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A Comparison Between Recursive Least-Squares (RLS) and Extended Recursive Least-Squares (E-RLS) for Tracking Multiple Fast Time Variation Rayleigh Fading Channel

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Abstract

In order to select the optimal tracking of fast time variation of multipath fast time variation Rayleigh fading channel, this paper focuses on the recursive least-squares (RLS) and Extended recursive least-squares (E-RLS) algorithms and reaches the conclusion that E-RLS is more feasible according to the comparison output of the simulation program from tracking performance and mean square error over five fast time variation of Rayleigh fading channels and more than one time (send/receive) reach to 100 times to make sure from efficiency of these algorithms.

Keywords: *RLS algorithm, E-RLS algorithm, Rayleigh fading channels.*

1. Introduction

Data transmission over mobile communication channels suffers from fading. The amplitude of transmitted signal will changes randomly according to affect of Frequency flat fading. It is important to track variations of the amplitude to detect transmitted symbols optimally. In the nonexistence of the direct line of sight, is often made Rayleigh fading supposition, i.e., the faded amplitude (received amplitude) is a random sequence with a Rayleigh density. The fading rate of the received signal amplitude is administered by the Doppler bandwidth, which in turn is related to the speed of mobile or receiver and the transmission frequency. This fading rate must be treated through design of powerful receivers. Subsequently, in the fast single or multipath fading channels, cannot be neglect affected part of the amplitude of the signal over a symbol period. The sampling frequency of the received signal over slow fading channels is the Nyquist frequency of the transmitted signal and this sampling rate is twice transmitted signal bandwidth, that means the effects of the fast time varying Rayleigh fading channel be huge on the (faded) signal since the received signal have a bandwidth greater than the transmitted data bandwidth [1,2,3,4], so it must be there an efficient channel estimation to get the original data, here we are compared the performance of two algorithms, recursive least-squares (RLS) and extended recursive least-squares (E-RLS) algorithms to show which one has a good ability to track fast time variation of Rayleigh fading channels, to decrease the negative effect on the faded signal at the receiver. By pointing to the recent years, the comparison assessment of the tracking performances for the least mean square LMS and recursive least-squares RLS algorithms much have been described such as in [5]–[8]. The LMS algorithm shows a well tracking performance than the RLS algorithm because the LMS algorithm is model independent, while the RLS algorithm is model dependent.

The remainder of this paper is prepared as follows. We introduced the channel model In Section 2. In Section 3, We present mathematical model of RLS and E-RLS algorithms. Next, in Section 4, we compared the performance of the RLS and E-RLS by computer simulation. Section 5 concludes the paper.

2. Channel Model

The impulse response of a multipath fading channel can be describe as

$$h(n) = \sum_{k=1}^{L} \gamma_k \, x_k(n) \, \delta(n - n_k) \qquad \ldots(1)$$

Where L, γ_k , $x_k(n)$, are respectively the channel length, path loss and Rayleigh fading sequence of the k-th reflector groups, and n_k are the groups delay. In this event, for each n, assumed the amplitude $|x(n)|$ have a Rayleigh distribution as shown below [9]:

$$f_{|x_k(n)|}(|x_k(n)|) = |x_k(n)| \, e^{-|x_k(n)|^2/2}, \quad |x_k(n)| \geq 0 \qquad \ldots(2)$$

Whereas, the phase $\angle x(n)$ is uniformly distributed (assumed) with in$[-\pi, \pi]$:

$$f\left(\angle x_k(n)\right) = \frac{1}{2\pi} \quad , -\pi \leq \angle x_k(n) \leq \pi \qquad \ldots(3)$$

Also, the auto-correlation function of the $x_k(n)$ sequence, now considered as a random process, is modeled as a zeroth-order Bessel function of the first kind [9], i.e.,

$$r(k) \triangleq E \, x_k(n) \, x_k(n - n_o) = \mathcal{J}_o(2\pi f_D T_s n_o), \;\; n_o = \cdots, -1, 0, 1, \ldots \qquad \ldots(4)$$

