A Dynamic Investment Model with Profit-Sharing in an Interest-Free Economy: Methodological Issues

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The present paper is a contribution to the building blocks of an investment model within the framework of an integrated macroeconomic model of an Islamic economy. Investment behavior in the model is guided by an Islamic-ethical value system and profit-sharing financial contracts. The typical firm's investment decision is believed to emerge from a dynamic inter-temporal maximization exercise within an infinite time horizon. The method of Calculus of Variations is applied to arrive at the optimal investment and employment criteria for the firm. The result is then incorporated into a macroeconomic model to study the behavior of key endogenous variables like national income and the rate of profit-share. Comparative statics exercised within a general equilibrium framework reveal the potency of monetary policy but the neutrality of fiscal policy with respect to output and employment.

Introduction

The past decade has witnessed a tremendous outpouring of interest as well as effort in the formalization of economic models based on profit-sharing financial arrangements as an Islamic alternative to the conventional interest-based economic system. Several macroeconomic models for interest-free economies have been proposed (Anwar 1987; Habibi 1987; Metwally 1981 & 1983). The rigor of an integrated approach to such macroeconomic modelling depends on the rigor of the component models, namely, the consumption, investment, monetary, and fiscal relationships. Economists have written extensively on different aspects of consumer behavior in Islamic societies. Kahf (1978) and Khan (1984), among others, have contributed to the conceptual and analytical formulation of the consumption function under

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the rubric of Islamic rationality. Considerable work is also evident in the discussion of the fiscal and monetary economics of Islam (Arif 1982; Ziauddin 1983). For practical reasons, the most rigorous treatment we have seen deals with the precepts and practices of interest-free banking (Siddiqi 1983; Khan and Mirakhor 1987), as researchers have sought to grapple with the conceptual and practical aspects of this model of an Islamic financial system to which several Islamic countries have pledged their commitment.

A critical component of macroeconomic models of the Islamic economy that has hitherto not received the rigorous analytical treatment it deserves is the behavioral relation governing investment in physical capital. The exercise at hand is an attempt to contribute to the building blocks of an investment model within the framework of an integrated macroeconomic model. In particular, the paper recognizes investment as a dynamic rather than a static behavior of the firm, thus making it necessary to invoke dynamic optimization procedures to derive the investment model.

Throughout, the assumption is that of an efficient capital market such that firms are able to maintain the desired level of capital stock via transactions in the capital market. A wide array of Islamic financial instruments[1] effectively helps mediate between savers and investors. Finally, it is apt to note that the conceptual basis of the ensuing model owes much to the work of Tobin in his Dynamic Aggregative Model (Tobin 1955) with a methodological approach following Sargent's (1987) recent interpretation of Tobin. Most importantly, the model rests on the foundations of an Islamic-ethical value system based on the Qur'an and the Sunnah.

Ethico-Economic Institutional Framework

At the outset, it is necessary to characterize the Islamic environment in which a firm is to pursue its production, investment, and profit-seeking activity. As is customary, we will regard the Islamic economy as a "theoretical construct" in which individuals, firms, institutions, and the state are all governed by the laws of the Islamic Shari'ah while subscribing to the ethical-spiritual value system stemming from the Qur'an and the Sunnah. Moreover, individual behavior is tempered by the pursuit of *falāḥ*—a term involving the maximization of well-being not merely in the present but in the hereafter as well. *Falāḥ*, therefore, serves as the basis for "Islamic rationality," as opposed to the Smithian notion of individual self-interest.

In addition to the above general principles, the following specific features of an Islamic economic system will be assumed to obtain:

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[1]Examples of Islamic financial instruments are: *muḍārabah, murābaḥah, muqāraḍah, mushārakah, ijārah, bay' al salam*, and so on.

(a) The institution of interest in borrowing and lending ceases to exist.
(b) Zakah is imposed at a fixed rate on all Muslims owning property above a specified minimum level. This rate is called *niṣāb*. Zakah revenues are distributed among the poor who live below the level of *niṣāb*.
(c) Private enterprise and private ownership of the means of production exist alongside possibly significant public sector activities.
(d) Wages and prices are generally determined by free market forces, albeit under the watchful eyes of *ḥisbah* that may regulate these variables to ensure equity and harmony in society. The principle of a just wage is upheld.
(e) Since savers, investors, and entrepreneurs are different entities, their pursuit of positive returns on their financial undertakings gives rise to an efficient market for financial instruments (i.e., *muḍārabah*, *mushārakah*, *muqāraḍah*).
(f) The government intervenes in the market only when private endeavors fail to achieve the desired social goals.

