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Empirical Equations for Analysis of Two-Way Reinforced Concrete Slabs

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Abstract

There are many different methods for analysis of two-way reinforced concrete slabs. The most efficient methods depend on using certain factors given in different codes of reinforced concrete design. The other ways of analysis of two-way slabs are the direct design method and the equivalent frame method. But these methods usually need a long time for analysis of the slabs.

In this paper, a new simple method has been developed to analyze the two-way slabs by using simple empirical formulae, and the results of final analysis of some examples have been compared with other different methods given in different codes of practice.

The comparison proof that this simple proposed method gives good results and it can be used in analysis of two-way slabs instead of other methods.

Keywords: *Analysis, Two-way, Reinforced concrete slabs, Empirical equations, Inflection lines.*

1. Introduction

There are many methods to estimate the values of the bending moments occur in reinforced concrete slabs, and perhaps the most common methods which depend on coefficients taken from special tables available in codes such as the method of BS Code CP110 and the method of ACI Code 63. These methods are approximate but practical and were formed in such a way that the moments are conservative because these methods neglected many important factors to obtain positive and negative bending moments by simple and fast way without complexity. The high accuracy in design calculations of structures is undesirable because there is no capability of estimating many factors affecting on design results such as live loads, material properties and methods of analysis and many other factors.

2. Analysis of Two Way Slab System

The coefficients used in different codes depend on the aspect ratio of reinforced concrete slabs, the boundary conditions at their edges (method of restrained) and the continuity or discontinuity of the edges. And to find negative and positive bending moments, these coefficients are multiplied by the load per unit area by the square of the span. So it is important for the designer to use tables to find these coefficients.

The most common manual calculation methods for calculating bending moments in reinforced concrete slabs are:-

1. Method two in ACI 63 Code (1): This method is the most common method used in design because of the simplicity in spite of that this method gives high conservative results which leads to increase in the quantity of steel reinforcement.

2. Method three in ACI 63 Code (1): This method is recommended to use by the latest ACI codes because it is more accurate than method two but it is more complicated.

3. Method one in ACI 63 Code (1): This method is seldom used in spite of it is more accurate than the above mentioned two methods because it needs more effort calculations.

4. Method of BS CP110 (3 and 6).

5. ACI Direct design method (2, 4 and 5): This method is limited so it cannot be used in many cases.

6. Equivalent frame method: This method in spite of it is accuracy and adequacy but it needs much effort and time for calculations.

3. Description of the Proposed Method

The proposed method in this paper depends on evaluating the coefficient (C) from simple empirical equation. The numbers 0.26 & 0.67 mentioned below were predicted from curve fitting to suit the other methods given by the ACI-Code and CP110. This equation is expressed as:-

$$C = 0.67\ \frac{l_1}{l} - 0.26 \qquad \ldots (1)$$

Where:-

l: is the distance between inflection lines in direction of bending moment required.
l_1: is the distance between inflection lines in direction opposite to the direction of bending moment required.

To obtain positive and negative bending moments, the equation given below has been developed

$$M = C \times B \times w \times S^2 \quad \text{(as method 1 ACI- Code 63)} \qquad \ldots (2)$$

Where:-

B: is a coefficient extracted from ACI Code (1) or BS CP110 (3 and 6) which is dedicated to find flexural bending in beams or one-way slabs as shown in Fig. (1).
w: is the total applied load (dead plus live) per unit area.
S: is the clear span in the direction of the required bending moment.

The identification of inflection lines between the positive and negative bending moments usually depends on approximate methods. It was found that the location of these inflection lines depends mainly on dimensions of spans of the neighboring panels. Also, in case of continuous panels from both sides, the ratio of $\frac{l}{L}$ or ($\frac{l_1}{L_1}$) is 0.76. While, in the case of panel continuous from one side and discontinuous from opposite side, the ratio of $\frac{l}{L}$ or ($\frac{l_1}{L_1}$) is 0.87.

Where:-

L: is the clear span in direction of required bending moment (same as S).
L_1: is the clear span in direction opposite to the required bending moment

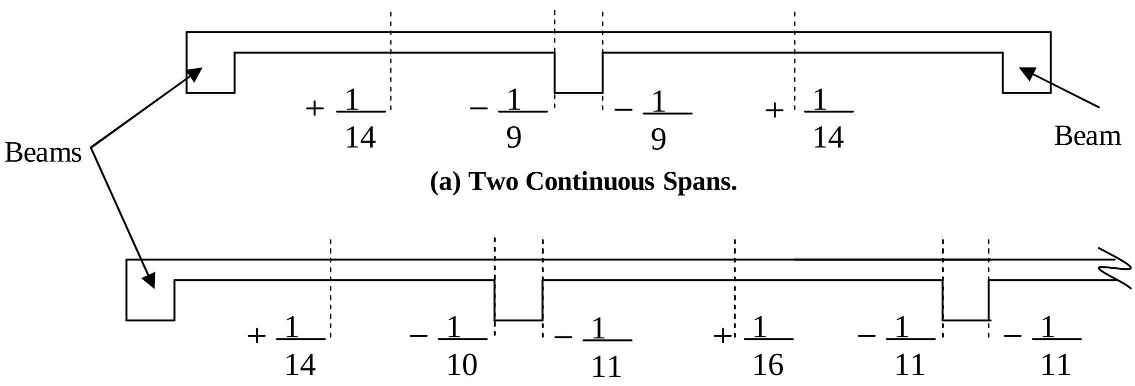
It is worth to mention that these values are accurate and acceptable if the ratio of the spans of the neighboring panels ranging between (2/3 - 3/2). Otherwise (when the ratio is beyond this range) the identification of the inflection lines may be evaluated according to the theory of structures. Fig. (2) Shows the inflection lines in panels according to ACI 63 Code (1).

Note: The values shown in Fig.(1) are not be applicable if the larger of two adjacent spans is greater than the shorter by 20%.

4. Comparison of Results with Other Methods

A comparison study was done to check the adequacy of the proposed method as it compared with the results obtained using the ACI Code (1) and the BS Standard methods (3). Tables (1, 2, 3 and 4) illustrate flexural positive and negative bending moments as functions to the applied load (w) for the proposed panels shown in Fig. (3, 4, 5 and 6) respectively. In these tables the symbols $(M^+)_x$, $(M^+)_y$, $(M^-)_x$ and $(M^-)_y$ are defined as below:-