Where T_s is the time sampling of the sequence $x_k(n)$, f_D is the maximum Doppler frequency of the Rayleigh fading channel, and the function $\mathcal{J}_o(\cdot)$ is determined by :

$$\mathcal{J}_o(y) \triangleq \frac{1}{\pi} \int_0^{\pi} \cos\,(y \sin\theta) d\theta \qquad \ldots(5)$$

The Doppler frequency f_D is the product of carrier frequency f_c by speed of mobile users (receiver),v, over to speed of light as given below:

$$f_D = \frac{v f_c}{c} \qquad \ldots(6)$$

Where $c = 3 * 10^8 \, m/s$.
Here, figure (1) show sequence magnitude and phase of Rayleigh fading channel at $f_D = 100 \, Hz$,$T_s = 1X10^{-3} s$, and number of channel samples = 200 samples.

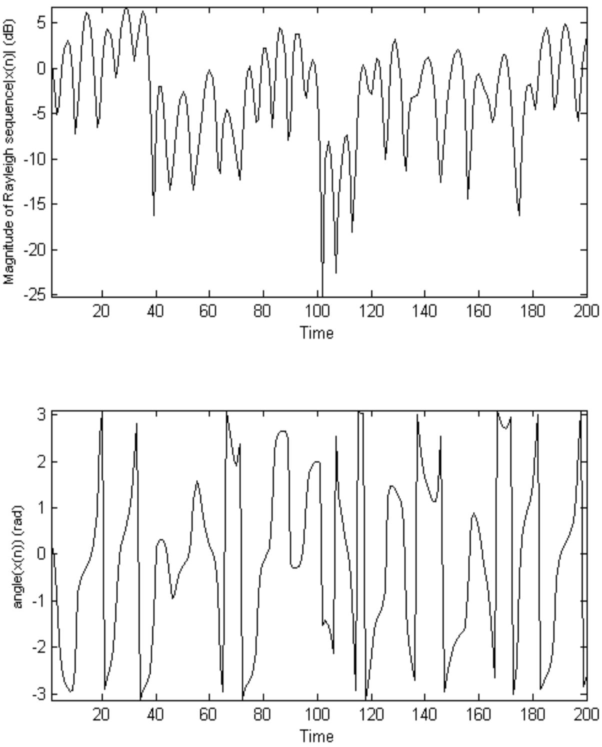

Fig .1. Sequence magnitude and phase of Rayleigh channel at $f_D = 100$ Hz .

3. Mathematical Model for Tracking Fast Time Variation of Rayleigh fading Channel

Here, we have explained the mathematical model of two algorithms as below [9]:

A. Tracking fast time variation of Rayleigh fading channel via RLS algorithm as given in equations below:

$$w_i = w_{i-1} + \frac{\lambda^{-1} \alpha P_{i-1} u_i^*}{1 + \lambda^{-1} u_i P_{i-1} u_i^*} [d(i) - u_i w_{i-1}] \quad \ldots(7)$$

$$P_i = \lambda^{-1} \left[P_{i-1} - \frac{\lambda^{-1} P_{i-1} u_i^* u_i P_{i-1}}{1 + \lambda^{-1} u_i P_{i-1} u_i^*} \right] \qquad \ldots(8)$$

B . Tracking fast time variation of Rayleigh fading channel via E-RLS algorithm as given in equations below :

$$w_i = \alpha w_{i-1} + \frac{\lambda^{-1} \alpha P_{i-1} u_i^*}{1 + \lambda^{-1} u_i P_{i-1} u_i^*} [d(i) - u_i w_{i-1}] \ldots(9)$$

$$P_i = \lambda^{-1} |\alpha|^2 \left[P_{i-1} - \frac{\lambda^{-1} P_{i-1} u_i^* u_i P_{i-1}}{1 + \lambda^{-1} u_i P_{i-1} u_i^*} \right] + qI \quad \ldots(10)$$

Where
w_i : Weight vector.
α : Some scalar value $|\alpha| \leq 1$.
λ: Some scalar value$1 \ll \lambda \leq 1$.

