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Common fixed points for generalized ψ-contractions in weak non-Archimedean fuzzy metric spaces

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Abstract

Fixed point theory in fuzzy metric spaces plays very important role in theory of nonlinear problems in applied science. In this paper, we prove an existence result of common fixed point of four nonlinear mappings satisfying a new type of contractive condition in a generalized fuzzy metric space, called weak non-Archimedean fuzzy metric space. Our main results can be applied to solve the existence of solutions of nonlinear equations in fuzzy metric spaces. Some examples supporting our main theorem are also given. Our results improve and generalize some recent results contained in Vetro (2011) [16] to generalized contractive conditions under some suitable conditions and many known results in the literature.

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1. Introduction

Fixed point theory in fuzzy metric spaces plays an important role in studying theory of equations in a fuzzy metric space. It can be applied to solve the existence problems of nonlinear equations in fuzzy metric spaces (see [2]-[6], [14]). Zadeh [17] was the first who introduced the concept of fuzzy sets. Since then, the concept of fuzzy metric spaces was introduced and study by many authors in different ways [15]-[16]. In 1974, George and Veeramani [2], [3] defined a Hausdorff topology in fuzzy metric spaces which improve from Kramosil and Michalek [9]. Especially, in 2008, Mihet [12] proved a fixed point theorem for fuzzy ψ-contractive mappings in complete non-Archimedean fuzzy metric spaces.

In 2011, Vetro [16] introduced a notion of a weak non-Archimedean fuzzy metric space which induces a Hausdorff topology and proved some common fixed point theorems for a pair of generalized contractive type mappings in this space. Recently, Martinez-Moreno [11] and others ([13],[6]) proved some common fixed point theorems for two contractive mappings with the $CLRg$-property in fuzzy metric spaces and other spaces.

In this paper, we aim to prove a common fixed point result of four nonlinear mappings satisfying a new type of contractive condition and some compatible conditions in weak non-Archimedean fuzzy metric spaces. The new type of contractive condition introduced in this paper is a new concept and it is suitable for studying a common fixed point problem of four nonlinear mappings while the previous ones cannot be used to study our problem because they can be used to study a common fixed point of only two nonlinear mappings. Our main result can be applied to study existence problem of nonlinear equations in weak non-Archimedean fuzzy metric spaces. Our results extend and generalize many results in literature, especially, Vetro [16], Mihet [12] and others. Also, we give some examples to illustrate our main results.

2. Preliminaries

In this section, we recall some notion and basic useful definitions.

Definition 2.1 ([15])**.** A binary operation $*: [0,1]\times[0,1]\to[0,1]$ is called a *t-norm* if it satisfies the following conditions:

(TN1) $*$ is associative and commutative;

(TN2) $a*1=a$ for any $a\in[0,1]$;

(TN3) $a*b\le c*d$ whenever $a\le c$ and $b\le d$ for all $a,b,c,d\in[0,1]$.

If $*$ is continuous, then $*$ is called the *continuous t-norm.* The following are some examples of a continuous t-norm; $a*b=\min\{a,b\}$, $a*b=ab/\max\{a,b,\lambda\}$, where $\lambda\in(0,1)$, $a*b=\max\{a+b-1,0\}$.

Definition 2.2 ([16])**.** A *fuzzy metric space* is a triple $(X,M,*)$, where X is a nonempty set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2\times[0,\infty)$, satisfying the following properties:

(FM1) $M(x,y,0)=0$ for all $x,y\in X$;

(FM2) $M(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$;
(FM3) $M(x, y, t) = M(y, x, t)$ for all $x, y \in X$ and $t > 0$;
(FM4) $M(x, y, \cdot) : [0, \infty) \to [0, 1]$ is left continuous for all $x, y \in X$;
(FM5) $M(x, z, t + s) \geq M(x, y, t) * M(y, z, s)$ for all $x, y, z \in X$ and $t, s > 0$.

From the above definition, if (FM5) is replaced by the following:
(NA) $M(x, z, \max\{t, s\}) \geq M(x, y, t) * M(y, z, s)$ for all $x, y \in X$ and $t, x > 0$,
then $(X, M, *)$ is called a *non-Archimedean fuzzy space*. Obviously, every non-Archimedean fuzzy metric space is itself a fuzzy metric space.

