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Characterization of Delamination Effect on Free Vibration of Composite Laminates Plate Using High Order Shear Deformation Theory

Nabil Hassan Hadi * **Kayser Aziz Ameen ****
Department of Mechanical Engineering/ College of Engineering/ University of Baghdad
* Email: nabilhha@yahoo.com
** Email: kayseraziz@yahoo.com

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Abstract

A dynamic analysis method has been developed to investigate and characterize embedded delamination on the dynamic response of composite laminated structures. A nonlinear finite element model for geometrically large amplitude free vibration intact plate and delamination plate analysis is presented using higher order shear deformation theory where the nonlinearity was introduced in the Green-Lagrange sense. The governing equation of the vibrated plate were derived using the Variational approach. The effect of different orthotropicity ratio, boundary condition and delamination size on the non-dimenational fundamental frequency and frequency ratios of plate for different stacking sequences are studied. Finally the discrepancy of the results was 17.4906% when the severe nonlinearity is considered.

Keywords: *Delamination, high order shear deformation, plate, free vibration, nonlinear finite element method.*

1. Introduction

Interlayer debonding or delamination is a prevalent form of damage phenomenon in laminated composites. Delamination can be often pre-existing or generated during service life. For example, delamination often occur at stress free edges due to the mismatch of properties at ply interfaces and it can also be generated inside the plate by external forces such as out of plane loading or impact during the service life. The existence of delamination not only alters the load carrying capacity of the structure, it can also affect its dynamic response. Thus the investigation of delamination is an important technology that must be addressed for the successful implementation and improved reliability of such structures. All type of damages in composite structure result in change in stiffness, strength and fatigue properties. Measurement of strength or fatigue properties during damage development is not feasible because destructive testing is required. However, stiffness reduction due to damage can be measured since damage directly affects structural response, which provides a promising method for identifying the occurrence, location and extent of the damage from measured structural dynamics characteristics. Existence of delaminations causes reduction in natural frequencies and increase in vibratory damping. Several authors studied the linear and an nonlinear free vibration and used different shear theory.

Malekzadeh 2007 studyied the effect of different parameters on the convergence and accuracy of natural vibration of the method a differential quadrature for large amplitude free vibration analysis of laminated composite skew plates, the governing equations are based on the thin plate theory (classical linear theory) and geometrical nonlinearity is modeled using Green's strain in conjunction with Von Karman assumption. On the other hand The Ganapathi, etal. 2009 investigation of the free vibration characteristics of simply supported anisotropic composite laminates using analytical approach the formulation is based on the first order shear deformation theory, the governing equation are obtained using energy method. Liangjin and Zhengneng 2000 presented a three dimensional ,model for the analysis of local buckling of stitched composite laminates with an embedded elliptical delamination near the surface, they investigated the effects of stitching, delamination size, delamination orientation and stacking sequence on buckling strains by using the Rayleigh Ritz energy method. Heung etal 2003 investigated and characterized the effect due to the presence of discrete single and multiple embedded delaminations on the dynamic response of composite laminated structures with balanced/unbalanced and arbitrary stacking sequences in terms of number placement, mode shapes and natural frequencies. A new generalized layerwise finite element model is developed by Dongwei and Christian 2004 An analytical solution to the free vibration of composite beams with two non overlapping delaminations is presented, the Euler Bernoulli beams using the delaminations as their boundaries, the continuity and the equilibrium conditions are satisfied between adjoining beams. Alberto and etal 2005 developed the analytical delamination of the strain energy release rates in a delaminated laminate by means of a model of plates which provides no singular stresses and they predicted the delamination criteria. Wang and Dong 2005 used the energy method to study hygrothermal effects on local buckling for different delaminated shapes near the surface of cylindrical laminated shells, the effect of non-linear obtained by considering transverse displacements of sub laminate shells and the young's modulus, thermal and humidity expansion coefficients of material are treaded as functions of temperature. Yang and Fu 2006 discussed the effects of delamination sizes, depths, boundary conditions, the material properties and the laminate stacking sequences on delamination growth, based on the variational principle of moving boundary and used classical theory for cylindrical shells. Christian and Dongwei 2007 studied the analytical models and numerical analysis for free vibration of delaminated composites, and they discussed the influence of delamination on the natural frequencies and the mode shapes of composite laminates in addition other factors affecting the vibration of the delaminated composites are discussed. Yang etal. 2007 discussed the effects of delamination sizes, depths, boundary conditions, the material properties and the laminate stacking sequences on delamination growth, based on the variational principle of moving boundary and considering the contact effect between delamination regions, the first order theory and nonlinear governing equations for the cylindrical shells are derived. Sang and Dae 2007 studied buckling behaviors of laminated composite structures with a delamination using the enhanced assumed strain (EAS), the EAS three dimensional finite element formulation described and focused on the significant effects of the local buckling for various parameters, such as size of delamination, aspect ratio, width to thickness ratio, stacking sequences and location delamination and multiple delaminations. Züleyha and Mustafa 2008 studied the effect of the size of beneath delaminations has no significant on the critical buckling load and compressive failure load of E-glass/epoxy composite laminates with multiple large delaminations, a numerical and experimental study is carried out to determine the buckling load of rectangular composite plates, for the experiments (0º/90º/0º/90º)s oriented cross-ply laminated plates with multiple large delaminations and without delamination are produced by using hand lay up technique and the results are compare with results obtained by ANSYS 11.0 software and good agreement obtained. Alnefaie 2009 developed a three dimensional finite element model of delaminated fiber reinforced composite plates, a classical plate theory is studied during the analyzing their dynamics natural frequencies and modal displacements calculated for various case studies with different dimensions and delamination characteristics.

This work is investigated theoretically by using finite element method the effect of delamination on the natural frequency of the composite material with considering the severe nonlinearity, which is studied the modal analysis of intact and delamination plate.

2. Mathematical Model

2.1. Displacement Field

A plate of length a, width b and thickness h is composed of N number of orthotropic layers of uniform thickness. The (α, β, ζ) was rectangular coordinate. The following displacement field for the laminated plate based on the high shear deformation theory (HSDT) is considered to derive the mathematical model. The displacement along the (α, β, ζ) coordinates is :

$$\left.\begin{aligned}\bar{u}(\alpha,\beta,\zeta,t)&=u+\zeta\phi_1+\zeta^2\psi_1+\zeta^3\theta_1\\ \bar{v}(\alpha,\beta,\zeta,t)&=v+\zeta\phi_2+\zeta^2\psi_2+\zeta^3\theta_2\\ \bar{w}(\alpha,\beta,t)&=w\end{aligned}\right\} \qquad \ldots(1)$$

where t is the time, (u,v,w) are the displacements of a point on the mid-plane and ϕ_1 and ϕ_2 are the rotations at $(\zeta=0)$ of normal to the mid-plane respect to the α and β-axes, respectively, $\psi_1,\psi_2,\theta_1,\theta_2$ are high order terms of Toyler series expansion defined at the mid-plane (Panda and Singh, 2009).

