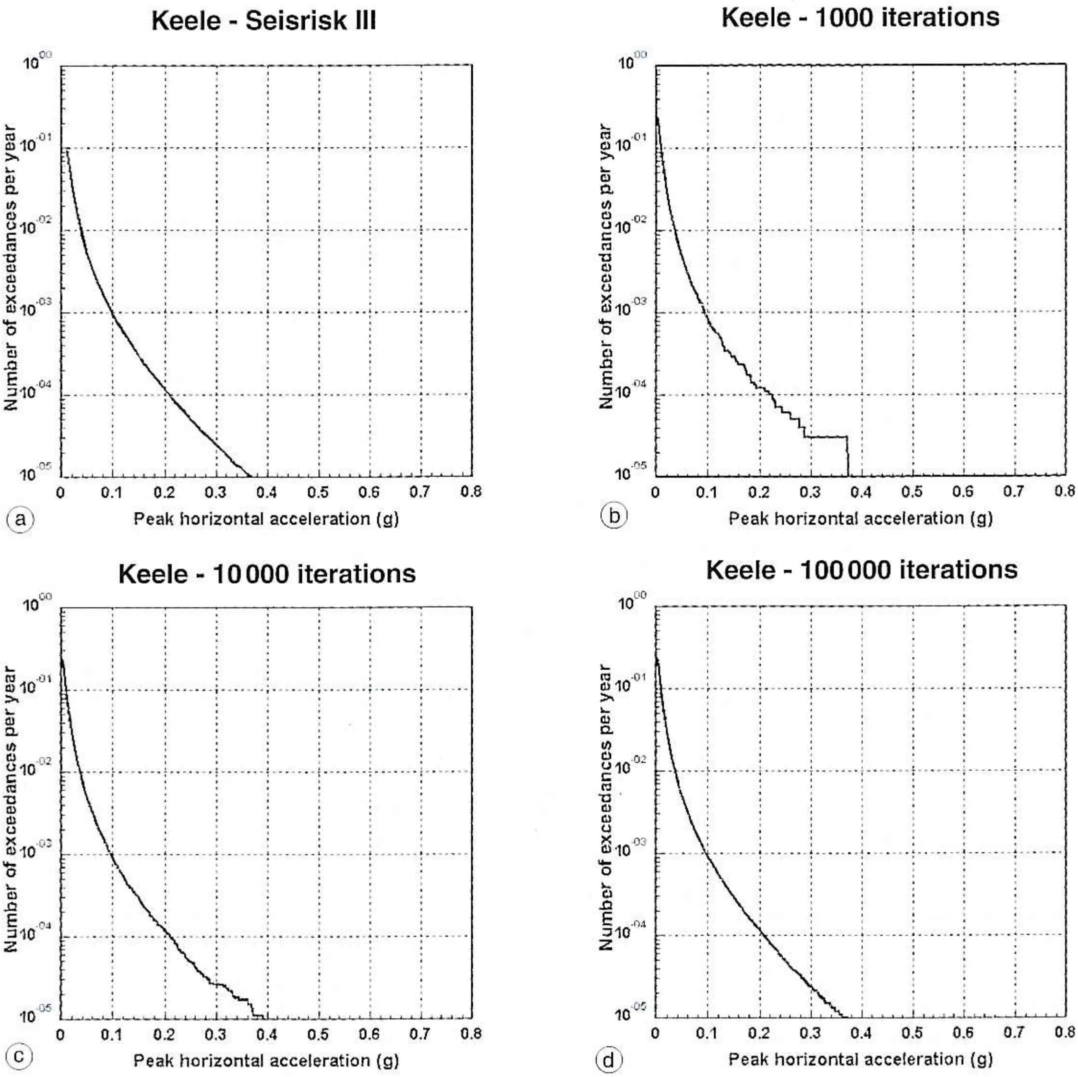

Fig. 4a-d. Seismic hazard curves for Keele with varying degrees of resolving power: a) from SEISRISK III; b) after 1000 simulations; c) after 10 000 simulations; d) after 100 000 simulations.

The results of the comparison are shown in fig. 4a-d, in which hazard curves obtained from the model are shown. Figure 4a shows the result from SEISRISK III; fig. 4b is the result from 1000 simulations of 100 years of data; fig. 4c is from 10000 simulations; and fig. 4d is from 100000 simulations. As can be seen, the resolving power of the Monte Carlo method improves with the number of simulations used. With 1000 simulations the curve is acceptable down to annual probabilities of about 10^{-3}, and the other results are in proportion. Figure 4d, at the highest resolution, is to all intents and purposes identical to fig. 4a from SEISRISK III.