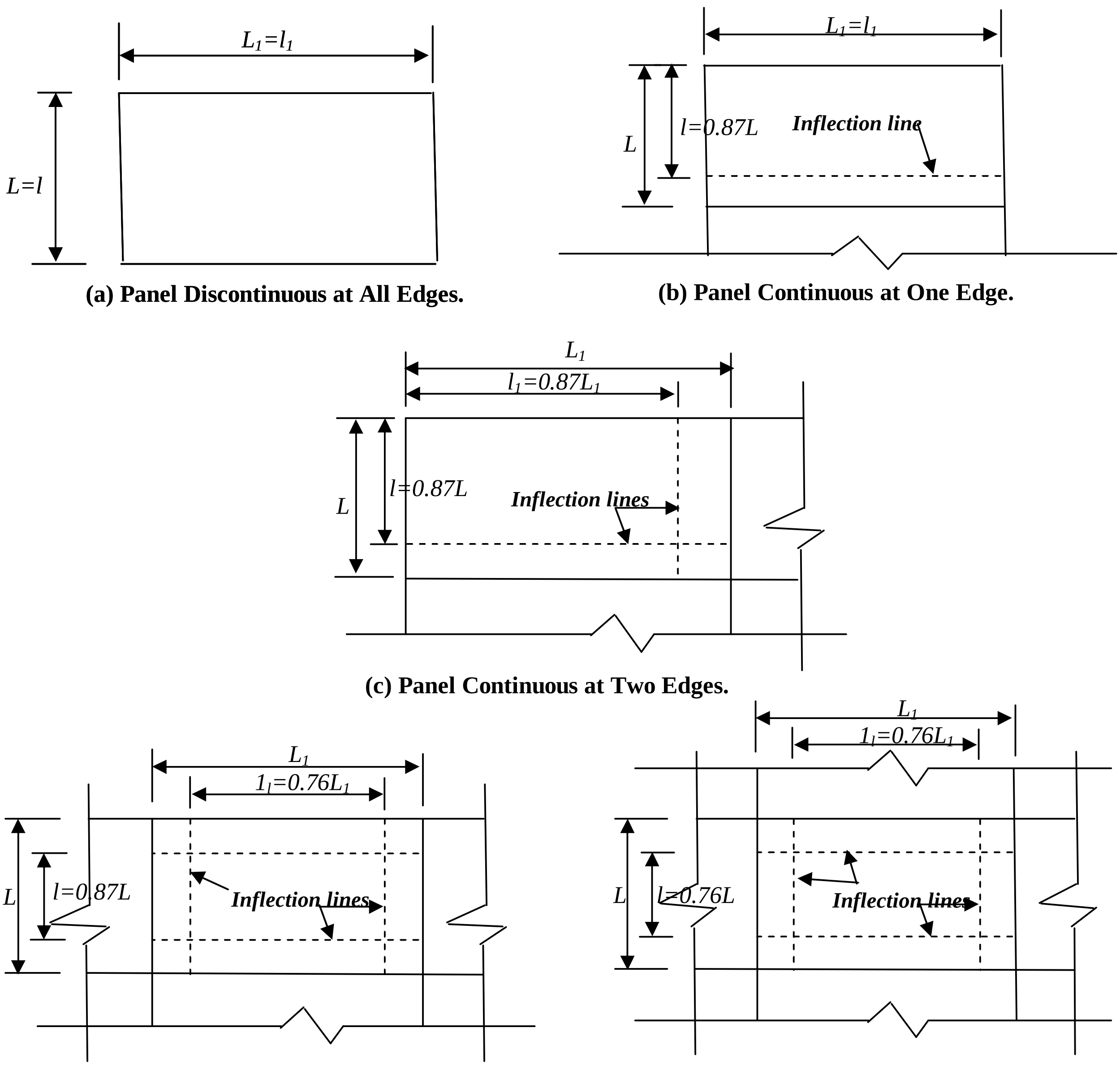
$(M^+)_x$ = Maximum positive bending moment in short direction.
$(M^-)_x$ = Maximum negative bending moment in short direction at continuous edges.
$(M^+)_y$ = Maximum positive bending moment in long direction.
$(M^-)_y$ = Maximum negative bending moment in long direction at continuous edges.



(a) Two Continuous Spans.

(b) More Than Two Continuous Spans.

Fig. 1. Values of the Coefficient (B) Used in Equation (2).



(a) Panel Discontinuous at All Edges.

(b) Panel Continuous at One Edge.

(c) Panel Continuous at Two Edges.

(d) Panel Continuous at Three Edges.

(e) Panel Continuous at All Edges.

Fig. 2. Inflection Lines in Reinforced Concrete Panels According to ACI 63 Code.

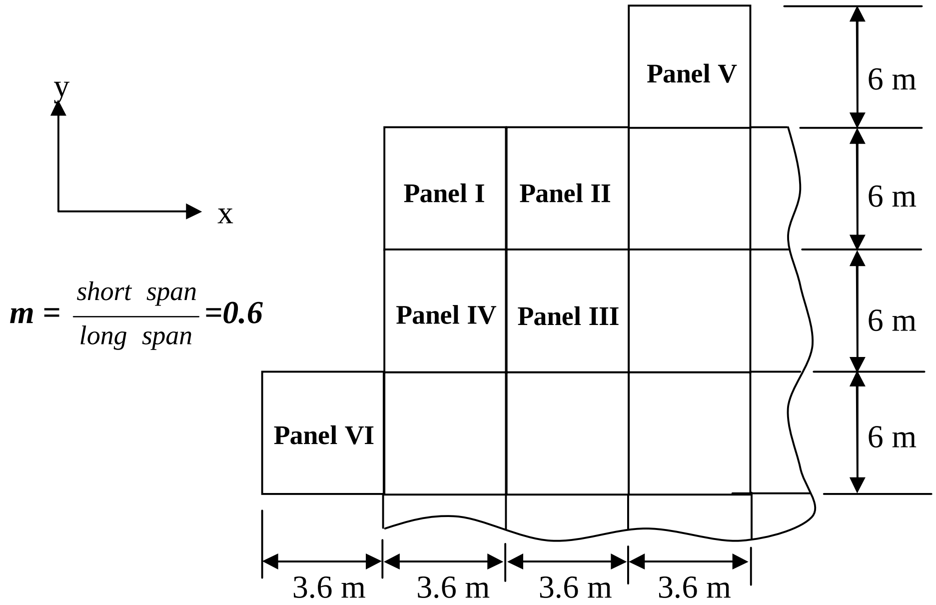


Fig. 3. Proposed Panels of m=0.6 Used for the Comparison between the Proposed Method and Other Methods.

Table 1,
Comparison among Different Methods.

Panel	Moment	BS Method	Method (II) ACI 63 Code	Method (III), ACI 63 Code			Average of Methods	Standard Deviation	Proposed Method
				Dead loads	Live loads	Average			
I	$(M^+)_x$	0.786	0.765	0.687	0.868	0.778	0.776	0.00865	0.793
	$(M^-)_x$	1.050	1.011	1.153	1.153	1.153	1.071	0.05990	1.110
	$(M^+)_y$	0.454	0.480	0.252	0.324	0.288	0.407	0.08505	0.365
	$(M^-)_y$	0.609	0.635	0.396	0.396	0.396	0.547	0.10706	0.511
II	$(M^+)_x$	0.605	0.674	0.467	0.765	0.616	0.632	0.03027	0.825
	$(M^-)_x$	0.799	0.894	1.102	1.102	1.102	0.932	0.12653	1.200
	$(M^+)_y$	0.363	0.402	0.144	0.252	0.198	0.321	0.08842	0.234
	$(M^-)_y$	0.480	0.531	0.216	0.216	0.216	0.409	0.13805	0.328
III	$(M^+)_x$	0.566	0.609	0.441	0.752	0.597	0.591	0.01812	0.694
	$(M^-)_x$	0.747	0.816	1.050	1.050	1.050	0.871	0.12967	1.009
	$(M^+)_y$	0.311	0.324	0.144	0.252	0.198	0.278	0.05658	0.320
	$(M^-)_y$	0.415	0.428	0.360	0.360	0.360	0.401	0.02947	0.465
IV	$(M^+)_x$	0.730	0.674	0.622	0.842	0.732	0.712	0.02688	0.662
	$(M^-)_x$	0.955	0.894	1.037	1.037	1.037	0.962	0.05859	0.927
	$(M^+)_y$	0.363	0.402	0.252	0.324	0.288	0.351	0.04731	0.450
	$(M^-)_y$	0.480	0.531	0.648	0.648	0.648	0.553	0.07033	0.655
V	$(M^+)_x$	1.080	0.881	0.946	0.998	0.972	0.978	0.08134	1.153
	$(M^-)_x$	-	-	-	-	-	-	-	-
	$(M^+)_y$	0.557	0.570	0.432	0.396	0.414	0.514	0.07068	0.520
	$(M^-)_y$	0.739	0.752	0.864	0.864	0.864	0.785	0.05611	0.727
VI	$(M^+)_x$	0.873	0.881	0.726	0.881	0.804	0.853	0.03457	0.947
	$(M^-)_x$	1.149	1.166	1.231	1.231	1.231	1.182	0.03534	1.326
	$(M^+)_y$	0.557	0.570	0.216	0.288	0.252	0.460	0.14694	0.404
	$(M^-)_y$	-	-	-	-	-	-	-	-