2.2. Strain –Displacement Relation

The nonlinear Green Lagrange strain displacement relation for the laminated plate can be expressed as follows (Panda and Singh, 2009).

$$\{\varepsilon\}=\begin{Bmatrix}\varepsilon_{\alpha,\alpha}\\ \varepsilon_{\beta,\beta}\\ \gamma_{\alpha,\beta}\\ \gamma_{\alpha,\zeta}\\ \gamma_{\beta,\zeta}\end{Bmatrix}=\begin{Bmatrix}\left(\dfrac{\partial\bar{u}}{\partial\alpha}\right)\\ \left(\dfrac{\partial\bar{v}}{\partial\beta}\right)\\ \left(\dfrac{\partial\bar{u}}{\partial\beta}+\dfrac{\partial\bar{v}}{\partial\alpha}\right)\\ \dfrac{\partial\bar{u}}{\partial\zeta}+\left(\dfrac{\partial\bar{w}}{\partial\alpha}\right)\\ \dfrac{\partial\bar{v}}{\partial\zeta}+\left(\dfrac{\partial\bar{w}}{\partial\beta}\right)\end{Bmatrix}+$$

$$\frac{1}{2}\begin{Bmatrix}\left[\left(\dfrac{\partial\bar{u}}{\partial\alpha}\right)^2+\left(\dfrac{\partial\bar{v}}{\partial\alpha}\right)^2+\left(\dfrac{\partial\bar{w}}{\partial\alpha}\right)^2\right]\\ \left[\left(\dfrac{\partial\bar{v}}{\partial\beta}\right)^2+\left(\dfrac{\partial\bar{u}}{\partial\beta}\right)^2+\left(\dfrac{\partial\bar{w}}{\partial\beta}\right)^2\right]\\ 2\left[\left(\dfrac{\partial\bar{u}}{\partial\alpha}\right)\left(\dfrac{\partial\bar{u}}{\partial\beta}\right)+\left(\dfrac{\partial\bar{v}}{\partial\alpha}\right)\left(\dfrac{\partial\bar{v}}{\partial\beta}\right)+\left(\dfrac{\partial\bar{w}}{\partial\alpha}\right)\left(\dfrac{\partial\bar{w}}{\partial\beta}\right)\right]\\ 2\left[\left(\dfrac{\partial\bar{u}}{\partial\alpha}\right)\left(\dfrac{\partial\bar{u}}{\partial\zeta}\right)+\left(\dfrac{\partial\bar{v}}{\partial\alpha}\right)\left(\dfrac{\partial\bar{v}}{\partial\zeta}\right)+\left(\dfrac{\partial\bar{w}}{\partial\alpha}\right)\left(\dfrac{\partial\bar{w}}{\partial\zeta}\right)\right]\\ 2\left[\left(\dfrac{\partial\bar{v}}{\partial\beta}\right)\left(\dfrac{\partial\bar{v}}{\partial\zeta}\right)+\left(\dfrac{\partial\bar{u}}{\partial\alpha}\right)\left(\dfrac{\partial\bar{u}}{\partial\zeta}\right)+\left(\dfrac{\partial\bar{w}}{\partial\beta}\right)\left(\dfrac{\partial\bar{w}}{\partial\zeta}\right)\right]\end{Bmatrix}$$

or , $\{\varepsilon\}=\{\varepsilon_L\}+\{\varepsilon_{NL}\}$ …(2)

Where $\{\varepsilon_L\}$ and $\{\varepsilon_{NL}\}$ are the linear and nonlinear strain vectors respectively these terms can be considered when the thickness is increased or is equal to (a/10). Substituting equation (1) b equation (2), the strain – displacement relation of the laminated plate is further expressed as:

$$\{\varepsilon_L\}+\{\varepsilon_{NL}\}=\begin{Bmatrix}\varepsilon_1^0\\ \varepsilon_2^0\\ \varepsilon_6^0\\ \varepsilon_5^0\\ \varepsilon_4^0\end{Bmatrix}+\frac{1}{2}\begin{Bmatrix}\varepsilon_1^4\\ \varepsilon_2^4\\ 2\varepsilon_6^4\\ 2\varepsilon_5^4\\ 2\varepsilon_4^4\end{Bmatrix}+$$

$$\zeta\left[\begin{Bmatrix}\chi_1^1\\ \chi_2^1\\ \chi_6^1\\ \chi_5^1\\ \chi_4^1\end{Bmatrix}+\frac{1}{2}\begin{Bmatrix}\chi_1^5\\ \chi_2^5\\ 2\chi_6^5\\ 2\chi_5^5\\ 2\chi_4^5\end{Bmatrix}\right]+\zeta^2\left[\begin{Bmatrix}\chi_1^2\\ \chi_2^2\\ \chi_6^2\\ \chi_5^2\\ \chi_4^2\end{Bmatrix}+\frac{1}{2}\begin{Bmatrix}\chi_1^6\\ \chi_2^6\\ 2\chi_6^6\\ 2\chi_5^6\\ 2\chi_4^6\end{Bmatrix}\right]$$

$$+\zeta^3\left[\begin{Bmatrix}\chi_1^3\\ \chi_2^3\\ \chi_6^3\\ \chi_5^3\\ \chi_4^3\end{Bmatrix}+\frac{1}{2}\begin{Bmatrix}\chi_1^7\\ \chi_2^7\\ 2\chi_6^7\\ 2\chi_5^7\\ 2\chi_4^7\end{Bmatrix}\right]+\zeta^4\left[\frac{1}{2}\begin{Bmatrix}\chi_1^8\\ \chi_2^8\\ 2\chi_6^8\\ 2\chi_5^8\\ 2\chi_4^8\end{Bmatrix}\right]$$

$$+\zeta^5\left[\frac{1}{2}\begin{Bmatrix}\chi_1^9\\ \chi_2^9\\ 2\chi_6^9\\ 2\chi_5^9\\ 2\chi_4^9\end{Bmatrix}\right]+\zeta^6\left[\frac{1}{2}\begin{Bmatrix}\chi_1^{10}\\ \chi_2^{10}\\ 2\chi_6^{10}\\ 0\\ 0\end{Bmatrix}\right]$$

…(3)