P_i: Covariance matrix.
u_i: Input vector
q: A constant multiple of the identity matrix I, where is more supposed to be less than unit magnitude In order to ensure the stability of the model.

And the measurements $d(i)$ satisfy

$$d(i) = u_i w_i^o + v(i) \quad \ldots(11)$$

$$w_{i+1}^o = \alpha w_i^o + n_i \quad \ldots(12)$$

Where
w_i^o: Unknown weight vector.
$v(i)$: White noise measurement with unit variance.
n_i: Disturbance.

4. Simulation Result

In this simulation program, we consider the channels is multipath Rayleigh fading and fast time variation with length 5 taps (number of simulated channels), Doppler frequency $f_D = 100$Hz, sampling frequency 1.25MHz, number of samples for each channel equal to 1000 samples. Before displaying the results of simulation, we first describe the simulated algorithms parameters in Table (1).

Table 1,
Simulation Parameters.

Parameters	Specifications
All initial values	zero
Scalar value λ	0.995
Scalar value α	0.95
Constant value q	0.1
Number of experiments	100

Figure (2), it is explained the magnitude for the 1st tap among five channels, it is illustrate three lines as described in top left of each figure. Furthermore the difference between performance of RLS and E-RLS algorithms to track fast time variation of the Rayleigh channel from start to end clearly appeared in figure (2), that means, E-RLS algorithm has a good ability to track fast variation than RLS algorithm.

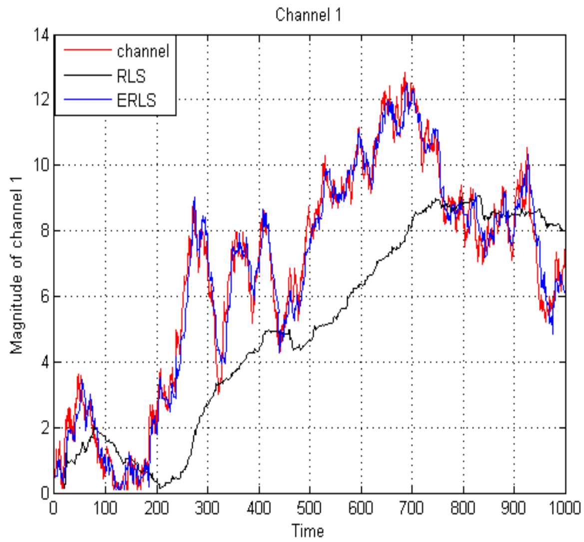

Fig. 2. Tracking performance of RLS and E-RLS algorithms for Rayleigh channel 1.

As well as, we explained tracking performance of each algorithm for all other channels as shown in Figure (3), Figure (4), Figure (5), And Figure (6), still E-RLS superiorly on RLS algorithm.

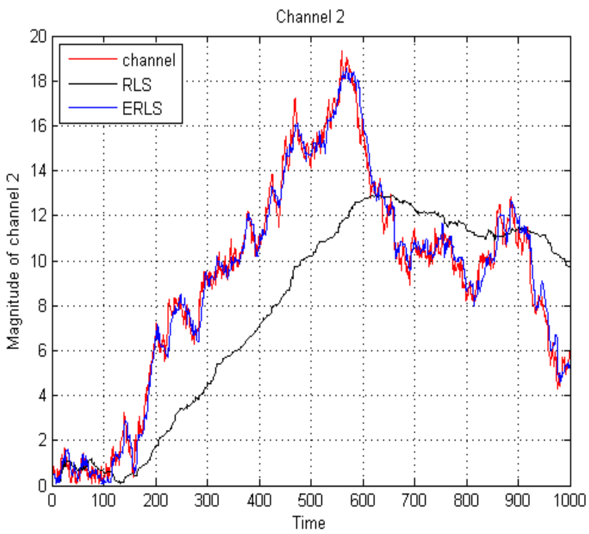

Fig. 3. Tracking performance of RLS and E-RLS algorithms for Rayleigh channel 2.