Figure 5 shows what happens when the model is freed from the constraints of maintaining compatibility with SEISRISK III and full use of the data is made. The model is enriched by the addition of uncertainties in the Gutenberg-Richter *a* and *b* values, computed separately for each zone; an arbitrary uncertainty for the maximum magnitude (standard deviation of 0.1) was added; the zone boundaries were smoothed by adding a scatter to the epicentral location (actually SEISRISK III allows this also); and each zone was given its own depth distribution based on the observed depths of earthquakes within that zone. The «simple» model in fig. 5 is shown by the dotted line, and is fig. 4d repeated; the «full» model is shown by the solid line, and is the result of including the uncertainties as shown.

The result shows that for higher probabilities of occurrence the hazard is slightly lower with the full model, almost certainly because the depth distribution is on average deeper than the fixed 10 km previously assumed. However, for lower probabilities (below annual probabilities of 10^{-4} in this case) the hazard is higher with the full model, as at such low probabilities the effect is felt of extreme cases made possible by the fluctuation in values caused by adding standard deviations to the various input parameters.

Turning to the matter of design earthquakes, that is, the characteristic (using the word here in a general sense) earthquake that actually produces the acceleration associated with a particular probability level, this is difficult to handle adequately using conventional methods. In this example, a hazard value of 0.21 *g* was obtained for the 10^{-4} annual probability level. Merely inverting the attenuation equation to find which combinations of magnitude and distance produce this acceleration does not help, as this takes no account of the critical effect of the scatter modelled by the sigma parameter.

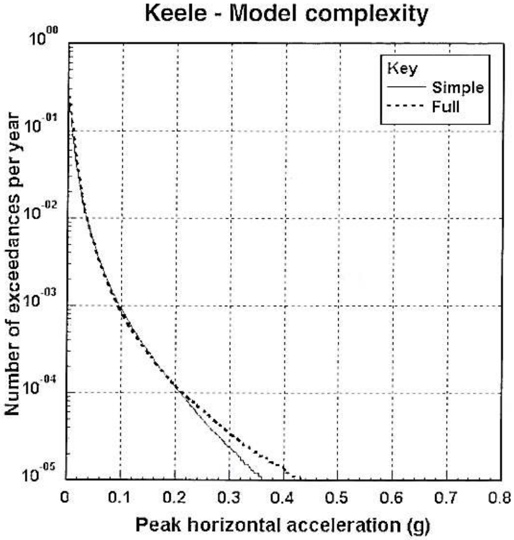

Fig. 5. Comparison of the results from the simplified model, with no uncertainties and fixed depth, and the full model in which all data are utilised.

However, with the Monte Carlo method it is a simple matter to extract from the synthetic catalogues all occurrences of 0.21 *g* plus/minus a tolerance level, and examine the earthquakes responsible. In this case a tolerance level of 0.01 *g* was used (so all events with accelerations 0.20-0.22 *g* were examined). The run of 100000 simulations produced 257 values within these limits. These are plotted in fig. 6, together with the curve showing the predicted magnitude/distance combinations that should produce 0.21 *g*. The points are marked with different symbols according to their source zone of origin. As can be seen (and should be expected), the majority of all the points are below/right of the attenuation line, representing cases where much higher accelerations than predicted have been caused by upward scatter.

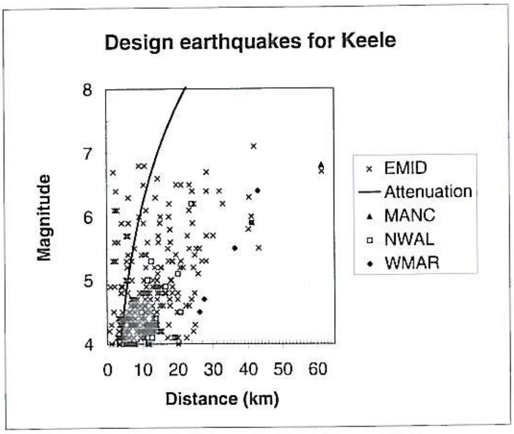

Fig. 6. Plot of 257 events causing the design acceleration of $0.21g \pm 0.01$, coded by source zone of origin. The line labelled «attenuation» is the inversion of the Ambraseys and Bommer (1991) attenuation formula for 0.21 g.