Hence the above equation can be rearranged as:

$$\{\varepsilon\}=\{\varepsilon_L\}+\{\varepsilon_{NL}\}=[\Im]_L\{\bar{\varepsilon}\}_L+\frac{1}{2}[\Im]_{NL}\{\bar{\varepsilon}\}_{NL}$$

…(4)

Where

$$\{\bar{\varepsilon}\}_L=\{\varepsilon_1^0\ \varepsilon_2^0\ \varepsilon_6^0\ \varepsilon_5^0\ \varepsilon_4^0\ \chi_1^1\ \chi_2^1\ \chi_6^1\ \chi_5^1\ \chi_4^1\ \chi_1^2\ \chi_2^2\ \chi_6^2\ \chi_5^2\ \chi_4^2\ \chi_1^3\ \chi_2^3\ \chi_6^3\ \chi_5^3\ \chi_4^3\}$$

$$\{\bar{\varepsilon}\}_{NL}=\{\varepsilon_1^4\ \varepsilon_2^4\ \varepsilon_6^4\ \varepsilon_5^4\ \varepsilon_4^4\ \chi_1^5\ \chi_2^5\ \chi_6^5\ \chi_5^5\ \chi_4^5\ \chi_1^6\ \chi_2^6\ \chi_6^6\ \chi_5^6\ \chi_4^6\ \chi_1^7\ \chi_2^7\ \chi_6^7\ \chi_5^7\ \chi_4^7\ \chi_1^8\ \chi_2^8\ \chi_6^8\ \chi_5^8\ \chi_4^8\ \chi_1^9\ \chi_2^9\ \chi_6^9\ \chi_5^9\ \chi_4^9\ \chi_1^{10}\ \chi_2^{10}\ \chi_6^{10}\}$$

2.3. Stress - Strain Relations

In the analysis of composite laminated materials, the assumption of plane stress is usually used for each layer because fiber reinforced material is utilized in beam, plate, cylinders, spherical and other structural shapes which have at least one characteristic geometric dimension in an order of magnitude less than the other two dimensions. In this case the stress components $(\sigma_3, \tau_{23}, \tau_{13})$ are set to zero. Then the strain displacement relations, for any general k[th] orthotropic composite lamina with an arbitrary fiber orientation angle with reference to the coordinate axes (α, β, ζ) is written as (Panda and Singh, 2009):

$$\begin{Bmatrix} \sigma_\alpha \\ \sigma_\beta \\ \sigma_{\alpha\beta} \\ \sigma_{\alpha\zeta} \\ \sigma_{\beta\zeta} \end{Bmatrix}^k = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} & 0 & 0 \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} & 0 & 0 \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \overline{Q}_{55} & \overline{Q}_{45} \\ 0 & 0 & 0 & \overline{Q}_{45} & \overline{Q}_{44} \end{bmatrix}^k \begin{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \\ \varepsilon_5 \\ \varepsilon_4 \end{Bmatrix} \end{bmatrix}^k \quad \ldots(5)$$

Where :

$$\overline{Q}_{11} = Q_{11} \cdot \cos^4 \vartheta + 2(Q_{12} + 2Q_{66}) \cdot \sin^2 \vartheta \cdot \cos^2 \vartheta + Q_{22} \cdot \sin^4 \vartheta$$

$$\overline{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66}) \cdot \sin^2 \vartheta \cdot \cos^2 \vartheta + Q_{11} (\cos^4 \theta + \sin^4 \vartheta)$$

$$\overline{Q}_{22} = Q_{11} \sin^4 \vartheta + 2(Q_{12} + 2Q_{66}) \sin^2 \vartheta \cdot \cos^2 \vartheta + Q_{22} \cos^4 \vartheta$$

$$\overline{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66}) \sin\vartheta \cdot \cos^3 \vartheta + (Q_{12} - Q_{22} + 2Q_{66}) \sin^3 \vartheta \cdot \cos\vartheta$$

$$\overline{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66}) \sin^3 \vartheta \cdot \cos\vartheta + (Q_{12} - Q_{22} + 2Q_{66}) \sin\vartheta \cdot \cos^3 \vartheta$$

$$\overline{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}) \sin^2 \vartheta \cdot \cos^2 \vartheta + Q_{66} (\sin^4 \vartheta + \cos^4 \vartheta)$$

$$\overline{Q}_{44} = Q_{44} \cos^2 \vartheta + Q_{55} \sin^2 \vartheta$$

$$\overline{Q}_{45} = (Q_{55} - Q_{44}) \cos\vartheta \cdot \sin\vartheta$$

$$\overline{Q}_{55} = Q_{55} \cos_2 \vartheta + Q_{44} \sin_2 \vartheta$$

2.4. Strain Energy of The Laminate

Energy and variational principle offered great simplification to many derivations of fundamental equations in elasticity. Also have been used to introduce and implement approximation techniques for structural systems. Strain energy is defined as the work done by the internal stresses which caused elongation or shear strains. The strain energy of the plate can be expressed as:

$$U = \frac{1}{2} \int_V \{\varepsilon\}_i^T \cdot \{\sigma_i\} dV \quad \ldots(6)$$

By substituting the strains from equation (2) and stresses from equation (5) into equation (6) the strain energy can be expressed as :

$$U = \frac{1}{2} \int_V \{\varepsilon_L + \varepsilon_{NL}\}_i^T \left[\overline{Q}\right] \{\varepsilon_L + \varepsilon_{NL}\}_i dV$$

$$= \frac{1}{2} \int \begin{pmatrix} \{\varepsilon_L\}_i^T [D_1] \{\varepsilon_L\}_i \\ + \frac{1}{2} \{\varepsilon_L\}_i^T [D_2] \{\varepsilon_{NL}\}_i \\ + \frac{1}{2} \{\varepsilon_{NL}\}_i^T [D_3] \{\varepsilon_L\}_i \\ + \frac{1}{4} \{\varepsilon_{NL}\}_i^T [D_4] \{\varepsilon_{NL}\}_i \end{pmatrix} dA \quad \ldots(7)$$

Where :

$$[D_1] = \sum_{k=1}^{N} \int_{\zeta_{k-1}}^{\zeta_k} [\Im]_L^T \left[\overline{Q}\right] [\Im]_L \, d\zeta$$

$$[D_2] = \sum_{k=1}^{N} \int_{\zeta_{k-1}}^{\zeta_k} [\Im]_L^T \left[\overline{Q}\right] [\Im]_{NL} \, d\zeta$$

$$[D_3] = \sum_{k=1}^{N} \int_{\zeta_{k-1}}^{\zeta_k} [\Im]_{NL}^T \left[\overline{Q}\right] [\Im]_L \, d\zeta$$

$$[D_4] = \sum_{k=1}^{N} \int_{\zeta_{k-1}}^{\zeta_k} [\Im]_{NL}^T \left[\overline{Q}\right] [\Im]_{NL} \, d\zeta$$

where : N is the numbers of layers.