Definition 2.3 ([16])**.** A *weak non-Archimedean fuzzy metric space* is a triple $(X, M, *)$, where X is a nonempty set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times [0, \infty)$, satisfying (FM1)-(FM4) and
(WNA) $M(x, z, t) \geq \max\{M(x, y, t)*M(x, z, t/2), M(x, y, t/2)*M(y, z, t/2)\}$ for all $x, y, z \in X$ and $t > 0$.

Remark 2.4. (1) Every non-Archimedean fuzzy metric spaces is itself a weak non-Archimedean fuzzy metric space.
(2) A weak non-Archimedean fuzzy metric space is not necessarily a fuzzy metric space.

Example 2.5 ([16])**.** Let $X = [0, \infty)$ and $a * b = ab$ for all $a, b \in [0, 1]$. Define a mapping $M : X^2 \times [0, \infty) \to [0, 1]$ by: $M(x, y, 0) = 0$, $M(x, x, t) = 1$ for all $t > 0$, $M(x, y, t) = t$ for $x \neq y$ and $0 < t \leq 1$, $M(x, y, t) = t/2$ for $x \neq y$ and $1 < t \leq 2$, $M(x, y, t) = 1$ for $x \neq y$ and $t > 2$. Then $(X, M, *)$ is a weak non-Archimedean fuzzy metric space, but it is not a fuzzy metric space.

Definition 2.6 ([16])**.** Let $(X, M, *)$ be a weak non-Archimedean fuzzy metric space. We define an *open ball* in X by

$$B(x, r, t) = \{y \in X : M(x, y, t) > 1 - r\}$$

for any $x \in X$, $r \in (0, 1)$ and $t > 0$.

Proposition 2.7 ([16])**.** *Let $(X, M, *)$ be a weak non-Archimedean fuzzy metric space. Then we have the following:*

(1) *Every open ball is an open set;*
(2) *The family*

$$\tau = \{A \subset X : x \in A \text{ iff there exist } t > 0 \text{ and } r \in (0, 1) \text{ with } B(x, r, t) \subset A\}$$

is a topology on X;
(3) *Every weak non Archimedean fuzzy metric space $(X, M, *)$ is a Hausdorff space.*

Proposition 2.8 ([16])**.** *Let $(X, M, *)$ be a weak non-Archimedean fuzzy metric space. A sequence $\{x_n\}$ in a weak non-Archimedean fuzzy metric space $(X, M, *)$ is convergent to $x \in X$ if and only if $\lim_{n\to\infty} M(x_n, x, t) = 1$ for all $t > 0$.*

Proposition 2.9 ([16]). *Let $(X, M, *)$ be a weak non-Archimedean fuzzy metric space and $(x_n) \subset X$ be a sequence convergent to $x \in X$. Then*

$$\lim_{n\to\infty} M(y, x_n, t) = M(y, x, t)$$

for all $y \in X$ and $t > 0$.

Proposition 2.10. *Let $(X, M, *)$ be a weak non-Archimedean fuzzy metric space. Suppose that $\{x_n\}$ and $\{y_n\}$ are two sequences in X such that $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$. Then*

$$\lim_{n\to\infty} M(x_n, y_n, t) = M(x, y, t)$$

for all $t > 0$.

Proof. Let $\{x_n\}$ and $\{y_n\}$ be two sequences in X such that $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$. Then, by the condition (WNA) and Proposition 2.9, we have

$$M(y, x_n, t) \geq M(y, y_n, t/2) * M(y_n, x_n, t)$$

and

$$M(x_n, y_n, t) \geq M(x_n, x, t/2) * M(x, y_n, t).$$

It follows that

$$M(x, y, t) \leq \liminf_{n\to\infty} M(x_n, y_n, t) \leq \limsup_{n\to\infty} M(x_n, y_n, t) \leq M(x, y, t).$$

Therefore, $\lim_{n\to\infty} M(x_n, y_n, t) = M(x, y, t)$ for all $t > 0$. This completes the proof. □

Definition 2.11 ([16]). Let $(X, M, *)$ be a weak non-Archimedean fuzzy metric space. A sequence $\{x_n\}$ in X is called a *Cauchy sequence* if, for each $\epsilon \in (0, 1)$ and $t > 0$, there exists $N \in \mathbb{N}$ such that $M(x_n, x_m, t) > 1 - \epsilon$ for all $m, n \geq N$.