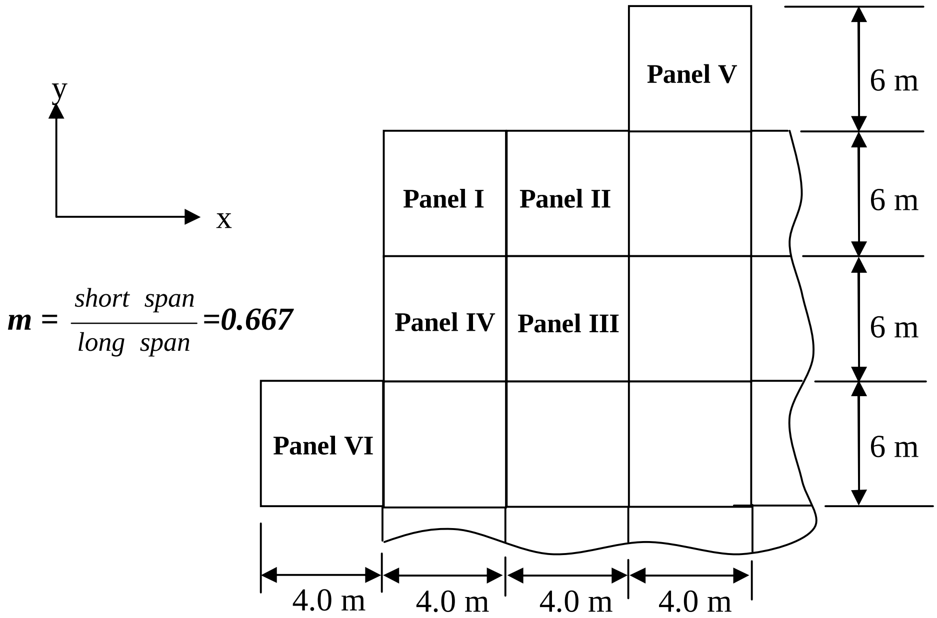


Fig. 4. Proposed Panels of m=0.667 Used for the Comparison between the Proposed Method and Other Methods.

Table 2,
Comparison among Different Methods.

Panel	Moment	BS Method	Method (II) ACI 63 Code	Method (III), ACI 63 Code			Average of Methods	Standard Deviation	Proposed Method
				Dead loads	Live loads	Average			
I	$(M^+)_x$	0.896	0.891	0.779	0.965	0.872	0.886	0.01034	0.851
	$(M^-)_x$	1.200	1.173	1.339	1.339	1.339	1.237	0.07273	1.192
	$(M^+)_y$	0.560	0.592	0.348	0.432	0.390	0.514	0.08865	0.480
	$(M^-)_y$	0.752	0.784	0.588	0.588	0.588	0.708	0.08585	0.672
II	$(M^+)_x$	0.704	0.779	0.539	0.843	0.691	0.725	0.03878	0.890
	$(M^-)_x$	0.912	1.029	1.317	1.317	1.317	1.086	0.17018	1.295
	$(M^+)_y$	0.448	0.496	0.192	0.348	0.270	0.405	0.09722	0.335
	$(M^-)_y$	0.592	0.656	0.324	0.324	0.324	0.524	0.14382	0.469
III	$(M^+)_x$	0.656	0.688	0.501	0.827	0.664	0.669	0.01360	0.745
	$(M^-)_x$	0.848	0.923	1.216	1.216	1.216	0.996	0.15878	1.084
	$(M^+)_y$	0.384	0.400	0.228	0.384	0.306	0.363	0.04106	0.420
	$(M^-)_y$	0.512	0.528	0.540	0.540	0.540	0.527	0.01147	0.611
IV	$(M^+)_x$	0.816	0.779	0.683	0.917	0.800	0.798	0.01515	0.706
	$(M^-)_x$	1.072	1.029	1.152	1.152	1.152	1.084	0.05097	0.989
	$(M^+)_y$	0.448	0.496	0.348	0.432	0.390	0.445	0.04334	0.565
	$(M^-)_y$	0.592	0.656	0.924	0.924	0.924	0.724	0.14382	0.822
V	$(M^+)_x$	1.216	1.024	1.003	1.083	1.043	1.094	0.08638	1.229
	$(M^-)_x$	-	-	-	-	-	-	-	-
	$(M^+)_y$	0.688	0.704	0.540	0.540	0.54	0.644	0.07383	0.652
	$(M^-)_y$	0.912	0.928	1.200	1.200	1.200	1.013	0.13216	0.912
VI	$(M^+)_x$	1.024	1.024	0.848	1.003	0.926	0.991	0.04620	1.023
	$(M^-)_x$	1.344	1.355	1.477	1.477	1.477	1.392	0.06027	1.432
	$(M^+)_y$	0.688	0.704	0.276	0.396	0.336	0.576	0.16983	0.579
	$(M^-)_y$	-	-	-	-	-	-	-	-

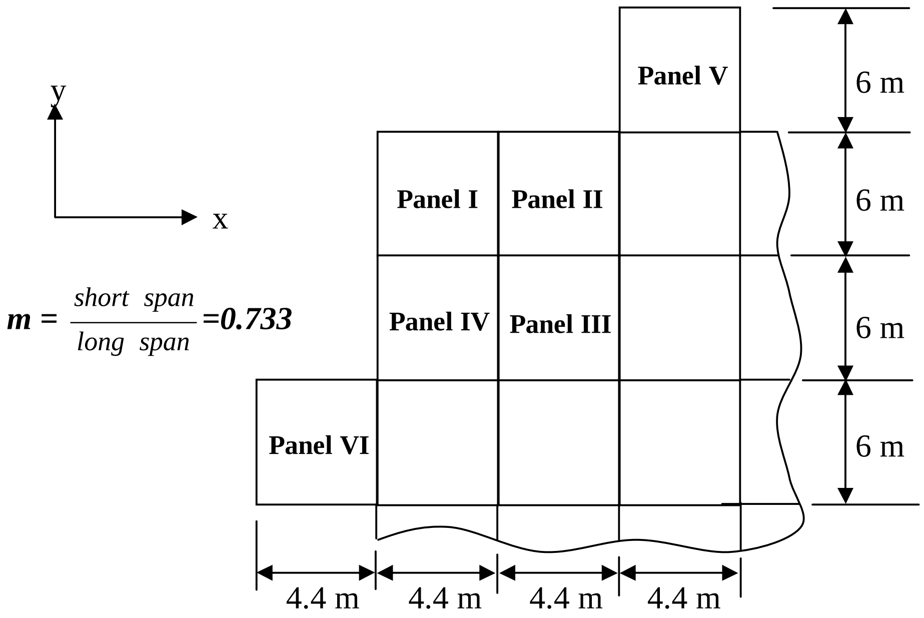


Fig. 5. Proposed Panels of m=0.733 used for the Comparison between the Proposed Method and Other Methods.