2.5. Kinetic Energy of the Vibrating Plate

The kinetic energy expression of a vibrated plate can be expressed as

$$T = \frac{1}{2} \int_V \rho \left\{\dot{\bar{\delta}}\right\}^T \left\{\dot{\bar{\delta}}\right\} dV \quad \ldots(8)$$

Where, ρ and $\dot{\bar{\delta}}$ are the density, displacement vector which is first derivative with respect to time, respectively. The Nodal velocity vector can be expressed as:

$$\left\{\dot{\bar{\delta}}\right\}_i = \begin{Bmatrix} \dot{\bar{u}} \\ \dot{\bar{v}} \\ \dot{\bar{w}} \end{Bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & \zeta & 0 & \zeta^2 & 0 & \zeta^3 & 0 \\ 0 & 1 & 0 & 0 & \zeta & 0 & \zeta^2 & 0 & \zeta^3 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{\phi}_1 \\ \dot{\phi}_2 \\ \dot{\psi}_1 \\ \dot{\psi}_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{Bmatrix}$$

$$= [f]\{\dot{\delta}\}$$

…(9)

Where, [*f*] is the function of the thickness coordinate. Then the kinetic energy for 'N' number of orthotropic layered composite plate obtained by substituting equation (9) into equation (8) gives:

$$T = \frac{1}{2}\int_A \left(\sum_{k=1}^{N} \int_{\zeta_{k-1}}^{\zeta_k} \{\dot{\delta}\}^T [f]^T \rho^k [f]\{\dot{\delta}\} \mathrm{d}\zeta \right) \mathrm{dA}$$

$$T = \frac{1}{2}\int_{\mathrm{A}} \{\dot{\delta}\}^T [m] \{\dot{\delta}\} \mathrm{dA} \qquad \text{…(10)}$$

where, $[m] = \sum_{k=1}^{N} \int_{\zeta_{k-1}}^{\zeta_k} \left([f]^T \rho^k [f]\right) \mathrm{d}\zeta$ is the inertia matrix.

3. Solution Technique

The displacement vector can be conceded to the form by employing the FEM

$$\{\delta\} = [N_i]\{\delta_i\} \qquad \text{…(11)}$$

where :

$$\{\delta_i\} = [u_i \quad v_i \quad w_i \quad \phi_1 \quad \phi_2 \quad \psi_1 \quad \psi_2 \quad \theta_1 \quad \theta_2]^T$$

The equations of strain for linear and nonlinear of large deflections, are studied in equation (4) and nonlinear displacement is shadied in equation (1), When they are substituted into equation (7), the strain energy can be written as:

$$U = \frac{1}{2}\int_A \begin{pmatrix} [B_L]_i^T \{\delta\}_i^T [D_1][B_L]_i \{\delta\}_i \\ + \frac{1}{2}[B_L]_i^T \{\delta\}_i^T [D_2][B_{NL}]_i \{\delta\}_i \\ + \frac{1}{2}[B_{NL}]_i^T \{\delta\}_i^T [D_3][B_L]_i \{\delta\}_i \\ + \frac{1}{4}[B_{NL}]_i^T \{\delta\}_i^T [D_4][B_{NL}]\{\delta\}_i^T \end{pmatrix} dA$$

…(12)

Where $[B_{NL}]_i = [A]_i [G]_i$, $[A]_{33\times27}$ is function to the displacements and $[G]_{27\times9}$ is the product form of differential operator and shape function in the nonlinear strain terms. $[B_L]_{20\times9}$ is the product form of the differential operator and nodal interpolation function in the linear terms.

The final form of governing equation for the nonlinear free vibration laminated plate panel is obtained by using Hamilton's principle. It can be viewed and axiom, from which other axioms like Newton's second law, Let the potential energy be defined as ($\Pi = U - W$), where U is the strain energy and W is the work done and the Lagrangian as the function L where *(L = (T – U+W)).*

Hamilton's principle states that the actual displacement that the body actually goes through from instant (t_1) to instant (t_2) out of many possible paths, is that which achieves an extremum of the line integral of the Lagrangian function. This is achieved if the variation of the time integral of the Lagrangian is set to zero:

$$\delta \int_{t_1}^{t_2} L \, dt = 0 \qquad \text{…(13)}$$

Hamilton's principle can be used to find the compatible set of equations of motion and boundary conditions for given stresses and strains. This is done by substituting the equations for strain energy equation (12) and kinetic energy equation (10) into the equation (13), performing the integration by parts, and setting the coefficients of the displacement variations (also called virtual displacement) to zero. The Lagrangian becomes (Marco, 2008).

$$[M]\{\ddot{\delta}\} + ([K_L] + [K_{NL}])\{\delta\} = 0 \qquad \text{…(14)}$$

Where $\{\delta\}$ is the displacement vector, *[M]*, *[K_L]* and *[K_{NL}]* are the global mass matrix and global linear stiffness matrix and nonlinear stiffness matrix depend on the displacement vector respectively.

4. Damage Modeling

Defect and damages cause the laminates to lose their strength and rigidity and also the safe working life is reduced. Defects and damages can propagate in at any time like manufacturing, in service or in design due to discontinuities such as cut out and play drops. The investigation of defects and damages are presented. By using the following expansion of the displacements through the thickness of the plate:

$$\left.\begin{aligned}\bar{u}(\alpha,\beta,\zeta,t) &= u+\zeta\phi_1+\zeta^2\psi_1+\zeta^3\theta_1 + \sum_{1}^{ND} H'(z)U'(\alpha,\beta)\\ \bar{v}(\alpha,\beta,\zeta,t) &= v+\zeta\phi_2+\zeta^2\psi_2+\zeta^3\theta_2 + \sum_{1}^{ND} H'(z)V'(\alpha,\beta)\\ \bar{w}(\alpha,\beta,t) &= w+\sum_{1}^{ND} H'(z)W'(\alpha,\beta)\end{aligned}\right\} \quad (15)$$

Where the step functions H' are computed in terms of Heaviside functions H' as :

$$H'(z)=H'(z-z_j)=1 \quad \text{for} \quad z \ge z_j$$
$$H'(z)=H'(z-z_j)=0 \quad \text{for} \quad z < z_j \qquad \ldots(16)$$

And the ND is the number of delaminations. The jumps in the displacements at the j^th delaminated interface are given by U', V' and W'. Using the step functions $H'(z)$, can model any number of delaminations through thickness; the number of additional variable is equal to the number of delaminations considered(Barbero and Reddy, 1991).