Figure 6 tells one a lot about the hazard. It can be seen that the source zone (EMID) containing the site dominates the hazard almost completely; that zone NWAL to the north-west has a small influence, and the rest little or no influence at this probability level. Furthermore, the hazard is largely controlled by small earthquakes (less than 5.0 M_L) occurring very close

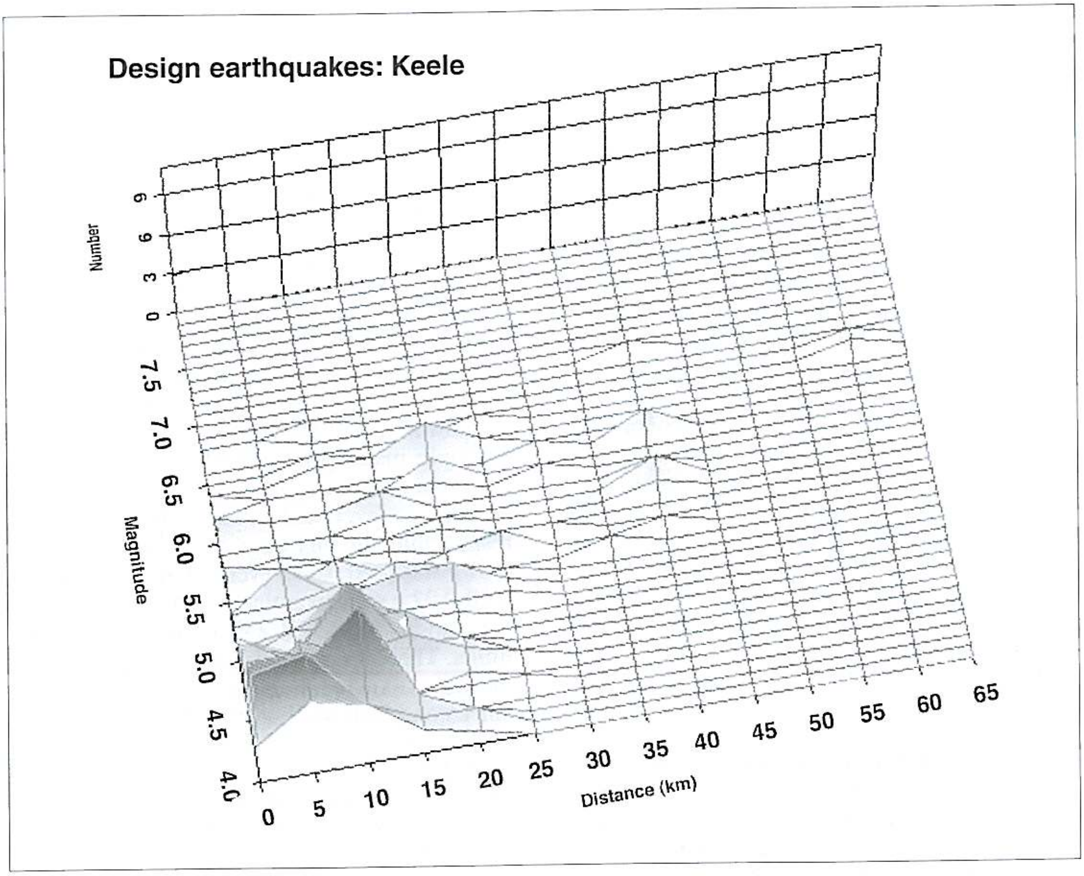

Fig. 7. The data from fig. 6 expressed as a surface to show the modal observations more clearly.

to the site, large earthquakes being relatively less important. The median distance of all observations is 11 km and the median magnitude is 4.7 M_L. A clearer view of the density function is obtained by the surface plot in fig. 7, where it can be seen that the modal peak is for a magnitude 4.2 earthquake at between 10-15 km.

5. Conclusions

A sample analysis of seismic hazard for a site at Keele, Staffordshire, U.K., shows that, given the same input data, Monte Carlo simulation techniques produce the same output as conventional PSHA methods, and can thus be safely used as an alternative. An advantage of this method is the flexibility obtained in handling uncertainty in the input parameters, which can be handled as standard deviations of a normal distribution. The effect of introducing such uncertainties is felt more strongly at lower probabilities, rather than being a perceptible increase across the whole curve. The method gives much more information about the actual nature of the hazard in terms of earthquakes and distances than can be obtained from other types of probabilistic hazard analysis. In the worked example, the peak ground acceleration at site with annual probability of 10^{-4} was found to be 0.21 g, which was most likely to be produced by an earthquake of magnitude around 4.2 M_L at a distance between 10-15 km.