Table 3,
Comparison among Different Methods.

Panel	Moment	BS Method	Method (II) ACI 63 Code	Method (III), ACI 63 Code			Average of Methods	Standard Deviation	Proposed Method
				Dead loads	Live loads	Average			
I	$(M^+)_x$	0.998	1.007	0.852	1.039	0.946	0.984	0.02689	0.904
	$(M^-)_x$	1.332	1.330	1.504	1.504	1.504	1.389	0.08156	1.265
	$(M^+)_y$	0.678	0.716	0.444	0.552	0.498	0.631	0.09508	0.595
	$(M^-)_y$	0.910	0.949	0.804	0.804	0.804	0.888	0.06127	0.833
II	$(M^+)_x$	0.780	0.871	0.613	0.916	0.765	0.805	0.04684	0.951
	$(M^-)_x$	1.037	1.155	1.529	1.529	1.529	1.240	0.20973	1.383
	$(M^+)_y$	0.542	0.600	0.240	0.444	0.342	0.495	0.11052	0.435
	$(M^-)_y$	0.716	0.794	0.468	0.468	0.468	0.659	0.13899	0.609
III	$(M^+)_x$	0.734	0.761	0.555	0.897	0.726	0.740	0.01497	0.791
	$(M^-)_x$	0.959	1.020	1.368	1.368	1.368	1.116	0.18016	1.150
	$(M^+)_y$	0.465	0.484	0.300	0.480	0.390	0.446	0.04058	0.521
	$(M^-)_y$	0.620	0.639	0.732	0.732	0.732	0.664	0.04894	0.757
IV	$(M^+)_x$	0.889	0.871	0.723	0.981	0.852	0.871	0.01511	0.744
	$(M^-)_x$	1.177	1.155	1.226	1.226	1.226	1.186	0.02968	1.042
	$(M^+)_y$	0.542	0.600	0.444	0.552	0.498	0.547	0.04177	0.681
	$(M^-)_y$	0.716	0.794	1.212	1.212	1.212	0.907	0.21777	0.990
V	$(M^+)_x$	1.332	1.162	1.033	1.129	1.081	1.192	0.10460	1.294
	$(M^-)_x$	-	-	-	-	-	-	-	-
	$(M^+)_y$	0.832	0.852	0.684	0.684	0.684	0.789	0.07493	0.784
	$(M^-)_y$	1.104	1.123	1.512	1.512	1.512	1.246	0.18802	1.097
VI	$(M^+)_x$	1.140	1.162	0.949	1.097	1.023	1.108	0.06101	1.093
	$(M^-)_x$	1.521	1.536	1.723	1.723	1.723	1.593	0.09189	1.530
	$(M^+)_y$	0.832	0.852	0.396	0.540	0.468	0.717	0.17649	0.754
	$(M^-)_y$	-	-	-	-	-	-	-	-

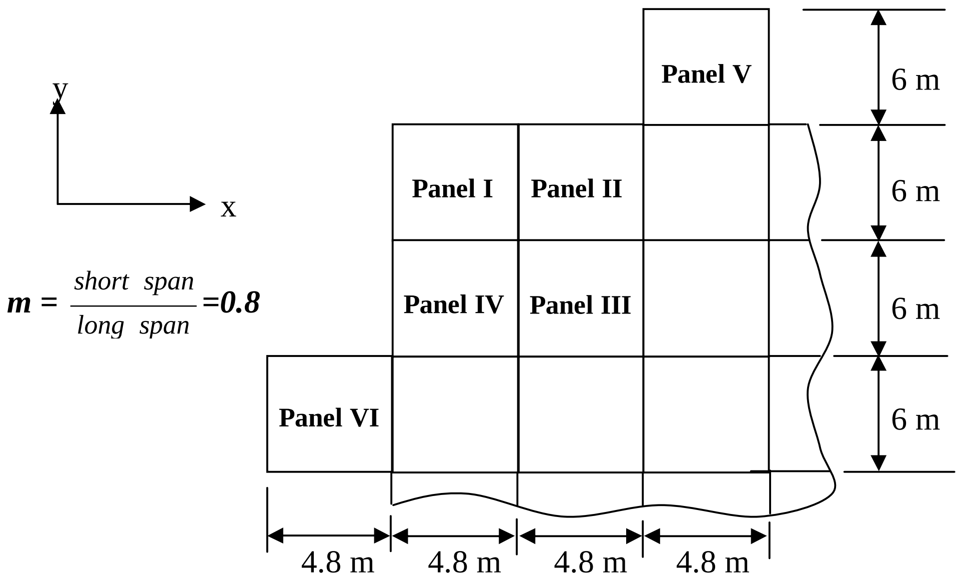


Fig. 6. Proposed Panels of m=0.8 used for the Comparison between the Proposed Method and Other Methods.

Table 4,
Comparison among Different Methods.

Panel	Moment	BS Method	Method (II) ACI 63 Code	Method (III), ACI 63 Code			Average of Methods	Standard Deviation	Proposed Method
				Dead loads	Live loads	Average			
I	$(M^+)_x$	1.083	1.106	0.899	1.106	1.003	1.064	0.04414	0.950
	$(M^-)_x$	1.440	1.475	1.636	1.636	1.636	1.517	0.08535	1.331
	$(M^+)_y$	0.806	0.852	0.576	0.720	0.648	0.769	0.08737	0.710
	$(M^-)_y$	1.083	1.129	1.044	1.044	1.044	1.085	0.03474	0.994
II	$(M^+)_x$	0.864	0.945	0.668	0.968	0.818	0.876	0.05250	1.006
	$(M^-)_x$	1.140	1.267	1.728	1.728	1.728	1.378	0.25263	1.463
	$(M^+)_y$	0.645	0.714	0.360	0.612	0.486	0.615	0.09547	0.535
	$(M^-)_y$	0.852	0.945	0.612	0.612	0.612	0.803	0.14029	0.750
III	$(M^+)_x$	0.783	0.829	0.599	0.945	0.772	0.795	0.02469	0.832
	$(M^-)_x$	1.037	1.106	1.498	1.498	1.498	1.214	0.20302	1.210
	$(M^+)_y$	0.553	0.576	0.396	0.612	0.504	0.544	0.03003	0.621
	$(M^-)_y$	0.737	0.760	0.972	0.972	0.972	0.823	0.10578	0.903
IV	$(M^+)_x$	0.956	0.945	0.737	1.014	0.876	0.926	0.03541	0.776
	$(M^-)_x$	1.256	1.267	1.267	1.267	1.267	1.263	0.00519	1.087
	$(M^+)_y$	0.645	0.714	0.540	0.684	0.612	0.657	0.04250	0.796
	$(M^-)_y$	0.852	0.945	1.476	1.476	1.476	1.091	0.27487	1.157
V	$(M^+)_x$	1.428	1.290	1.037	1.175	1.106	1.275	0.13190	1.350
	$(M^-)_x$	-	-	-	-	-	-	-	-
	$(M^+)_y$	0.991	1.014	0.792	0.828	0.810	0.938	0.09123	0.916
	$(M^-)_y$	1.313	1.336	1.836	1.836	1.836	1.495	0.24131	1.282
VI	$(M^+)_x$	1.267	1.290	1.037	1.175	1.106	1.221	0.08186	1.156
	$(M^-)_x$	1.693	1.705	1.981	1.981	1.981	1.793	0.13303	1.619
	$(M^+)_y$	0.991	1.014	0.540	0.684	0.612	0.872	0.18432	0.928
	$(M^-)_y$	-	-	-	-	-	-	-	-

Figures (7, 8, 9, 10, 11 and 12) show the relation between the aspect ratio (*m*) and the bending moments as a function to (*w*) for panels (I, II, III, IV, V and VI) respectively.