A weak non-Archimedean fuzzy metric space $(X, M, *)$ is said to be *complete* if every Cauchy sequence is convergent.

Definition 2.12 ([10]). An element $x \in X$ is called a *common fixed point* of the mappings $S, T, A, B : X \to X$ if

$$x = Sx = Tx = Ax = Bx.$$

Definition 2.13 ([16]). The self-mappings S and T of a weak non-Archimedean fuzzy metric space $(X, M, *)$ are said to be *compatible* if

$$\lim_{n\to\infty} M(STx_n, TSx_n, t) = 1$$

for all $t > 0$ whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = u$$

for some $u \in X$.

Definition 2.14 ([8])**.** The self-mappings S and T of a nonempty set X are said to be *weak compatible* if

$$STz = TSz$$

whenever $Sz = Tz$ for some $z \in X$.

3. Common fixed points for compatible mappings

In this section, we introduce the notion of ψ-contractions of four self-mappings and prove some common fixed points theorems for a ψ-contraction in complete non-Archimedean fuzzy metric spaces.

Let $\psi : [0,1] \to [0,1]$ be a function such that

(a) ψ is nondecreasing and left continuous;

(b) $\psi(t) > t$ for all $t \in (0,1)$.

We denote $\Psi := \{\psi : [0,1] \to [0,1] : \psi$ satisfies (a) $-$ (b)$\}$.

Lemma 3.1 ([8])**.** *If $\psi \in \Psi$, then*

(1) $\lim_{n\to\infty} \psi^n(t) = 1$ *for all* $t \in (0,1)$*;*

(2) $\psi(1) = 1$.

Definition 3.2. Let X be a nonempty set and M be a fuzzy set on $X^2 \times [0,\infty)$. Let $A, B, S, T : X \to X$ be four mappings. The four couple $(A, B; S, T)$ is called *ψ-contractive mappings* if there exists $\psi \in \Psi$ such that, for all $x, y \in X$ and $t \in (0,\infty)$ with $M(x, y, t) > 0$, the following condition holds:

$$M(Ax, By, t) \geq \psi(m(x, y, t)),$$

where

$$m(x, y, t) = \min\{M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t)\}.$$

Let $A, B, S, T : X \to X$ be four mappings such that $A(X) \subset T(X)$ and $B(X) \subset S(X)$. Suppose that there exist $x_0, x_1 \in X$ such that $Ax_0 = Tx_1$ and $M(x_0, x_1, t) > 0$. Let $x_0 \in X$. Since $A(X) \subset T(X)$ and $B(X) \subset S(X)$, there exist $x_0, x_1 \in X$ such that $Ax_0 = Tx_1 = y_0$. Also, since $A(X) \subset T(X)$ and $B(X) \subset S(X)$, there exists $x_2 \in X$ such that $Bx_1 = Sx_2 = y_1$. Inductively, we can define a sequence $\{y_n\}$ in X such that

$$(\Omega) \qquad Ax_{2n} = Tx_{2n+1} = y_{2n}, \quad Bx_{2n+1} = Sx_{2n+2} = y_{2n+1}$$

for each $n \geq 0$.

Lemma 3.3. *Let $(X, M, *)$ be a weak non-Archimedean fuzzy metric space and A, B, S, T be four self-mappings of X satisfying the following conditions:*

(1) *$A(X) \subset T(X)$ and $B(X) \subset S(X)$;*

(2) *The pairs A, S and B, T are compatible;*

(3) *One of S, T, A and B is continuous;*

(4) *The four couple $(A, B; S, T)$ is ψ-contractive mappings.*

Suppose that there exist $x_0, x_1 \in X$ such that $Ax_0 = Tx_1$ and $M(x_0, x_1, t) > 0$. Then the sequence $\{y_n\}$ in X generated by (Ω) with initial point x_0, x_1 is a Cauchy sequence.