Then substituting equation (16) into the nonlinear Green Lagrange strain displacement relation for the laminated shell, equation (2) will be:

$$\{\varepsilon_L\}+\{\varepsilon_{NL}\}=\begin{Bmatrix}\varepsilon_1^0\\ \varepsilon_2^0\\ \varepsilon_6^0\\ \varepsilon_5^0\\ \varepsilon_4^0\end{Bmatrix}+\begin{Bmatrix}\varepsilon_{d1}^0\\ \varepsilon_{d2}^0\\ \varepsilon_{d6}^0\\ \varepsilon_{d5}^0\\ \varepsilon_{d4}^0\end{Bmatrix}+\frac{1}{2}\begin{Bmatrix}\varepsilon_1^4\\ \varepsilon_2^4\\ 2\varepsilon_6^4\\ 2\varepsilon_5^4\\ 2\varepsilon_4^4\end{Bmatrix}+\frac{1}{2}\begin{Bmatrix}\varepsilon_{d1}^4\\ \varepsilon_{d2}^4\\ 2\varepsilon_{d6}^4\\ 2\varepsilon_{d5}^4\\ 2\varepsilon_{d4}^4\end{Bmatrix}$$

$$+\zeta\left[\begin{Bmatrix}\chi_1^1\\ \chi_2^1\\ \chi_6^1\\ \chi_5^1\\ \chi_4^1\end{Bmatrix}+\frac{1}{2}\begin{Bmatrix}\chi_1^5\\ \chi_2^5\\ 2\chi_6^5\\ 2\chi_5^5\\ 2\chi_4^5\end{Bmatrix}+\frac{1}{2}\begin{Bmatrix}\chi_{d1}^5\\ \chi_{d2}^5\\ 2\chi_{d6}^5\\ 2\chi_{d5}^5\\ 2\chi_{d4}^5\end{Bmatrix}\right]$$

$$+\zeta^2\left[\begin{Bmatrix}\chi_1^2\\ \chi_2^2\\ \chi_6^2\\ \chi_5^2\\ \chi_4^2\end{Bmatrix}+\frac{1}{2}\begin{Bmatrix}\chi_1^6\\ \chi_2^6\\ 2\chi_6^6\\ 2\chi_5^6\\ 2\chi_4^6\end{Bmatrix}+\frac{1}{2}\begin{Bmatrix}\chi_{d1}^6\\ \chi_{d2}^6\\ 2\chi_{d6}^6\\ 2\chi_{d5}^6\\ 2\chi_{d4}^6\end{Bmatrix}\right]$$

$$+\zeta^3\left[\begin{Bmatrix}\chi_1^3\\ \chi_2^3\\ \chi_6^3\\ \chi_5^3\\ \chi_4^3\end{Bmatrix}+\frac{1}{2}\begin{Bmatrix}\chi_1^7\\ \chi_2^7\\ 2\chi_6^7\\ 2\chi_5^7\\ 2\chi_4^7\end{Bmatrix}+\frac{1}{2}\begin{Bmatrix}\chi_{d1}^7\\ \chi_{d2}^7\\ 2\chi_{d6}^7\\ 0\\ 0\end{Bmatrix}\right]$$

$$+\zeta^4\left[\frac{1}{2}\begin{Bmatrix}\chi_1^8\\ \chi_2^8\\ 2\chi_6^8\\ 2\chi_5^8\\ 2\chi_4^8\end{Bmatrix}\right]+\zeta^5\left[\frac{1}{2}\begin{Bmatrix}\chi_1^9\\ \chi_2^9\\ 2\chi_6^9\\ 2\chi_5^9\\ 2\chi_4^9\end{Bmatrix}\right]+\zeta^6\left[\frac{1}{2}\begin{Bmatrix}\chi_1^{10}\\ \chi_2^{10}\\ 2\chi_6^{10}\\ 0\\ 0\end{Bmatrix}\right] \qquad \ldots(17)$$

The generalized displacements (DOF) in this case are:

$$(u, v, w, \phi_1, \phi_2, \psi_1, \psi_2, \theta_1, \theta_2, U', V', W').$$

The effect of delamination on the stiffness is much higher compared to the mass, hence it can be considered the same mass matrix in equation (10).

5. Numerical Results and Discussion

A nonlinear finite element code using the present displacement field plate model in Green-Lagrange sense in the framework of the HSDT is presented. The validation and accuracy of the present algorithm are examined by comparing the results with those available in the literature. The effect of different combinations of the thickness ratio (a/h), amplitude ratio (W_{max}/h), where W_{max} is the maximum deflection of plate and h is the thickness, and various boundary conditions on the composite plate response are also examined.

The following sets of boundary conditions are used for the present analysis;

a) Simply support boundary conditions (S):

$v = w = \phi_2 = \psi_2 = \theta_2 = 0$ at x=0,a

$u = w = \phi_1 = \psi_1 = \theta_1 = 0$ at y=0,b

b) Clamped boundary conditions (C):

$u = v = w = \phi_1 = \phi_2 = \psi_1 = \psi_2 = \theta_1 = \theta_2 = 0$ at x= 0,a

$u = v = w = \phi_1 = \phi_2 = \psi_1 = \psi_2 = \theta_1 = \theta_2 = 0$ at y= 0,b

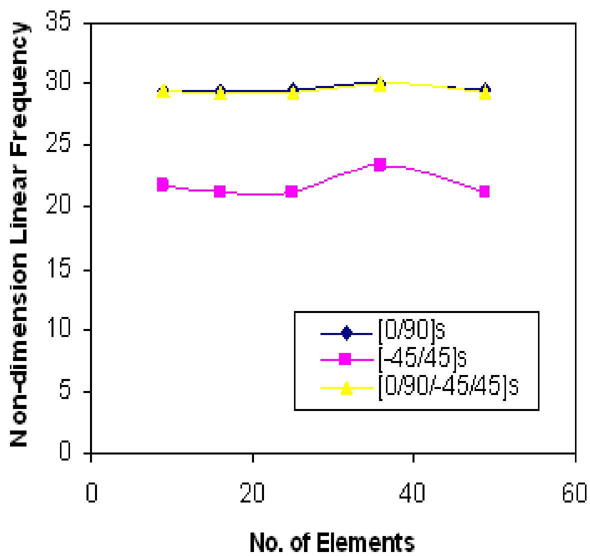

Fig.1. Convergence Study of Non-Dimensional Frequency for Square Plate Having Ssss Boundary Condition with Different Stacking Sequences.