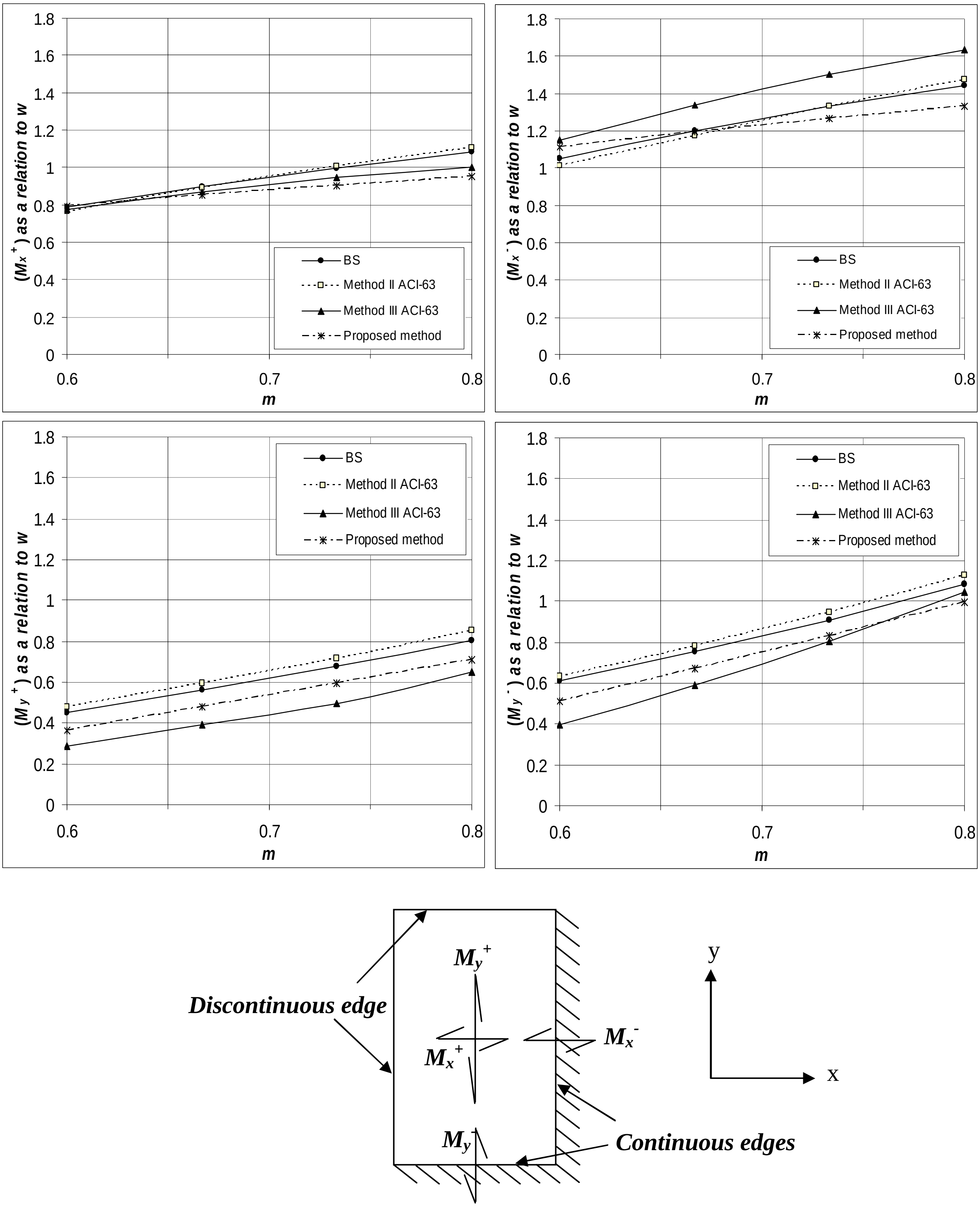


Fig. 7. Relationships between (m = short span/long span) and Bending Moments as Functions to w for Panel I.

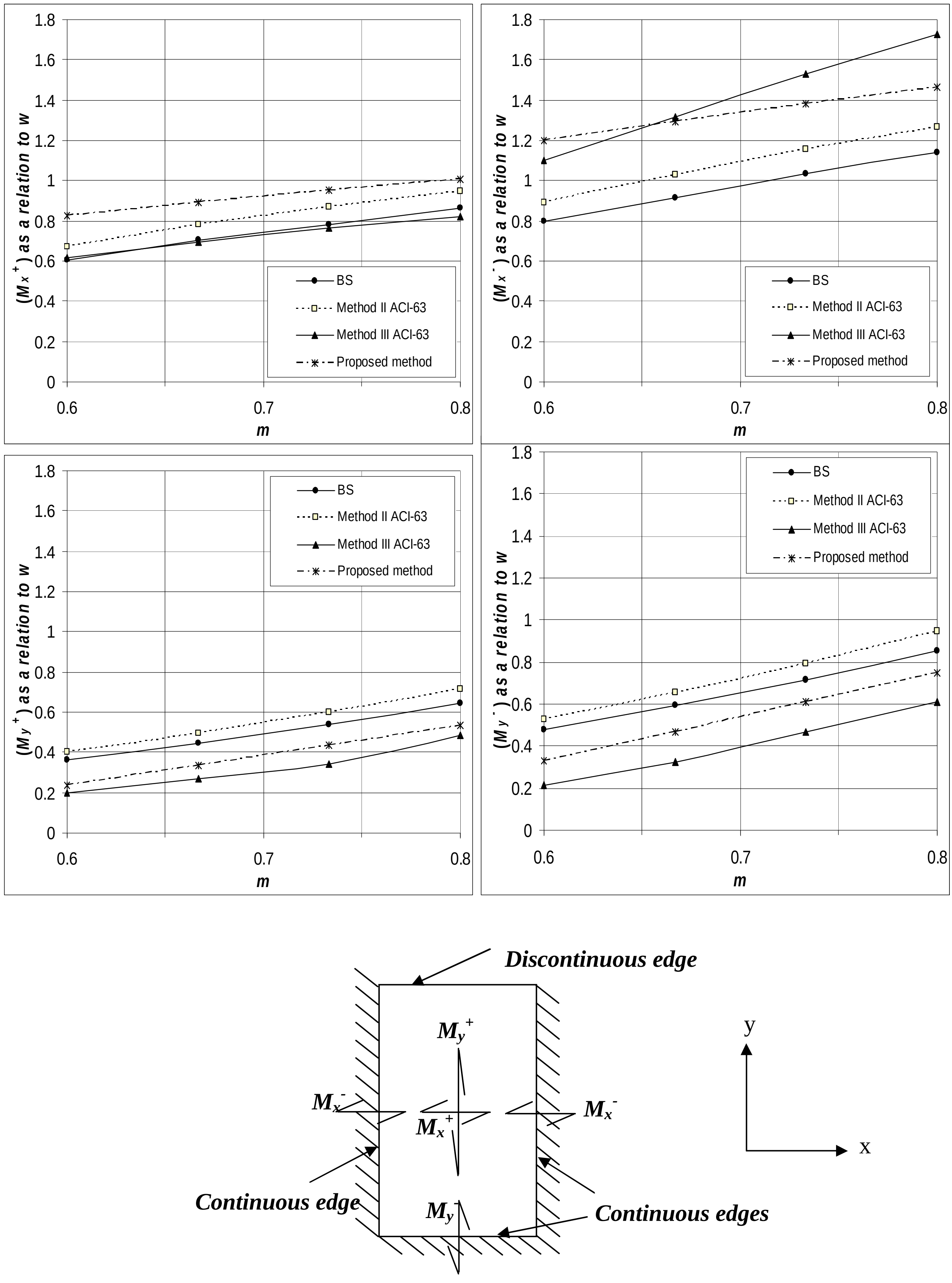


Fig. 8. Relationships between (m = short span/long span) and Bending Moments as Functions to w for panel II.