It is in the light of the foregoing ethico-economic environment that we shall investigate a given firm's investment behavior in an Islamic economy.

A Firm's Equilibrium Capital Stock

In keeping with the idea of an aggregate production function, it will be assumed that there exists a single aggregate commodity that is sold at a standardized price, p, which will serve as the numeraire for the valuation of the portion of output that is consumed and that which is invested (physical capital). The assumption is that capital stock in the ownership of a typical firm will be regarded as a fungible resource that can be bought or sold in a perfectly competitive and efficient capital market. In other words, capital market equilibrium implies that each firm's holding of capital stock corresponds to the desired level. Any discrepancies between the desired and the actual levels of capital stock will prompt transactions in the capital market and a consequent adjustment towards equilibrium.[2] Under these conditions of equilibrium, it will be shown that the firm equates the marginal productivity of capital to the rental or user cost of capital. The adjustment process envisaged

[2]For the sake of simplicity, the assumption here is that of zero cost of adjustment. However, it is possible to incorporate a cost of adjustment function into the model as Lucas (1967) and Treadway (1969) have done.

in the proposed model differs from the one proposed by Tobin in his formulation of the Keynesian investment schedule (Tobin 1969). There, the crucial variable driving investments was Tobin's q, the ratio of the nominal value of the firm's equities to the current replacement cost of capital.[3] Positive investments would be forthcoming for $q > 1$, but the adjustment to equilibrium is not instantaneous, for it only takes place at some finite rate per unit of time.

A Firm's Investment Behavior under Profit-Sharing

The typical loanable funds market in an Islamic financial system would operate on an interest-free basis but permit the pursuit of profits, albeit under certain ethical constraints[4] as ordained in the Qur'anic principle: "God has permitted trade but forbidden *ribā*" (2:275).

The upshot of these principles is the emergence of a market for financial instruments that recognize some form of profit-sharing arrangements between financier and entrepreneur. The most relevant of these instruments for the investment demand function are *muḍārabah, mushārakah*, and *muqāraḍah*.

Muḍārabah involves interest-free loans from commercial banks under an arrangement whereby the banks share in the profits of the enterprise at a predetermined ratio, say, $(1 - k):k$, where k is the share of profits accruing to the financier. An alternative source of funds arises from equity participation (*mushārakah*) with the bank, where both equity capital and perhaps management is shared. The banks, in turn, might float equity bonds (*muqāraḍah*) to attract investors. The consequence is the emergence of a stock market comprised of transferrable instruments whose "price and rate of return are determined by market forces" (Khan and Mirakhor 1986). The critical variable in the loanable funds market that determines the rates of return in all of the various financial instruments is the profit-sharing ratio k. More important for the purpose at hand, it serves as the critical component of the user cost of capital. Note that unlike the interest cost of capital, which is a constant, the user cost based on the profit-sharing ratio and the profit rate is subject to variability in response to changes in the profit-rate. Thus, investment decisions must be based on some expected profit-rate, which might be arrived at in the following manner:

Let the expected rate of business profits in time t be given by:

$$(1) \qquad \bar{\phi} = E_t(\phi_{t+1}) = \sum_{i=0}^{n} w_i \phi_{t-i}$$

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[3]See Tobin and Brainard (1977) for more details on q.

[4]See Sattar (1988) for a discussion on ethical constraints on profits.

where $E_t(\phi_{t+1})$ is the expected value of profits in the next period and w_i are weights that follow a geometrically declining lag function, such that:

$$\sum_{i=0}^{n} w_i = \lambda \sum_{i=0}^{n} (1-\lambda)^i = 1 \text{ as } n \Rightarrow \alpha \text{ and } 0<\lambda<1.$$

The specification (1) implies that the expected profit-rate, $\bar{\phi}$, is a weighted average of past such profit-rates with declining weights accorded to rates in the more distant past. Thus, the expected user cost of capital is given by $\alpha = k\bar{\phi}$ (the product of the profit-sharing ratio and the expected profit-rate). Ex post values of α might diverge from ex ante values as actual ϕ_t differs from $\bar{\phi}$. It has been shown by Siddiqi (1983) and others that k, the profit-sharing ratio, is determined by the demand for and supply of savings in the loanable funds market. For an enterpreneur, however, k is an agreed pre-determined rate. The ex post value of α might diverge from its ex ante value if the actual profit rate, ϕ, differs from its expected rate, $\bar{\phi}$. Hence the uncertainty in the financier's rate of return in an interest-free economy.[5]

Prices and Wages

The existence of ethical constraints on profit-making suggest certain upper bounds for prices in relation to unit or marginal costs. Profit-optimization is therefore subject to such exogenous constraints on prices. Similarly, the notion of a just wage on the grounds of equity in Islamic societies is analogous to the idea of a wage-floor, thus introducing an element of Keynesian downward wage-rigidity into the labor market of an Islamic economy. Further, the absence of money illusion ensures that the real wages of workers are protected by the proportionate adjustment of wages to price changes.