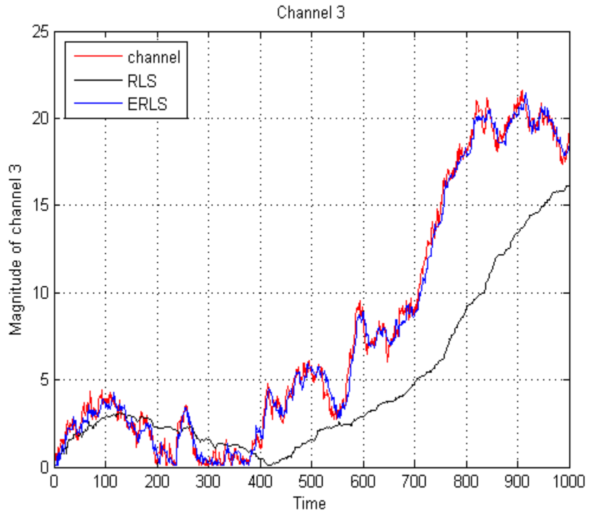

Fig. 4. Tracking performance of RLS and E-RLS algorithms for Rayleigh channel 3.

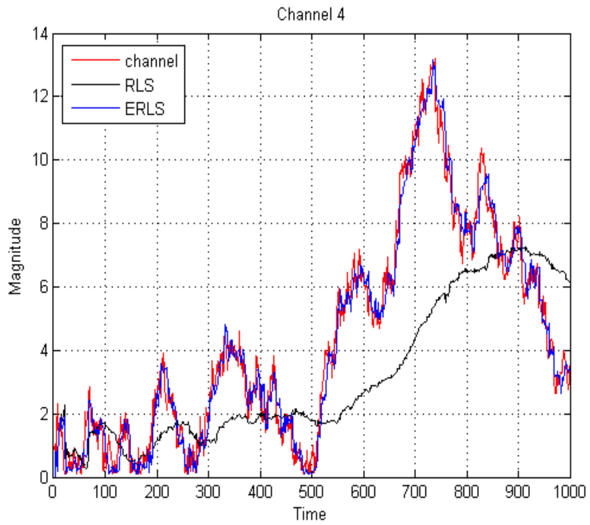

Fig. 5. Tracking performance of RLS and E-RLS algorithms for Rayleigh channel 4.

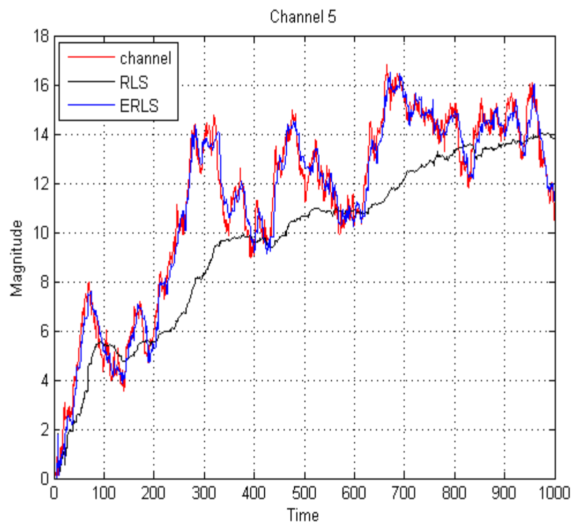

Fig. 6. Tracking performance of RLS and E-RLS algorithms for Rayleigh channel 5.