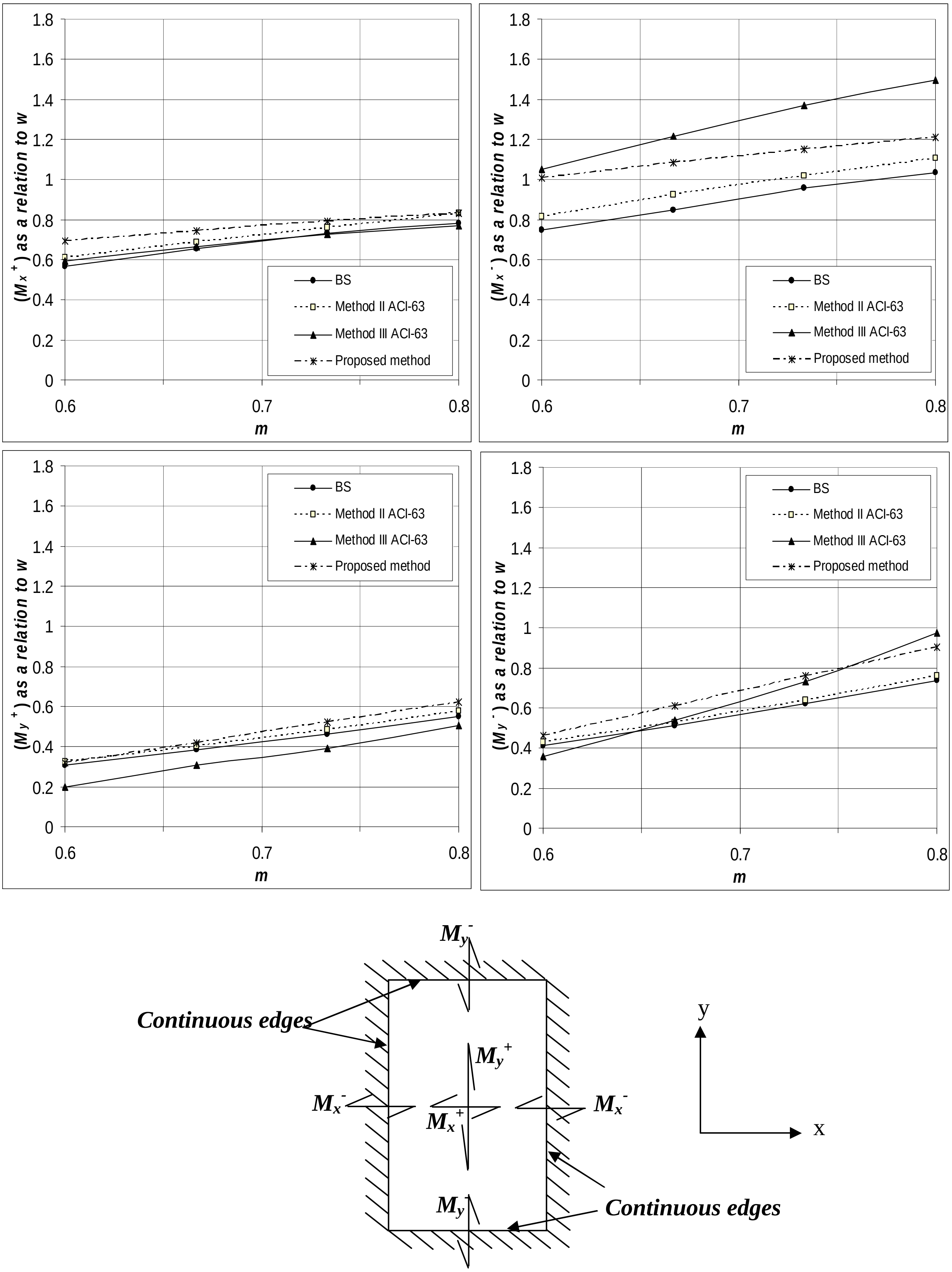


Fig. 9. Relationships between (m = short span/long span) and bending Moments as Functions to w for panel III.