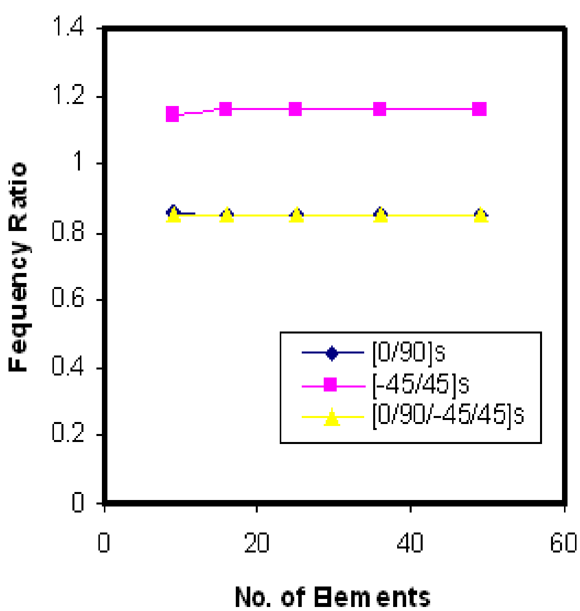

Fig.2. Convergence Study for Frequency Ratio of Square Laminated Plate with Ssss Boundary Condition with Different Stacking Sequences.

A convergence of the mathematical model developed for laminated plate is presented in Figures (1) and (2) which shown the nondimensional fundamental frequency ($\varpi = \omega n_L \left(\frac{a^2}{h}\right)\sqrt{\left(\frac{\rho}{E_2}\right)}$), and frequency ratio $\left(\omega_{NL}/\omega_L\right)$ against mesh division respectively for simply support boundary condition and for different stacking sequences, The results are plotted using the material properties (*E_1=181 GPa, E_2=7.17GPa, G_{23}=6.71GPa, υ_{12}=0.28*, and the geometry parameters are *a/b=1, a/h=10*). Figures show that the convergence is a (5X5) mesh, and that it is used to compute the results throughout the study.

A comparison of the linear free vibration with out defect results with Classical Plate Theory (CPT) and the Layerwise plate theory (LWST) and the Generalized Lamination Plate Theory (GLPT) is also included. (GLPT, though identical in formulation with the Layerwise plate theory (LWST), does not consider the variation of the transverse displacement, w, through the thickness) (Samuel ,1992); and the results were obtained from a 3-D orthotropic elasticity theory (Noor,1975). The material properties and the geometries details of this comparison are:

$$E_2 = E_3 = 1.0 \times 10^6\ psi,\quad \frac{E_1}{E_2} = 3,\ 10,\ 20,\ 30,$$

$$G_{12} = G_{13} = 0.6 \times 10^6\ psi,\quad G_{23} = 0.5 \times 10^6\ psi,$$

$$\upsilon_{12} = \upsilon_{13} = 0.25,\quad \upsilon_{23} = 0.24,\quad a = 20\ \ inch,$$

$$b = 20\ inch,\quad \frac{a}{h} = 10.$$

The results are summarized in Table (1). The differences are more pronounced between the present results and the results existing in (Samuel, 1992 and Noor, 1975), because the present work used the framework of the high order shear theory and the other results used the framework of the first order theory and classical theory.

When the plate containing some defect such as delamination, this causes decreasing in natural frequency with diverge in some results because the present work used in the framework of the high order shear theory and geometrical nonlinearity modeled using Green's strain. The results are shown in Table (2). The linear fundamental frequency is increasing with the increase in modular ratio. The frequency ratio is decreasing with the increase in modular ratio.

Table 1,
Nondimensional Fundamental Frequency of (0/90/0) and (0/90/0/90/0) Cross-Ply Simply Support Plates.

No. of layers	Method	E_{11}/E_{22}			
		3	10	20	30
3	LWST	0.2671	0.338	0.3897	0.4197
	GLPT	0.2645	0.3368	0.3897	0.4205
	3-D Elasticity	0.2647	0.3284	0.3824	0.4109
	CPT	0.2919	0.4126	0.5404	0.6433
	Present work	0.25699	0.343019	0.395955	0.4245764
5	LWST	0.2671	0.3429	0.4012	0.4354
	GLPT	0.2645	0.3415	0.4012	0.4367
	3-D Elasticity	0.2659	0.3409	0.3997	0.4314
	CPT	0.2919	0.4126	0.5404	0.6433
	Present work	0.332695	0.44010713	0.506355	0.5417735

Table 2,
Effect of Material Orthotropy on Nonlinear Free Vibration Ratio $\left(\frac{\varpi_{NL}}{\varpi_L}\right)$ of Delamination Laminated Plate.

	W_{max}/h	E_1/E_2							
		3		5		10		15	
		κ_{dNL}	ϖ_{dNL}/ϖ_{dL}	κ_{dNL}	ϖ_{dNL}/ϖ_{dL}	κ_{dNL}	ϖ_{dNL}/ϖ_{dL}	κ_{dNL}	ϖ_{dNL}/ϖ_{dL}
[0/90/0/90]s	0.5	0.271253341	1.439972862	0.496753765	1.005498833	0.463875194	1.074907011	0.479950624	1.114002269
	1	0.377780936	1.22934653	0.567228255	0.854560912	0.67313799	0.689274524	0.635394303	0.812535895
	1.5	0.518695812	1.010721	0.648812609	0.701566829	0.662164365	0.735156446	0.53275242	1.031948538
	2	0.512059386	1.008796066	0.649101227	0.700989768			0.540102051	1.00545388
	ϖ_{dL}	56.05608734		80.72544339		52.7409818		56.3617073	
	κ_{dL}	0.231787883		0.031989607		0.379276273		0.336781921	
[45/-45/45/-45]s	0.5	0.524938484	1.008676236	0.346213883	1.488284445	0.302825431	1.568922348	0.288723595	1.591158038
	1	0.52655556	1.005162538	0.535180065	1.122745485	0.612533832	0.984962028	0.585499164	1.008888609
	1.5	0.547574918	1.000895833	0.670324913	0.823119111	0.569620939	1.094049155	0.600627397	1.00808513
	2	0.549970942	0.995595058	0.597129584	1.005869819	0.583307751	1.093122543	0.618881411	1.007317623
	ϖ_{dL}	56.04256347		56.17882202		51.77412417		56.20827023	
	κ_{dL}	0.169927471		0.175307231		0.248169694		0.188396351	
[0/45/-45/90]s	0.5	0.770996366	1.439424677	0.31815017	1.44382817	0.341425355	1.320595998	0.457053434	1.09130257
	1	0.321721145	1.38750057	0.427610259	1.231845439	0.449647998	1.084938451	0.516577858	1.006602312
	1.5	0.530488979	1.009934922	0.546070899	1.008082255	0.546266667	0.985149923	0.537217179	1.013647779
	2	0.529218539	1.008795481		1.007125703	0.669394579	0.705624484		0.898161014
	ϖ_{dL}	56.05506261		56.1860639		52.25566967		56.36350015	
	κ_{dL}	0.203232959							

The variation of the frequency ratio for different support conditions and the amplitude ratios are analyzed for anti-symmetric and symmetric lamination schemes. The results are figured in table (3) . The effects of three different support conditions are examined for the frequency ratio such as all sides simple support (SSSS), all sides clamped (CCCC), and two sides simply support and two sides clamped (SCSC). The material properties and other parameters are ($E_1/E_2 = 15$, $G_{12}/E_2 = 0.5$, $G_{23}/E_2 = 0.2$, $\upsilon = 0.25$, $a/b = 1$, $a/h = 10$, and. It is found that the fundamental frequency increasing with increase the amplitude ratio in all typs of boundary condition.