Table 2 shows comparison between the mean square error MSE values (selected randomly) for the recursive least-squares (RLS) and extended recursive least-squares (E-RLS) algorithms and the difference it is clearly appeared, MSE of the E-RLS (Blue Line) nearest to zero level than MSE of the RLS (Black Line).

Table 2,
The Comparison between Mean square error (MSE) of RLS values and E-RLS values.

MSE Sample Number	MSE-RLS Value	MSE-E-RLS Value	Smaller Value
20	6.1479	3.5628	E-RLS
200	15.0826	4.3558	E-RLS
500	16.5332	4.0328	E-RLS
800	16.9514	2.6490	E-RLS
1000	17.2023	3.3763	E-RLS

Figure (7) described mean square error (MSE) for each algorithm. The blue line represent MSE of E-RLS algorithm, and it is nearest to zero level than red line (MSE of RLS). Naturally after the results that appeared in Figure (3,4), the error ratio for E-RLS should be less than the error ratio of RLS algorithm, as shown in the mention figure.

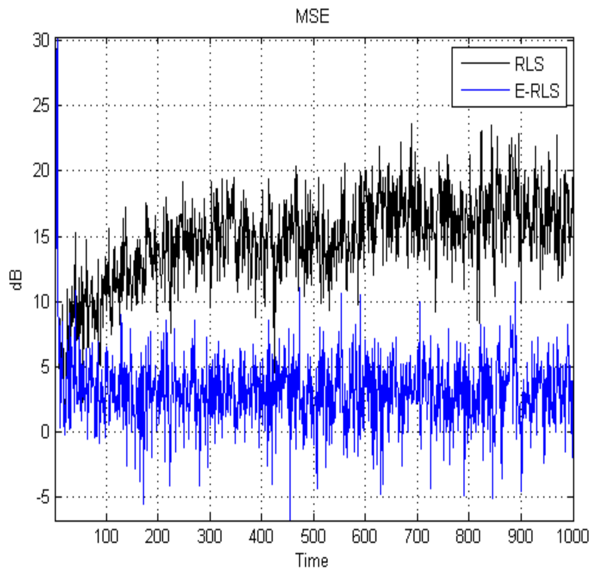

Fig. 7.Mean square error (MSE) of RLS and E-RLS algorithms.

5. Conclusion

In this paper, we have compared two Blind algorithms recursive least-squares (RLS) and Extended recursive least-squares (E-RLS), adaptive algorithms mean that, these algorithms try to estimate the channel without need any knowledge at the receiver (Blindly). Output Figures (2,3,4,5,6) in the simulation section have shown a good performance and higher ability for E-RLS than RLS to track fast time variation of Rayleigh fading channel since the E-RLS blue line more close to the original channel red line than RLS black line, as well table 2 Explain E-RLS superiority by comparing output value for each algorithms also the simulation program have been plotted in the Figure (7). The future work for this paper, apply these algorithms to track other channels, i.e, finite impulse response complex (FIR) channel or Rician fading channels to see which algorithm has a good ability to track fast amplitude variation than other.

6. References

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المقارنة بين RLS و E-RLS في تتبع الوسط الناقل المسمى رايلاي من نوع سريع التغير مع الزمن

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الخلاصة

من أجل تحديد التتبع الأمثل للناقل سريع التغير مع الزمن والمسمى قناة رايلي، هذا البحث يركز على الخوارميات (RLS) و(E-RLS) التي تقوم بتخمين الوسط الناقل من غير الاحتياج الى اية معلومات عن الوسط اي مايسمى بالتخمين الاعمى وتوصل هذا البحث إلى استنتاج مفاده E -RLS هو أكثر جدوى من RLS لما يمتلكه من دقه عالية في تتبع وتخمين الوسط الناقل سريع التغير (قناة رايلي) مع الزمن على وفق النتائج التي تم حسابها في برنامج محاكاة من تتبع الأداء ولأكثر من مرة واحدة (إرسال / استقبال) تصل إلى ١٠٠ مرة للتأكد من كفاءة هذه الخوارزميات.