Proof. First, from the sequence $\{y_n\}$ in X generated by (Ω), we show that

$$\lim_{n\to\infty} M(y_n, y_{n+1}, t) = 1$$

for all $t > 0$. Assume that $M(y_n, y_{n+1}, t) < 1$ for all $n \geq 1$. Since $M(Ax_0, Bx_1, t) = M(y_0, y_1, t) > 0$, we obtain

$$\begin{aligned} M(y_2, y_1, t) &= M(Ax_2, Bx_1, t) \\ &\geq \psi(m(x_2, x_1, t)) \\ &= \psi(\min\{M(Sx_2, Tx_1, t), M(Ax_2, Sx_2, t), M(Bx_1, Tx_1, t)\}) \\ &= \psi(\min\{M(y_1, y_0, t), M(y_2, y_1, t), M(y_1, y_0, t)\}) \\ &= \psi(\min\{M(y_1, y_0, t), M(y_2, y_1, t)\}). \end{aligned}$$

Suppose that $M(y_1, y_0, t) > M(y_2, y_1, t)$ then

$$M(y_2, y_1, t) \geq \psi(M(y_2, y_1, t)) > M(y_2, y_1, t),$$

which is a contradiction. Therefore $M(y_1, y_0, t) \leq M(y_2, y_1, t)$, which implies that

$$M(y_2, y_1, t) \geq \psi(M(y_1, y_0, t)) > 0.$$

Again, we consider

$$\begin{aligned} M(y_2, y_3, t) &= M(Ax_2, Bx_3, t) \\ &\geq \psi(m(x_2, x_3, t)) \\ &= \psi(\min\{M(Sx_2, Tx_3, t), M(Ax_2, Sx_2, t), M(Bx_3, Tx_3, t)\}) \\ &= \psi(\min\{M(y_1, y_2, t), M(y_2, y_1, t), M(y_3, y_2, t)\}) \\ &= \psi(\min M(y_2, y_1, t), M(y_3, y_2, t)\}) \end{aligned}$$

Suppose that $M(y_2, y_1, t) > M(y_3, y_2, t)$ then

$$M(y_2, y_3, t) \geq \psi(M(y_2, y_3, t)) > M(y_2, y_3, t),$$

which is a contradiction. Therefore $M(y_2, y_1, t) \leq M(y_2, y_3, t)$, which implies that

$$M(y_2, y_3, t) \geq \psi(M(y_2, y_1, t)) \geq \psi^2(M(y_0, y_1, t)) > 0.$$

Therefore, for any $n \in \mathbb{N}$ we have

$$M(y_{n+1}, y_n, t) \geq \psi^n(M(y_0, y_1, t)) > 0.$$

By Lemma 3.1, as $n \to \infty$, we obtain

$$\lim_{n\to\infty} M(y_{n+1}, y_n, t) = 1.$$

Next, we show that the sequence $\{y_n\}$ is a Cauchy sequence. If $\{y_n\}$ is not Cauchy, then there exist $\epsilon \in (0, \frac{1}{2})$ and $t > 0$ such that, for each $k \geq 1$, there exist $m(k), n(k) \in \mathbb{N}$ such that $m(k) > n(k) \geq k$ and $M(y_{m(k)}, y_{n(k)}, t) \leq 1-2\epsilon$. By (WNA), we have

$$1 - 2\epsilon \geq M(y_{m(k)}, y_{n(k)}, t) \geq M(y_{m(k)}, y_{n(k)+1}, t) * M(y_{n(k)+1}, y_{n(k)}, t/2),$$

which implies, by $k \to \infty$, that

$$\begin{aligned} 1 - 2\epsilon &\geq \limsup_{k\to\infty} M(y_{m(k)}, y_{n(k)+1}, t) * M(y_{n(k)+1}, y_{n(k)}, t/2) \\ &= \limsup_{k\to\infty} M(y_{m(k)}, y_{n(k)+1}, t). \end{aligned}$$