Discounting Future Values

Discounting future values is necessitated by considerations of inter-temporal economic efficiency as well as the pursuit of the Islamic objective of avoiding waste in consumption or production (the principle of *isrāf*). The fact that current investments yield a positive return makes it necessary to discount the future yields by an appropriate factor. Zarqa (1981) furnishes

[5]Note that α, the expected rate of return for the financier, is a combination of the profit-sharing ratio and the expected profit rate.

a rigorous argument for considering the expected rate of return on equities (*mushārakah*) as the appropriate discount factor. Since, in equilibrium, this rate coverages to the rate of profit-share, α, this latter coefficient serves to replace the interest rate as the discounting factor in an Islamic economy.

The Investment Model

In this section, we set out to lay the microfoundations of the given firm's investment behavior over time. It is hypothesized that the typical firm's investment decisions are brought about by an inter-temporal maximization problem within an infinite time horizon. The firm employs capital and labor, and its output at any instant in time is given by a linear homogeneous production function:

$$(2) \qquad Y(t) = F(K(t), L(t)) \qquad t \in [0,\alpha)$$

with all the standard marginal productivity conditions, including the Euler theorem, duly satisfied. We also assume a one-sector model such that output[6] and capital are both evaluated at the price of the homogeneous good, Y.

The firm's objective is to maximize its present value subject to the condition that it will have to share its net cash flow with the financier in accord with a preagreed profit-sharing ratio, k. Thus, the firm's net cash flow at time t is given by:

$$(3) \qquad (1-k)CF(t) = (1-k)[p(t)Y(t) - w(t)L(t) - p(t)\dot{K}(t)]$$

where: $p(t)Y(t)$ is the firm's revenue,
$w(t)L(t)$ is its wage bill, and
$p(t)\dot{K}(t)$ is its current expenditures on capital goods, i.e., its financial costs of adding to existing capital stock at the rate of $\dot{K}(t)$ per unit of time.[7] Thus, the firm's present value at time t_0 is given by:

$$(4) \qquad PV = \int_o^\alpha e^{-\int_o^\alpha \alpha(s)ds} (1-k)CF(t)dt$$

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[6]Introducing additional goods and making a distinction between output and capital goods does not alter the outcome of the optimization exercise, but rather adds unnecessary complexity. Hence it is avoided.

[7]Although the depreciation term is ignored, it may be noted that the standard assumption of a constant rate of depreciation has no effect on the optimal criteria for investment.

In specification (4), α is expected to vary over time, thus introducing a complexity to the computation of present values with a variable discounting factor. For the sake of simplicity and mathematical tractability, we therefore use the expected rate of profit share ($\alpha = k\bar{\phi}$) as a constant discounting factor. Then, the firm's maximization problem is reduced to:

$$\text{(5)} \qquad \underset{\substack{L(t)\\K(t)}}{\text{MAX}} \int_o^\alpha e^{-\alpha t}\,(1-k)CF(t)dt$$

or:

$$\text{(5')} \qquad \underset{\substack{L(t)\\K(t)}}{\text{MAX}} \int_o^\alpha e^{-\alpha t}\,(1-k)[p(t)F(K(t),L(t)) - w(t)L(t) - p(t)\dot{K}(t)]dt$$

Further, we will assume that given the economy's anticipated rate of inflation, π, prices are adjusted along the path: $p(t) = p\ e^{\pi t}$. Ensuring a just wage in periods of inflation requires that wages are also adjusted along the path: $w(t) = w\ e^{\pi t}$.