Table 3,
Effect of Variable Boundary Condition with Delamination Defect on the Frequency Ratio.

	W_{max}/h	Boundary Condition					
		SSSS		SCSC		CCCC	
		κ_{dNL}	ϖ_{dNL}/ϖ_{dL}	κ_{dNL}	ϖ_{dNL}/ϖ_{dL}	κ_{dNL}	ϖ_{dNL}/ϖ_{dL}
[0/90/0/90]s	0.5	0.491758607	1.088579493	0.538991224	0.987414065	0.062150024	2.104234903
	1	0.643715012	0.793992918	0.220360475	1.73745255	0.158652081	1.8765753
	1.5	0.543415532	1.008398321	0.102742944	1.981654155	0.050560733	2.100182456
	2	0.550597436	0.9825083	0.393293891	1.326413856	0.074813047	2.027026282
	ϖ_{dL}	56.3617073		57.49825598		57.67798328	
	κ_{dL}	0.336781921		0.323434162		0.321415743	
[45/-45/45/-45]s	0.5	0.133385624	1.591158038	0.879263163	0.952480486	0.023840315	2.190273943
	1	0.494974975	1.008888609	0.393103954	1.353254954	0.000917226	2.23183492
	1.5	0.513407113	1.00808513	0.210310709	1.745408323	0.419374326	1.283481406
	2	0.535647682	1.007317623	0.00169324	2.184253397	0.553357876	0.977484197
	ϖ_{dL}	56.20827023		56.4956763		57.02122715	
	κ_{dL}	0.333873817		0.335140781		0.329024622	
[0/45/-45/90]s	0.5	0.457053433	1.09130257	0.055274753	2.119779963	0.018531167	2.202208867
	1	0.516577858	1.006602312	8986.481765	2.004210859	0.171728119	1.847245944
	1.5	0.537217179	1.013647779	0.365342642	1.403137192	0.213629079	1.739024909
	2	0.58431569	0.898161014	0.432516003	1.242230958	0.245671236	1.651955386
	ϖ_{dL}	56.36350015		56.98452359		57.32010532	
	κ_{dL}	0.336764848		0.329470127		0.325554679	

The effect of delamination on the natural frequency, decreasing the natural frequency with increases the size of defect, because of the increase in the delamination size this leads to an increase in the reduction in stiffness matrix the results are shown in table (4).

The Mode shape of the intact plate and delamination plate show in figures (3) and (4) the decreasing in the mode shape when the plate containing the delamination.

Table 4,
Effect of Delamination Size on the Frequency Ratio.

	W_{max}/h	Delamination Size					
		B(8X8)cm		A(6.6X6.6)cm		C(4X4)cm	
		κ_{dNL}	ϖ_{dNL}/ϖ_{dL}	κ_{dNL}	ϖ_{dNL}/ϖ_{dL}	κ_{dNL}	ϖ_{dNL}/ϖ_{dL}
[0/90/0/90]s	0.5	0.479950624	1.114002269	0.193951872	2.55757464	0.378470183	2.95283291
	1	0.635394303	0.812535895	0.223217546	2.72598089	0.299101517	2.895090839
	1.5	0.53275242	1.031948538	0.201466386	2.653521464	0.313329257	2.900578336
	2	0.540102051	1.00545388	0.036672763	2.266430313	0.281039999	2.800679241
	ϖ_{dL}	56.3617073		57.51626797		57.8864755	
	κ_{dL}	0.336781921		0.323196004		0.318839707	
[45/-45/45/-45]s	0.5	0.133385625	1.591158038	0.144176768	2.100779902	0.24478903	2.28551029
	1	0.494974977	1.008888609	0.093482347	2.184449945	0.386776317	2.770363377
	1.5	0.513407114	1.00808513	0.096838416	2.272344147	0.48287703	3.072108791
	2	0.535647682	1.007317623	0.011386133	2.144596099	0.387957769	3.010891227
	ϖ_{dL}	56.20827023		56.31581296		56.68828199	
	κ_{dL}	0.333873817		0.332599325		0.328185181	
[0/45/-45/90]s	0.5	0.457053434	1.09130257	0.302822548	1.401300962	0.345981887	2.705373944
	1	0.516577858	1.006602312	0.066301456	2.220298611	0.316935026	2.742178577
	1.5	0.537217179	1.013647779	0.171090219	1.815587184	0.238173784	2.712011012
	2	0.58431569	0.898161014	0.174845634	1.782895011	0.229407794	2.656357541
	ϖ_{dL}	56.36350015		56.91225624		57.28434623	
	κ_{dL}	0.336764848		0.330307578		0.325929157	

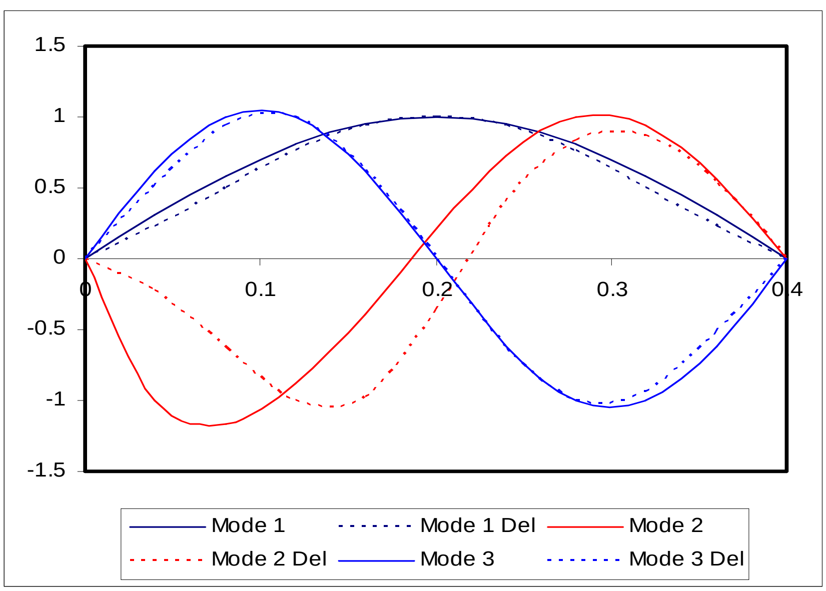

Figure.3. The Mode Shape of Plate of the Intact and Delamination Plate [0/90/0/90]s.