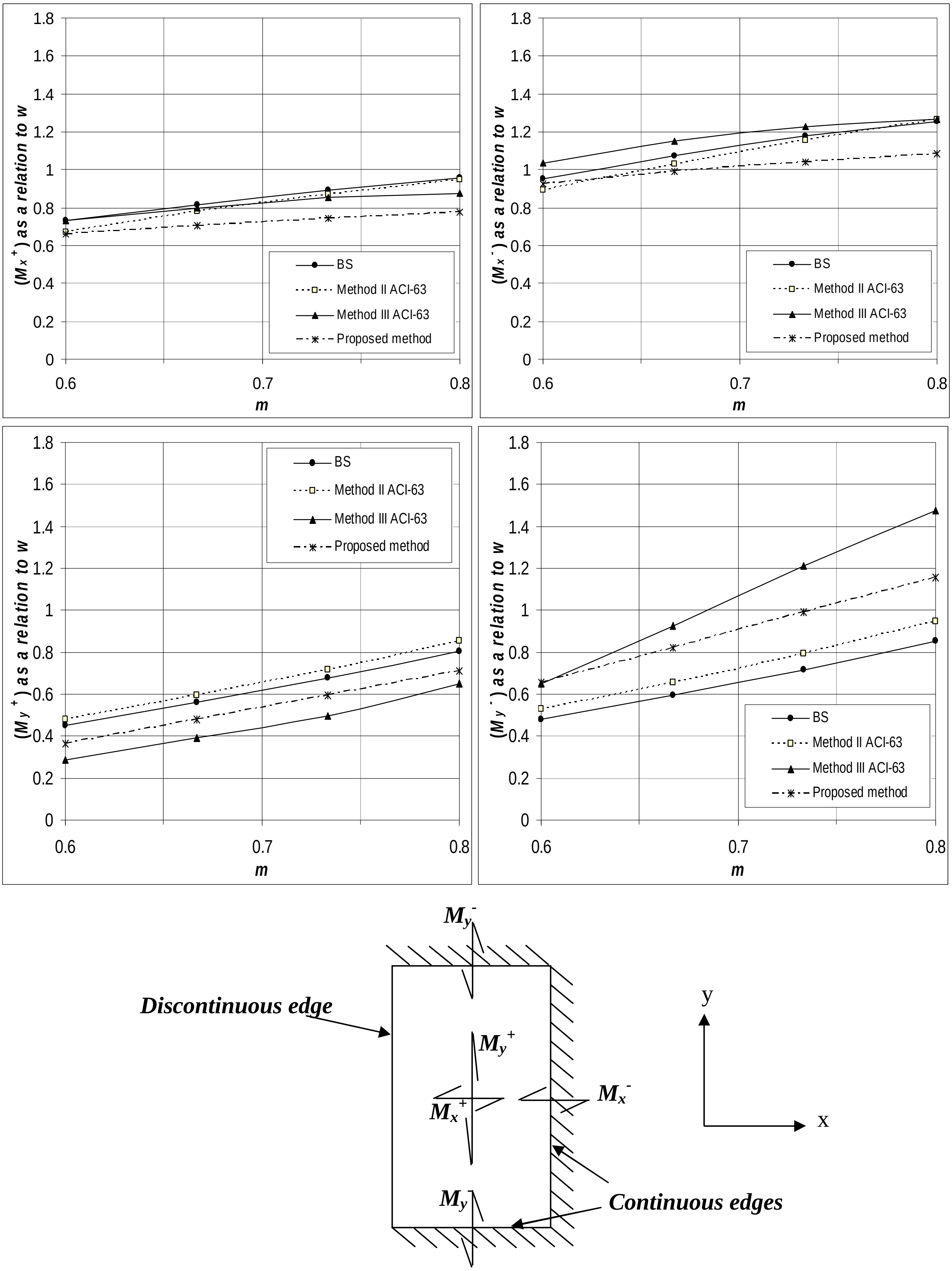


Fig. 10. Relationships between (m = short span/long span) and bending Moments as Functions to w for Panel IV.

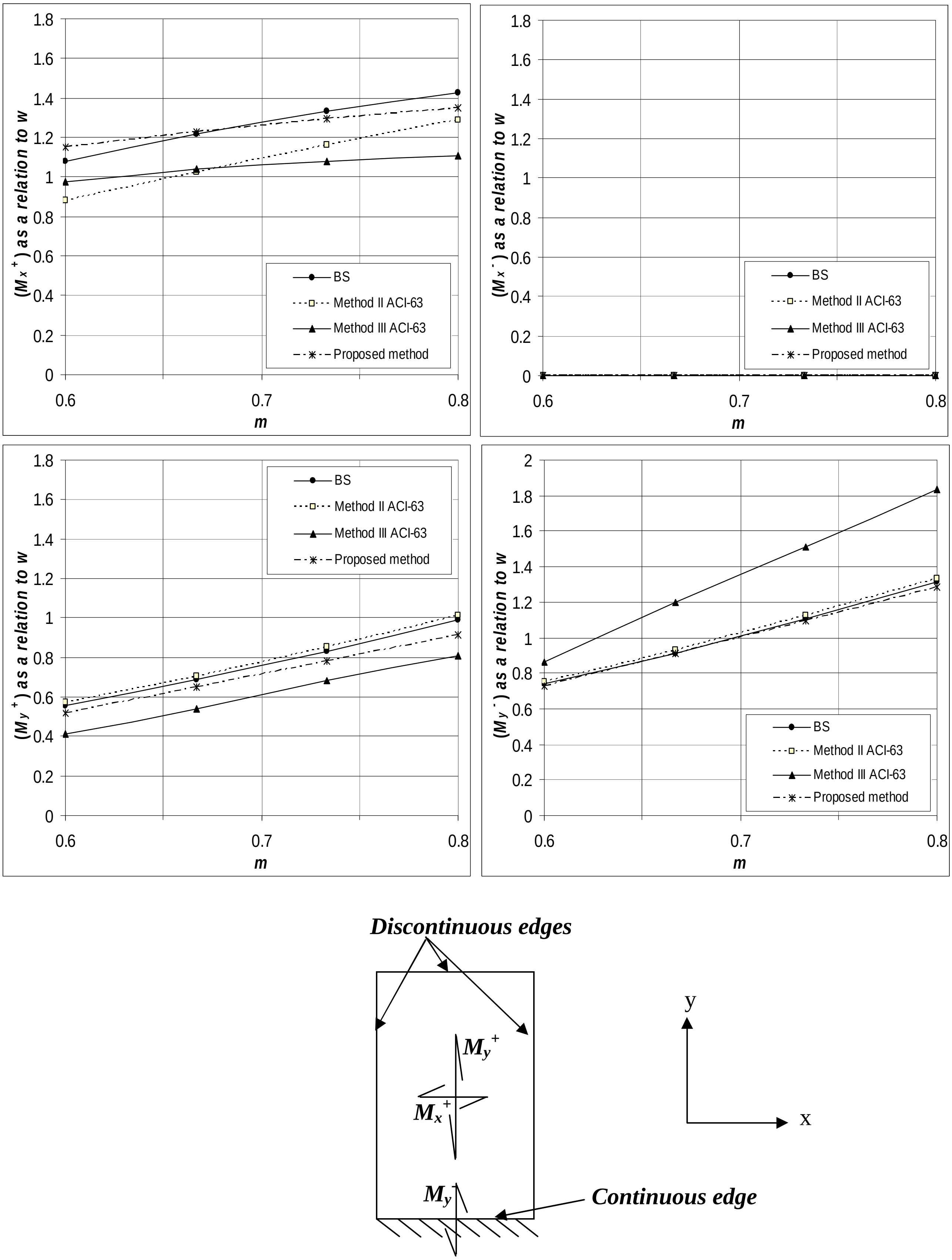


Fig. 11. Relationships between (m = short span/long span) and Bending Moments as Functions to w for Panel V.