Similarly, we obtain

$$1 - 2\epsilon \geq \limsup_{k\to\infty} M(y_{m(k)+1}, y_{n(k)}, t),$$

$$1 - 2\epsilon \geq \limsup_{k\to\infty} M(y_{m(k)+1}, y_{n(k)+1}, t).$$

So, we can assume that $m(k)$ are odd numbers, $n(k)$ are even numbers and

$$M(x_{m(k)}, x_{n(k)}, t) \leq 1 - \epsilon$$

for all $k \geq 1$. Define $q(k) = \min\{m(k) : M(y_{m(k)}, y_{n(k)}, t) \leq 1-\epsilon,\ m(k) \text{ is odd number}\}$. By (WNA), we have

$$\begin{aligned} 1 - \epsilon &\geq M(y_{q(k)}, y_{n(k)}, t) \geq M(y_{q(k)}, y_{q(k)-2}, t/2) * M(y_{q(k)-2}, y_{n(k)}, t) \\ &\geq M(y_{q(k)}, y_{q(k)-1}, t/4) * M(y_{q(k)-1}, y_{q(k)-2}, t/2) * (1 - \epsilon). \end{aligned}$$

Letting $k \to \infty$, we obtain $\lim_{k\to\infty} M(y_{q(k)}, y_{n(k)}, t) = 1 - \epsilon$. By (WNA) and the condition (4), we have

$$\begin{aligned} & M(y_{q(k)}, y_{n(k)}, t) \\ & \geq M(y_{q(k)}, y_{n(k)+1}, t) * M(y_{n(k)+1}, y_{n(k)}, t/2) \\ & \geq M(y_{q(k)+1}, y_{n(k)+1}, t) * M(y_{q(k)+1}, y_{q(k)}, t) * M(y_{n(k)+1}, y_{n(k)}, t/2) \\ & = M(Sx_{q(k)+1}, Tx_{n(k)+1}, t) * M(y_{q(k)+1}, y_{q(k)}, t) * M(y_{n(k)+1}, y_{n(k)}, t/2) \\ (*) \quad & \geq \psi(m(x_{q(k)+1}, x_{n(k)+1}, t)) * M(y_{q(k)+1}, y_{q(k)}, t) * M(y_{n(k)+1}, y_{n(k)}, t/2), \end{aligned}$$

where

$$\begin{aligned} & m(x_{q(k)+1}, x_{n(k)+1}, t) \\ & = \min\{M(Sx_{q(k)+1}, Tx_{n(k)+1}, t), M(Ax_{q(k)+1}, Sx_{q(k)+1}, t), \\ & \qquad M(Bx_{n(k)+1}, Tx_{n(k)+1}, t)\} \\ & = \min\{M(y_{q(k)}, y_{n(k)}, t), M(y_{q(k)+1}, y_{q(k)}, t), M(y_{n(k)+1}, y_{n(k)}, t)\}. \end{aligned}$$

Since

$$\begin{aligned} & \lim_{k\to\infty} m(x_{q(k)+1}, x_{n(k)+1}, t) \\ & = \min\{\lim_{k\to\infty} M(y_{q(k)}, y_{n(k)}, t), \lim_{k\to\infty} M(y_{q(k)+1}, y_{q(k)}, t), \lim_{k\to\infty} M(y_{n(k)+1}, y_{n(k)}, t)\} \\ & = \min\{1 - \epsilon, 1, 1\} \\ & = 1 - \epsilon \end{aligned}$$

By taking $k \to \infty$ in (*), we obtain

$$1 - \epsilon \geq \psi(1 - \epsilon) * 1 * 1 > 1 - \epsilon,$$

which is a contradiction. Therefore, $\{y_n\}$ is a Cauchy sequence. This completes the proof. □

Now, we are ready to state and prove our main results.

Theorem 3.4. *Let $(X, M, *)$ be a complete weak non-Archimedean fuzzy metrci space and A, B, S, T be the self-mappings of X satisfying the following conditions:*

(1) *$A(X) \subset T(X)$ and $B(X) \subset S(X)$;*
(2) *The pairs A, S and B, T are compatible;*
(3) *One of A, B, S and T is continuous;*
(4) *The four couple $(A, B; S, T)$ is a ψ-contractive mapping.*

Suppose that there exist $x_0, x_1 \in X$ such that $Ax_0 = Tx_1$ and $M(x_0, x_1, t) > 0$. Assume that, for any $x, y \in X$ with $x \neq y$, there exists $t > 0$ such that $0 < M(x, y, t) < 1$. Then A, B, S and T have a unique common fixed point.