In addition to the practical advantages of this method, it is much easier to explain the principles behind it to a layman, than it is to explain the intricacies of the total probability theorem. It is possible that planners and politicians may react better to results obtained in a readily understandable way, and treat them with more confidence. This benefit should not be overlooked.

Acknowledgements

This paper was supported by the Natural Environment Research Council (NERC) and is published with the permission of the Director of the British Geological Survey (NERC).

REFERENCES

AHORNER, L. and W. ROSENHAUER (1993): Seismiche risiko-analyse, in *Naturkatastrophen und katastrophenvorbeugung, Bericht zur Int. Decade Natural Disaster Reduction*, Deutsche Forschungsgemeinschaft, Verlag, Weinheim, 177-190.

AMBRASEYS, N.N., and J.J. BOMMER (1991): The attenuation of ground accelerations in Europe, *Earthquake Eng. Struct. Dyn.*, **20**, 1179-1202.

BASHAM, P., J. ADAMS, S. HALCHUK and D. WEICHERT (1997): A new «robust» seismic hazard assessment for Canada, in *Abstracts of the 29th IASPEI General Assembly, Thessaloniki, 18-28 August 1997*, p. 294.

BENDER, B.K. and D.M. PERKINS (1987): SEISRISK III: a computer program for seismic hazard estimation, *USGS Bulletin*, 1772.

CHADWICK, R.A., T.C. PHARAOH, J.P. WILLIAMSON and R.M.W. MUSSON (1996): Seismotectonics of the UK, *BGS Technical Report No. WA/96/3C.*

COPPERSMITH, K.J. and R.R. YOUNGS (1986): Capturing uncertainty in probabilistic seismic hazard assessments within intraplate tectonic environments, in *Proceeding Third US National Conference Earthquake Eng., Charleston*, vol. 1, 301-312.

CORNELL, C.A. (1968): Engineering seismic risk analysis, *Bull. Seismol. Soc. Am.*, **58** (5), 1583-1606.

DAHLE, A., H. BUNGUM and L.B. KVAMME (1990): Attenuation models inferred from intraplate earthquake recordings, *Earthquake Eng. Struct. Dyn.*, **19**, 1125-1141.

JOHNSON, C.E. and R.Y. KOYANAGI (1988): A Monte-Carlo approach applied to the estimation of seismic hazard for the state of Hawaii, *Seismol. Res. Lett.*, **59** (1), 18.

MUSSON, R.M.W. (1994): A catalogue of British earthquakes, *BGS Global Seismology Report No. WL/94/04.*

MUSSON, R.M.W. (1997): Seismic hazard studies in the UK: source specification problems of intraplate seismicity, *Natural Hazards*, **15**, 105-119.

MUSSON, R.M.W. (1999): Determination of design earthquakes in seismic hazard analysis through Monte Carlo simulation, *J. Earthquake Eng.*, **3**, 463-474.

MUSSON, R.M.W. and P.W. WINTER (1997): Seismic hazard maps for the UK, *Natural Hazards*, **14**, 141-154.

REITER, L. (1990): *Earthquake Hazard Analysis* (Columbia University Press, New York).

ROSENHAUER, W. (1983): Methodological aspects encountered in the Lower Rhine area seismic hazard analysis, in *Seismicity and Seismic Risk in the Offshore North Sea Area*, edited by A.R. RITSEMA and A. GÜRPINAR (Reidel, Dordrecht), 385-396.

SCOTT, S.G., J.J. BOMMER and S.K. SARMA (1998): Definition of hazard-consistent ground motions through multi-parameter seismic hazard assessment, in *Seismic Design Practice into the Next Century*, edited by E. BOOTH (Balkema, Rotterdam), 229-236.

SHAPIRA, A. (1983): Potential earthquake risk estimations by application of a simulation process, *Tectonophysics*, **95**, 75-89.

TERA CORPORATION (1980): Seismic hazard analysis: a methodology for the Eastern United States, *US Nuclear Regulatory Commission Report No. NUREG/CR-1582.*

(received October 20, 1998;
accepted August 19, 1999)