Incorporating these adjustments into (5′) yields[8]:

$$\text{(6)} \qquad \underset{\substack{L(t)\\K(t)}}{\text{MAX}} \int_o^\alpha e^{-(\alpha-\pi)t}\,(1-k)[pF(K(t),L(t)) - wL(t) - p\dot{K}(t)]dt$$

Addressing the above maximization problem by way of Calculus of Variatons yields the first order conditions summarized by the following set of Euler equations:

$$\text{(A)} \qquad \partial I/\partial L = 0,\ d/dt(\partial I/\partial \dot{L}) = 0 \qquad t \in [0,\alpha)$$

The above condition applied to the objective functional gives the result:

$$e^{-(\alpha-\pi)t}(1-k)\ [p(\partial F/\partial L - w] = 0$$

[8]The objective functional in this maximization or control problem corresponds to an intermediate function of the form:

$$I(K,\dot{K},L,t)$$

lending the maximization problem to be addressed by way of Calculus of Variations in order to obtain the extremum solutions. For more details, see Intrilligator (1971, chapter 12).

which then yields the optimal criteria for the employment of labor:

(7) (A) $\partial F/\partial L = F_L = w/p$

Thus, the requirements for dynamic optimality with respect to the firm's hiring decisions are identical with the static condition that firms equate the marginal productivity of labor to the real wage.

The second Euler equation, related to the capital stock, is given by:

$$\text{(B)}\quad \partial I/\partial K - d/dt(\partial I/\partial \dot{K}) = 0, \qquad t \in [0,\alpha)$$

$$\partial I/\partial K = e^{-(\alpha-\pi)t}\,(1-k)pF_K$$

$$d/dt(\partial I/\partial \dot{K}) = p(\alpha-\pi)(1-k)e^{-(\alpha-\pi)t}$$

Thus:

$$pe^{-(\alpha-\pi)t}(1-k)F_K - p(\alpha-\pi)\,(1-k)e^{-(\alpha-\pi)t} = 0$$

Therefore:

$$p(1-k)e^{-(\alpha-\pi)t}[F_K - \alpha + \pi] = 0$$

which yields the optimal investment criteria:

(8) $F_K = \alpha - \pi$

The above criteria indicates that in an interest-free economy with an equilibrating capital market, firms will attain their desired capital stock by borrowing from the capital market and investing up to the point where the marginal productivity of capital is equalized with the user cost of capital[9]: $(\alpha - \pi)$.

The rationale for such investment behavior is simple. Firms will find it profitable to borrow and invest as long as the marginal product of capital exceeds the user cost, i.e., $\dot{K}(t) > 0$ if $F_K > \alpha - \pi$. Firms will decumulate their capital stock when $F_K < \alpha - \pi$. Thus, the more fundamental relationship takes the form:

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[9]Following the work of Metwally (1983), an additional source of rigidity needs to be borne in mind—a minimum floor for α. Metwally has shown that for investments to continue, the rate of profit share must exceed the rate of zakah on idle balances (i.e., $\alpha > .025$).

$$(9) \qquad I(t) = I\left(\frac{F_K}{\alpha - \pi}\right) \qquad I' > 0$$

showing that investment is an increasing function of the marginal productivity of capital and a decreasing function of the user cost of capital.[10] In practical terms, the marginal productivity of capital is difficult to estimate. So, to obtain an estimable investment function, this argument needs to be replaced. Given the linear homogeneity of the production function $Y = F(K,L)$, we have $F_1 > 0$, $F_2 > 0$, and $F_{12} > 0$. The last inequality implies that the marginal productivity of capital is positively related to L, so that when L is high, so is F_K. But L, the employment of labor, is high precisely when output Y is high. Thus, the functional relationship (9) implies that investment is an increasing function of Y and a decreasing function of $(\alpha - \pi)$. The structural equation for the investment function may be written thus:

$$I = aY + b\,(\alpha - \pi)$$

Incorporating a constant and a stochastic term, we obtain the econometric formulation of the investment equation:

$$(10) \qquad I = a_0 + a_1 Y + a_2\,(\alpha - \pi) + \epsilon$$

$$a_0,\ a_1 > 0,\ a_2 > 0, \qquad \epsilon \simeq N(0,\sigma_\epsilon^2)$$

The following points about the preceding exercise on the derivation of an investment function in an interest-free economy are noteworthy:

(a) The marginal conditions for optimality are identical regardless of whether the firm seeks to maximize total cash flow or its own share of the cash flow. The term $(1-k)$ in the objective functional leaves no impact on the optimality criteria, as is evident from the Euler equations (A) and (B) and the subsequent derivation of the optimality criteria for employing labor or capital.