Proof. By Lemma 3.3, the sequence $\{y_n\}$ is a Cauchy sequence. Since X is complete, there exists $z \in X$ such that $\lim_{n\to\infty} y_n = z$. From

$$Ax_{2n} = Tx_{2n+1} = y_{2n} \text{ and } Bx_{2n+1} = Sx_{2n+2} = y_{2n+1}$$

for all $n \geq 1$, we obtain

$$\lim_{n\to\infty} Ax_{2n} = \lim_{n\to\infty} Tx_{2n+1} = \lim_{n\to\infty} Bx_{2n+1} = \lim_{n\to\infty} Sx_{2n+2} = z. \tag{3.1}$$

For the proof, we divide 4 Cases for the continuity of A, B, S and T.

Case 1. Suppose that S is continuous. Then we have

$$\lim_{n\to\infty} SAx_{2n} = \lim_{n\to\infty} S^2x_{2n} = Sz. \tag{3.2}$$

Since A and S are compatible mappings, $\lim_{n\to\infty} M(SAx_{2n}, ASx_{2n}, t) = 1$. Thus, from

$$M(ASx_{2n}, Sz, t) \geq M(ASx_{2n}, SAx_{2n}, t) * M(SAx_{2n}, Sz, t/2)$$

as $n \to \infty$, we obtain $\lim_{n\to\infty} M(ASx_{2n}, Sz, t) = 1$, i.e.

$$\lim_{n\to\infty} ASx_{2n} = Sz. \tag{3.3}$$

First, wet prove that z is a common fixed point of A and S. If $Sz \neq z$, then there exists $t > 0$ such that $0 < M(Sz, z, t) < 1$. By the condition (4), we have

$$M(ASx_{2n}, Bx_{2n+1}, t) \geq \psi(m(Sx_{2n}, x_{2n+1}, t)),$$

where

$$\begin{aligned}&m(Sx_{2n}, x_{2n+1}, t)\\ &= \min\{M(S^2x_{2n}, Tx_{2n+1}, t), M(ASx_{2n}, S^2x_{2n}, t), M(Tx_{2n+1}, Bx_{2n+1}, t)\}.\end{aligned}$$

Lettinge $n \to \infty$, by (3.1), (3.2) and (3.3), we obtain

$$\begin{aligned}M(Sz, z, t) &\geq \psi(\min\{M(Sz, z, t), M(Sz, Sz, t), M(z, z, t)\})\\ &= \psi(\min\{M(Sz, z, t), 1, 1\})\\ &= \psi(M(Sz, z, t))\\ &> M(Sz, z, t),\end{aligned}$$

which is a contradiction. Therefore, $Sz = z$. If $Az \neq z$, then there exists $t > 0$ such that $0 < M(Az, z, t) < 1$. By the condition (4), we have

$$M(Az, Bx_{2n+1}, t) \geq \psi(m(z, x_{2n+1}, t)),$$

where

$$m(z, x_{2n+1}, t) = \min\{M(Sz, Tx_{2n+1}, t), M(Az, Sz, t), M(Tx_{2n+1}, Bx_{2n+1}, t)\}.$$

Letting $n \to \infty$, by (3.1), (3.2) and (3.3), we obtain

$$\begin{aligned} M(Az, z, t) &\geq \psi(\min\{M(Sz, z, t), M(Az, Sz, t), M(z, z, t)\}) \\ &= \psi(\min\{M(z, z, t), M(Az, z, t), M(z, z, t)\}) \\ &= \psi(\min\{1, M(Az, z, t), 1\}) \\ &= \psi(M(Az, z, t)) \\ &> M(Az, z, t), \end{aligned}$$

which is a contradiction and hence $Az = z$. Therefore, z is a common fixed point of A and S. Since $A(X) \subset BT(X)$, there exists $z^* \in X$ such that $z = Az = Tz*$. If $z \neq Tz^*$, then there exists $t > 0$ such that $0 < M(z, Bz^*, t) < 1$. By the condition (4) and Lemma 3.1, we have

$$M(z, Bz^*, t) = M(Az, Bz^*, t) \geq \psi(m(z, z^*)) = \psi(M(z, Bz^*, t)) > M(z, Bz^*, t),$$

which is a contradiction. Then $z = Bz^*$. Since B and T are compatible, we obtain

$$M(Tz, Bz, t) = M(TBz^*, BTz^*t) = 1 \quad \text{for any } t > 0,$$

which implies $Tz = Bz$.