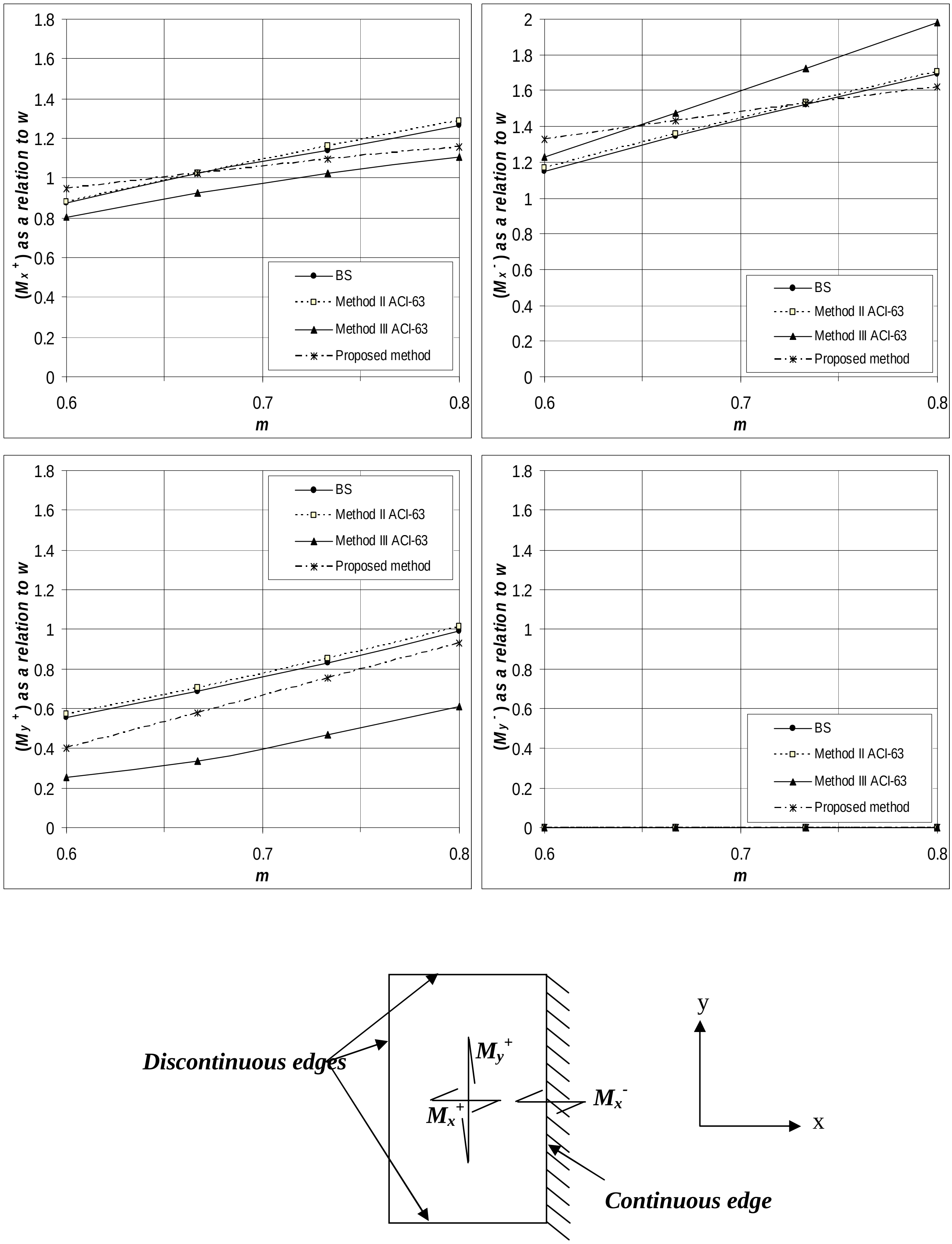


Fig. 12. Relationships between (m = short span/long span) and Bending Moments as Functions to w for Panel VI.

Example: Find the positive and negative flexural moments for panels I and II shown in Fig. (13):

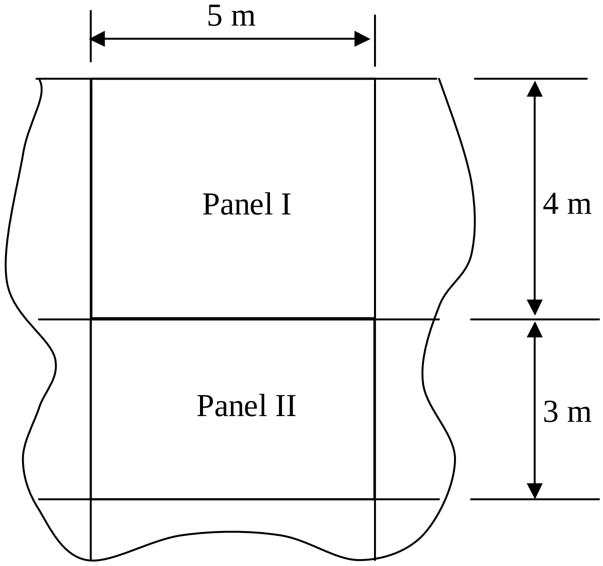


Fig. 13.

Solution

Panel I

Flexural moments in short direction

$$C = 0.67 \times \frac{5 \times 0.76}{4 \times 0.87} - 0.26 = 0.472$$

$$M^{+} = 0.472 \times \frac{1}{14} \times 4^2 \times w = 0.539w$$

$$M^{-} = 0.472 \times \frac{1}{10} \times 4^2 \times w = 0.755w$$

Flexural moments in long direction

$$C = 0.67 \times \frac{4 \times 0.87}{5 \times 0.76} - 0.26 = 0.354$$

$$M^{+} = 0.354 \times \frac{1}{16} \times 5^2 \times w = 0.552w$$

$$M^{-} = 0.354 \times \frac{1}{11} \times 5^2 \times w = 0.804w$$

Panel II

Flexural moments in short direction

$$C = 0.67 \times \frac{5 \times 0.76}{3 \times 0.76} - 0.26 = 0.857$$

$$M^{+} = 0.857 \times \frac{1}{16} \times 3^2 \times w = 0.482w$$

$$M^{-} = 0.857 \times \frac{1}{11} \times 3^2 \times w = 0.701w$$

Flexural moments in long direction

$$C = 0.67 \times \frac{3 \times 0.76}{5 \times 0.76} - 0.26 = 0.142$$

$$M^{+} = 0.142 \times \frac{1}{16} \times 5^2 \times w = 0.222w$$

$$M^{-} = 0.142 \times \frac{1}{11} \times 5^2 \times w = 0.323w$$

5. Discussion

1. In case of panel I (corner panel), the proposed method gives less value of $(M^+)_x$ & $(M^-)_x$, (maximum difference is 14.10% & 18.64% respectively) as compared with other methods for large values of *m*. While $(M^+)_y$ and $(M^-)_y$ are in good agreement with the other methods.
2. In case of panel II (discontinuous at one short edge), the proposed method gives more value of $(M^+)_x$ as compared with the other methods with maximum difference of 36.36%. While $(M^-)_x$, $(M^+)_y$ and $(M^-)_y$ obtained by proposed method are in good agreement with the other methods.
3. In case of panel IV (discontinuous at one long edge), the proposed method gives less value of $(M^+)_x$ & $(M^-)_x$ (maximum difference is 18.83% & 14.21% respectively) as compared with other methods in all values of *m*. While $(M^+)_y$ and $(M^-)_y$ are in good agreement with other methods.
4. In case of panel VI (discontinuous at three edges), the proposed method gives more value of $(M^-)_x$ for smaller value of m than the other methods by maximum difference of 17.79% , while it gives less value of $(M^-)_x$ for larger value of *m* compared other methods with maximum difference of 18.27%.

6. Recommendations

The proposed formulae presented in this paper can be used for analyzing of reinforced concrete two-way slabs supporting on beams. These slabs are square or rectangular with aspect ratio not exceeding 2. The coefficient *B* used in equation 2 can be taken from the coefficient used in ACI code for evaluating the values of flexural moment in beams or one-way slabs. If the difference between two adjacent spans exceeds by more than 20% from the shorter span, then the value of coefficient *B* can be estimated according to theory of structures as well as the inflection lines.

7. References

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معادلات وضعية لتحليل البلاطات الخرسانية المسلحة العاملة باتجاهين

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الخلاصة

تتوفر عدة طرق مثبتة في المراجع والمدونات لتحليل البلاطات الخرسانية العاملة باتجاهين. ان اغلب هذه الطرق تعتمد اما على استخدام ثوابت محددة ولحالات مختلفة من نسبة ابعاد البلاطة وطريقة تثبيتها عند الحافات. وهناك جداول لهذه الثوابت متوفرة في المدونات الشائعة الاستخدام. او تعتمد طريقه التحليل على طريقه التصميم المباشر او طريقة الهيكل المكافئ و هذه الطريقتين تحتاج على الاغلب الى وقت طويل لتحليل البلاطات.

ولاجل تبسيط حل المعادلات للبلاطات العاملة باتجاهين تم استنباط صيغ لمعادلات رياضية مبسطة. وللتاكد من صلاحية هذه المعادلات تم اجراء دراسة مقارنة للنتائج المستحصلة من هذه المعادلات مع الطرق المعتمدة في المدونات الشائعة الاستخدام والمتبعة حاليا في تحليل البلاطات العاملة باتجاهين وقد تم الحصول على نتائج جيده من هذه المقارنه مما يفسح المجال للاستفادة من هذه المعادلات في تحليل البلاطات باتجاهين بيسر وبوقت قصير دون الحاجه الى الرجوع الى الجداول الموجودة في المدونات.