(b) It is possible to demonstrate the equivalence of maximizing the firm's present value and maximizing profits at each point in time. Indeed, they lead to the same optimality conditions, as is shown below. Central to this issue is the imputation

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[10]Alternatively, the investment function may be specified as $I(t) = I(F_K, \alpha - \pi)$, $I_1 > 0$, $I_2 < 0$.

of the cost of capital (Anwar 1987, 31) which, in an interest-free economy, must involve the use of the expected real user cost of capital, $(\alpha - \pi)$. Thus the firm's profit maximization problem at time t is given by:

$$(11) \quad \underset{\substack{L(t)\\K(t)}}{\text{MAX}}\ \Pi(t) = p(t)F(K(t), L(t)) - w(t)L(t) - (\alpha - \pi)p(t)K(t)$$

The first order conditions are:

$$\Pi_L = p(t)F_L - w(t) = 0$$

which yields $F_L = w(t)/p(t)$ [same as (7)]

Again,

$$\Pi_K = p(t)F_K - (\alpha-\pi)p(t) = 0$$

so that $F_K = \alpha-\pi$ [same as (8)]

(c) The real return on new capital, $\alpha-\pi$, will vary over time if the actual profit rate itself diverges from the long-run expected rate of profit. However, where α ($=k\bar{\phi}$) is treated by financiers as the long-run expected rate of return on investible funds, the distinction between $(\alpha-\pi)$ in the interest-free economy and $(r-\pi)$ in an interest-based economy is given by the fact that r guarantees a certain positive return on financial capital while α does not. Given that $\bar{\phi}= E(\phi(t))$, i.e., $\bar{\phi}$ is the expected rate of profit at time t, it is possible that $\phi(t) \neq \bar{\phi}$. Then $\alpha(t) \neq \alpha$. However, r(t) involves a guaranteed nominal rate of return on capital such that $r(t) = E(r(t)) = r$.

(d) Regardless of the differences between α and r, it is important to note that since α substitutes for r in an interest-free economy, it appears as the substitute argument in investment as well as in money demand functions.

The Macroeconomic Model

We now incorporate the optimality conditions (7) and (8) derived from the preceding profit-maximization exercise into a macroeconomic model following a methodology attributable to Sargent (1987):

(i) Consumption: $C = c(Y^d,\alpha)$ $0<c_1<1;\ c_2<0;\ Y^d=Y-T$

(ii) Income Identity: $Y = C + I + G$

(iii) Production Function: $Y = F(K,L)$

(iv) Equilibrium condition for employment: $F_L = w/p,\ w = \bar{w}$

(v) Equilibrium condition for investment: $F_K = \alpha - \pi$

(vi) Money Market Equilibrium: $M/p = m(\alpha,Y),\quad m_1<0,\ m_2>0$

Endogenous variables: C, I, Y, L, α, p

Exogenous variables: G, T, M, w, $\dot{\pi}$, K

Equations (i), (ii), and (iii) represent conventional specifications of the consumption function, national income identity, and the production function, except that consumption is made responsive to changes in the rate of profit-share (a rise in such a rate increases the incentive to save and to reduce consumption).

Equation (iv) incorporates the optimal employment criteria (7), but introduces the condition of wage rigidity in view of the requirement that a "just" minimum wage must be ensured through an equitable wage policy in an Islamic economy. This condition (given by $w = \bar{w}$) introduces a notion of disequilibrium in the labor market that is typically Keynesian. Equation (v) is the capital market equilibrium condition following from the optimal investment criteria (8). Equation (vi) postulates money market equilibrium with α replacing the interest rate in a conventional money demand function [following al Jarhi (1981)].

Linearizing the system of equations and presenting the endogenous variables on the left-hand side and exogenous variables on the right-hand side, we get:

(i′) $dC - c_1dY - c_2d\alpha = -c_1dT$

(ii′) $dY - dC - dI = dG$

(iii′) $dY - F_LdL = 0$

(iv′) $F_{LL}dL + (w/p^2)dp = dw/p$

(v′) $F_{KL}dL = d\alpha - d\pi$

(vi′) $m_\alpha d\alpha + m_YdY + (M/p^2)dp = dM/p$

[See Appendix A for a compact presentation of the above system and its solution].

To study the behavior of the system and analyze the impact of policies, we depict the model graphically, first, to study the effect of fiscal policies and, next, to analyze the effects of monetary policies. This will be done, first, in a partial equilibrium framework followed by a general equilibrium analysis to enable a comparison of results.