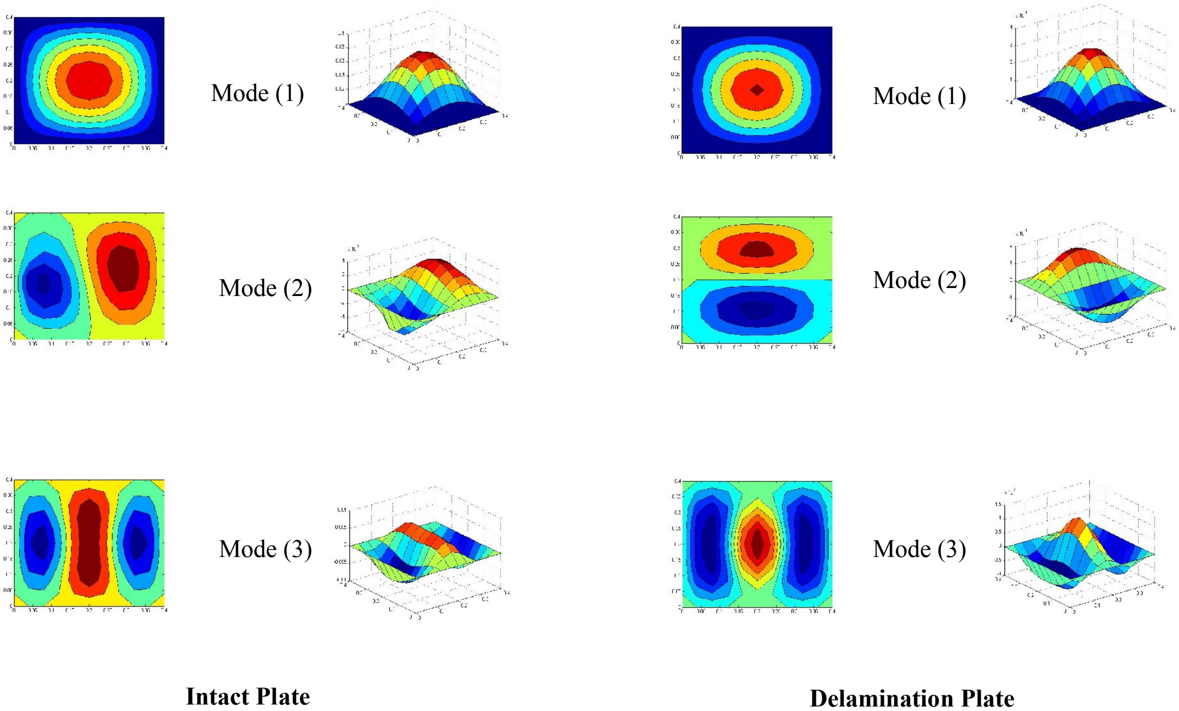

Figure.4. The Contour and Surface Mode Shape of Plate of the Intact and Delamination Plate [0/90/0/90]s.

6. Conclusion

The geometrically nonlinear free vibration analysis of composite plate with and without containing the delamination is investigated using nonlinear finite element method in the framework of a higher order shear deformation theory in Green-Lagrange sense. The governing equation of the vibrated plate is derived using the Variational approach. The frequency amplitude relations for the nonlinear free vibrated plate are computed using eigenvalue formulation and are solved employing a direct iterative procedure. Based on the numerical results the following conclusions are drawn.

- The validation shows the necessities of taking into account full nonlinearity.
- The two dimensional finite element model proposed can predict accurately the dynamic behaviors of a laminated composite plate with internal delamination at arbitrary location.
- The discrepancy of the results when considered the nonlinear Green Lagrange strain displacement was (17.4906% when the severe nonlinearity is considered).
- Local internal delamination has slight effect on the natural frequencies of the laminated composite plate although the extent of the natural frequency variation increase with both the delamination dimension and the order of the natural frequency.

Notations

α, β, ζ	Rectangular coordinate axes
$\overline{u}, \overline{v}, \overline{w}$	Displacement along the α, β, ζ coordinate
$\{\varepsilon_L\}, \{\varepsilon_{NL}\}$	Linear and nonlinear strain vectors
$[Q]$	Transferred reduced elastic constant
E	Young's modulus (GN/m^2)
G	Shear modulus (GN/m^2)
υ	Poisson's ratio
ρ	Density
T	Kinetic energy
W_{max}/h	Amplitude ratio
ω_{NL}/ω_L	Frequency ratio
κ_{dL}, κ_{dNL}	Damage ratio for linear and nonlinea respectively $(\omega_{\text{int}\,act} - \omega_{delamenation}) / \omega_{\text{int}\,act})$

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تاثير الانخلاعات على الاهتزازات الحرة للصفائح المركبة المتعددة باستعمال نظرية القص ذات الرتب العالية

نبيل حسن هادي * قيصر عزيز امين **

قسم الهندسة الميكانيكية/ كلية الهندسة/ جامعة بغداد

* البريد الالكتروني: nabilhha@yahoo.com

** البريد الالكتروني: kayseraziz@yahoo.com

الخلاصة

ان البحث واسع في الطرق التحليل الدينميكي لايجاد الخواص الديناماكية للمواد المركبة المتعددة الطبقات والحاوية على الانخلاعات. في هذا البحث تم تحليل الصفائح بواسطة طريقة العناصر المجددة اللاخطية بالنسبة للاشكال الهندسية وللسعات الاهتزازية العالية للصفائح السليمة وللصفائح الحاوية على الانخلاعات وباستعمال نظريات القص ذات المرتبات العالية وتم اشتقاق المعادلات الجامعة للصفائح المهتزة باستعمال طريقة المتغيرات التقريبية. تم البحث في هذا العمل دراسة تاثير تغير نسبة الخواص التعامدية للمواد المركبة، الشروط الحدية وحجم الانخلاعات الموجودة في الصفائح وتاثيرهم على الترداد الطبيعيه للصفائح المركبة. ولقد تم الحصول على تناقض في النتائج بمقدار (١٧,٤٩ %) عند الاخذ بنظر الاعتبار العوامل اللاخطية .