First, capital market equilibrium is analyzed as follows: Using the investment function:

(12) $$I = I(F_K, \alpha - \pi), \qquad I_1 > 0, \qquad I_2 > 0$$

and taking total derivatives, we obtain:

(13) $$dI = I_1 F_{KL} dL + I_2 d\alpha - I_2 d\pi$$

Using the condition $dI = 0$ for capital market equilibrium (CME), and equation (iii′):

(14) $$0 = I_1 (F_{KL}/F_L)\, dY + I_2 d\alpha - I_2 d\pi$$

which yields:

(15) $$\left.\frac{d\alpha}{dY}\right|_{CME} = \frac{-I_1}{I_2} (F_{KL}/F_L) > 0$$

Using equations (i′) and (ii′) coupled with (13) yields the result for goods market equilibrium (GME) [see Appendix B for details]:

(16) $$d\alpha = 1/\eta\ [1 - c_1 - I_1 (F_{KL}/F_L)] dY + 1/\eta\ [c_1 dT + {} + I_2 d\pi - dG]; \qquad \eta = c_2 + I_2$$

(17) $$\left.\frac{d\alpha}{dY}\right|_{GME} = 1/\eta\ [1 - c_1 - I_1\ (F_{KL}/F_L)] < 0$$

Figure 1 describes the interaction of the goods and the capital market.[11]

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[11]Detailed comparative statics results within a general equilibrium framework may be obtained from the reduced form equations for key endogenous variables. This has been done subsequently.

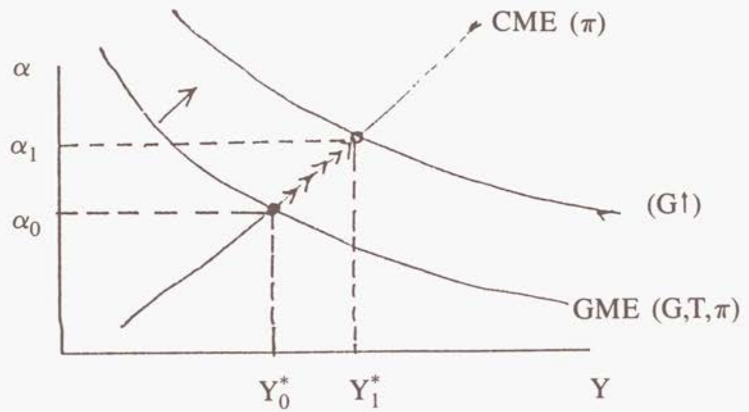

Figure 1

Thus, in a partial equilibrium analysis of the goods and capital markets, fiscal policies appear to have the conventional impact on output, while expansionary fiscal policies help to raise the rate of profit-share in the economy and contractionary policies have the opposite effect. This result is not difficult to imagine if we do not consider the money market: expansionary fiscal policies create excess demands in the goods market raising the profitability of investments. Any disequilibrium in the capital market is then resolved through a rise in the rate of profit-share, α.

To study the behavior in the money market, we postulate money market equilibrium (MME) through appropriate substitutions in equations (iii′) – (vi′). We obtain these results [see Mathematical Appendix C for details]:

$$(18) \qquad d\alpha = 1/m_\alpha[dM/p - \frac{M}{p}\frac{dw}{w} + (\frac{M}{p}\frac{F_{LL}}{F_L^2} - m_Y)\, dY]$$

Hence:

$$(19) \qquad \left.\frac{d\alpha}{dY}\right|_{MME} = 1/m_\alpha\ (\ \frac{M}{p}\ \frac{F_{LL}}{F_L^2} - m_Y\ ) > 0$$

$m_\alpha < 0,\ m_Y > 0,\ F_{LL} < 0$

Figure 2 describes the interaction of the money and capital markets:

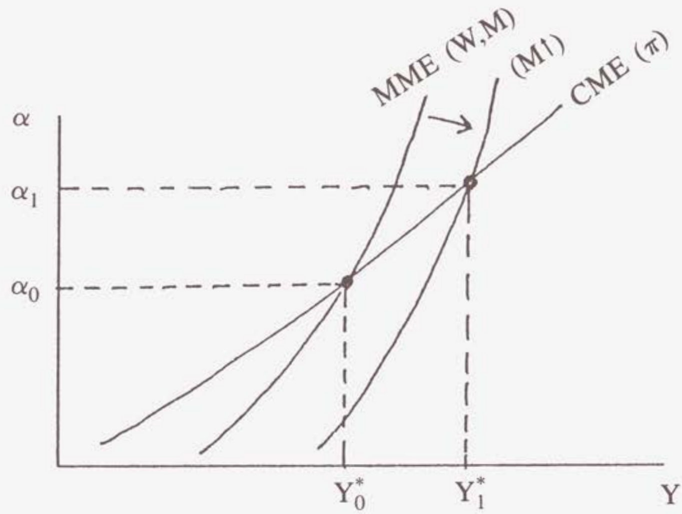

Figure 2

Since the slope of the CME and MME curves are both positive, a necessary condition for the stability of equilibrium requires that MME have a steeper slope than CME. It is demonstrated that monetary expansion, such as fiscal expansion, has output effects but that capital market equilibrium is maintained at a higher level of profit-share. Thus, the separate treatment of the interaction between the goods and the money markets with the capital market generate the outcome that fiscal and monetary expansion both raise the rate of profit-share by raising the profitability of investments as an aggregate demand and output both rise.

A general equilibrium analysis would be difficult to present graphically. Therefore, the comparative statics results will be derived from the reduced form solutions for the key endogenous variables, namely, Y and α.

Thus, substitution of (18) and (iii′) into (v′) yields reduced form for Y:

$$(20)\quad \left[\frac{F_{KL}}{F_L} - \frac{M}{p}\cdot\frac{F_{LL}}{F_L^2}\cdot\frac{1}{m_\alpha} + \frac{m_Y}{m_\alpha}\right] dY = \frac{1}{m_\alpha}\left[\frac{dM}{p} - \frac{M}{p}\cdot\frac{dw}{w}\right] - \pi$$

and:

(21) $$dY = \frac{1}{Sm_\alpha}\left[\frac{dM}{p} - \frac{M}{p}\cdot\frac{dw}{w}\right] - \frac{d\pi}{S}$$

$$S = \left[\frac{F_{KL}}{F} - \frac{M}{p}\cdot\frac{F_{LL}}{F_L^2}\cdot\frac{1}{m_\alpha} + \frac{m_Y}{m_\alpha}\right] \qquad \text{and } S < 0$$

The above produces the following comparative statics results:

$dY/dM > 0$

$dY/dw < 0$

$dY/d\pi > 0$

Notice that dY/dG, $dY/dT = 0$ implying that output changes are not responsive to fiscal policy. Only monetary shocks have output effects.

Substituting (21) into (18) yields the reduced form for $d\alpha$:

$$d\alpha = \frac{1}{m_\alpha}\left[\frac{dM}{p} - \frac{M}{p}\cdot\frac{dw}{w}\right] + \frac{1}{m_\alpha}\left(\frac{M}{p}\cdot\frac{F_{LL}}{F_L^2} - m_Y\right)$$

$$\left\{\frac{1}{Sm_\alpha}\left[\frac{dM}{p} - \frac{M}{p}\cdot\frac{dw}{w}\right] - \frac{d\pi}{S}\right\}$$

The comparative statics are:

$d\alpha/dM$ ambiguous

$d\alpha/dw < 0$

$d\alpha/d\pi > 0$

and $d\alpha/dG$, $d\alpha/dT = 0$

Again the neutrality of fiscal policy with respect to α is established, although the impact of monetary shocks on α appears ambiguous. In sum, it appears that the general equilibrium framework presents a comparative statics outcome that contrasts with the previous partial equilibrium analysis. The interesting result that emerges here is that of fiscal neutrality with respect to output and the rate of profit-share. On the contrary, the non-neutrality of monetary shocks has been demonstrated. We conclude, therefore, that in an Islamic economy based on profit-sharing financial arrangements, monetary policy would be a more potent instrument of stabilization than fiscal policy, provided that the assumptions of an efficient capital market hold.

The above result makes ample sense in view of the unique constraints faced in the area of fiscal management in an Islamic economy due to the absence of bond-financing. The choice for the fiscal authority, therefore, lies between balanced budgets or monetary financing of the deficit. The latter, clearly, is indistinguishable from pure monetary expansion. Thus, inflationary consequences notwithstanding, monetary policy remains the only potent instrument of stabilization. In the prevailing circumstances, pursuing a policy of noninflationary growth would require observing a monetary rule in the Friedmanian tradition while calling for strict limits on discretionary policy.

Conclusion

The present paper has presented an initial framework for an investment model—a key building block in an integrated macroeconomic model of an Islamic economic system. The key variable introduced in investment analysis is the rate of profit-share, which turns out to be a critical variable determining investment criteria. Moreover, as investment behavior is governed by expected profitability and demand growth over the long term, the dynamic optimization exercise adopted here appears to be a more realistic basis for studying the microfoundations of investment behavior. An econometric formulation of the investment model has also been proposed that could lend itself to an estimation of the availability of an adequate amount of cross-section or time-series data.

The macroeconomic model proposed has both classical and Keynesian attributes. Therefore, the indications regarding the effectiveness of activist stabilization policies are mixed. The main policy implications of the present exercise are essentially twofold: first, the neutrality of fiscal policy with respect to output, investment, and the rate of profit-share has been demonstrated, as has the greater potency of monetary policy as an instrument of economic stabilization. However, the pursuit of noninflationary growth would call for strict limits on the scope for discretionary policy. Second, the impact of fiscal or monetary policies on the rate of profit-share is either neutral or ambiguous. Prudence, therefore, dictates the need to avoid using the rate of profit-share either as an instrument or as a target variable for stabilization policies, since such a strategy might contribute to the volatility of investments without the benefit of higher output or productivity.

A related but critical issue that emerges is whether stabilization policies should be actively used in an Islamic economic system. The answer to this question can only be obtained after further investigation of the long-term relations between real and nominal variables in the system, the nature of interdependence between the component markets, formation of profit and other expectations in response to monetary and fiscal policies, and the desirability, for instance, of fulfilling one goal (employment) at the expense of another (investment stability). The present study attempts to provide only the preliminary answers to these broader questions.

Mathematical Appendices

Appendix A

The linearized system of equations can be presented compactly in matrix form:

$$\begin{pmatrix} 1 & 0 & -c_1 & 0 & -c_2 & 0 \\ -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -F_L & 0 & 0 \\ 0 & 0 & 0 & F_{LL} & 0 & w/p^2 \\ 0 & 0 & 0 & F_{KL} & -1 & 0 \\ 0 & 0 & m_Y & 0 & m_\alpha & M/p^2 \end{pmatrix} \begin{pmatrix} dC \\ dI \\ dY \\ dL \\ d\alpha \\ dp \end{pmatrix} = \begin{pmatrix} -c_1 dT \\ dG \\ 0 \\ dw/p \\ -d\pi \\ dM/p \end{pmatrix}$$

Inspection of the 6x6 matrix above shows the presence of a 4x2 null matrix in the bottom left-hand corner implying the characteristic of "block recursivity" of the model (a typically classical attribute). By implication, the subset of equations (iii) - (vi) can then independently determine the values of Y, L, α, and p. Substituting the value of Y in (i) solves C. I is then determined in (ii).

Appendix B

Derivation of goods market equilibrium is as follows:
Given:

(i′) $dC - c_1 dY - c_2 d\alpha = -c'dT$

(ii′) $dY - dC - dI = dG$

Substituting (i′) into (ii′):

$$dY - c_1 dY + c_1 dT - c_2 d\alpha - dI = dG$$

Using (iii′) and (13), and substituting for dI:

$$dY - c_1dY - c_2d\alpha + c_1dT - I_1\frac{F_K}{F_L}dY - I_2d\alpha + I_2d\pi = dG$$

$$[\ 1 - c_1 - I_1\frac{F_{KL}}{F_L}\ ]\ dY + c_1dT + I_2d\pi - dG = I_2d\alpha + c_2d\alpha$$

$$d\alpha = \frac{1}{\eta}[\ 1 - c_1 - I_1\frac{F_{KL}}{F_L}\ ]\ dY + \frac{1}{\eta}[c_1dT + I_2d\pi - dG\ ];$$

$$\eta = c_2 + I_2$$

Thus: $$\left.\frac{d\alpha}{dY}\right|_{GME} = \frac{1}{\eta}[\ 1 - c_1 - I_1\frac{F_{KL}}{F_L}] < 0$$

Appendix C

To obtain money market equilibrium, first collapse equations (iii′) to (vi′) into a system of two equations in $d\alpha$ and dY. Equations (iii′) and (iv′) then yield:

$$(w/p^2)\ dp = dw/p - F_{LL}dL$$

$$dp/p = dw/w - (p/w)F_{LL}dL$$

$$= dw/w - \frac{F_{LL}}{F_L}dL \qquad (F_L = w/p)$$

$$dp/p = dw/w - \frac{F_{LL}}{F_L^2}dY$$

Substituting into (vi′):

$$m_\alpha d\alpha + m_YdY + \frac{M}{p}(\frac{dw}{w} - \frac{F_{LL}}{F_L^2}dY) = \frac{dM}{p}$$

$$d\alpha = \frac{1}{m}[\frac{dM}{_\alpha p} - \frac{M}{p}\cdot\frac{dw}{w} + (\frac{M}{p}\cdot\frac{F_{LL}}{F_L^2} - m_Y)\ dY\ ]$$

$$\left.\frac{d\alpha}{dY}\right|_{MME} = \frac{1}{m_\alpha}(\frac{M}{p}\cdot\frac{F_{LL}}{F_L^2} - m_Y) > 0$$

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