Next, if $z \neq Bz$, then there exists $t > 0$ such that $0 < M(z, Bz, t) < 1$. By the condition (4), we have

$$M(z, Bz, t) = M(Az, Bz, t) \geq \psi(m(z, z)) = \psi(M(z, Bz, t)) > M(z, Bz, t),$$

which is a contradiction. Hence $z = Tz$ and so $z = Tz = Bz = Sz = Az$.

Case 2. Suppose that T is continuous. In the same way as in Case 1, we can obtain the result.

Case 3. Suppose that A is continuous. Then

$$\lim_{n\to\infty} ASx_{2n} = \lim_{n\to\infty} A^2x_{2n} = Az. \tag{3.4}$$

Since S and A are compatible mappings, $\lim_{n\to\infty} M(ASx_{2n}, SAx_{2n}, t) = 1$. By (WNA), we have

$$M(SAx_{2n}, Az, t) \geq M(SAx_{2n}, ASx_{2n}, t) * M(ASx_{2n}, Az, t/2).$$

Letting $n \to \infty$, we obtain $\lim_{n\to\infty} M(SAx_{2n}, Az, t) = 1$, i.e.,

$$\lim_{n\to\infty} SAx_{2n} = Az. \tag{3.5}$$

If $Az \neq z$, then there exists $t > 0$ such that $0 < M(Az, z, t) < 1$. By the condition (4), we have

$$M(A^2x_{2n}, Bx_{2n+1}, t) \geq \psi(m(Ax_{2n}, x_{2n+1}, t)),$$

where

$$m(Ax_{2n}, x_{2n+1}, t)$$
$$= \min\{M(SAx_{2n}, Tx_{2n+1}, t), M(A^2x_{2n}, SAx_{2n}, t), M(Tx_{2n+1}, Bx_{2n+1}, t)\}.$$

Letting $n \to \infty$, by (3.1), (3.4) and (3.5), we obtain

$$\begin{aligned} M(Az, z, t) &\geq \psi(\min\{M(Az, z, t), M(Az, Az, t), M(z, z, t)\}) \\ &= \psi(\min\{M(Az, z, t), 1, 1\}) \\ &= \psi(M(Az, z, t)) \\ &> M(Az, z, t), \end{aligned}$$

which is a contradiction and hence $Az = z$. Since $A(X) \subset T(X)$, there exists $z^* \in X$ such that $z = Az = Tz*$. If $z \neq Bz^*$, then there exists $t > 0$ such that $0 < M(z, Bz^*, t) < 1$. By the condition (4), we have

$$M(z, Bz^*, t) = M(Az, Bz^*, t) \geq \psi(m(z, z^*, t)) = \psi(M(z, Bz^*, t)) > M(z, Bz^*, t),$$

which is a contradiction. Then $z = Bz^*$. Since B and T are compatible, we obtain

$$M(Tz, Bz, t) = M(TBz^*, BTz^*t) = 1 \quad \text{for any } t > 0,$$

which implies $Tz = Bz$.

Next, if $z \neq Bz$, then there exists $t > 0$ such that $0 < M(z, Bz, t) < 1$. By the condition (4) and Lemma 3.1, we have

$$M(z, Bz, t) = M(Az, Bz, t) \geq \psi(m(z, z, t)) = \psi(M(z, Bz, t)) > M(z, Bz, t),$$

which is a contradiction and hence $z = Bz$. Since $B(X) \subset S(X)$, there exists $z^{**} \in X$ such that $z = Bz = Sz^{**}$. If $z \neq Az^{**}$, then there exists $t > 0$ such that $0 < M(z, Az^{**}, t) < 1$. By the condition (4), we have

$$M(Az^{**}, z, t) = M(Az^{**}, Bz, t) \geq \psi(m(z^{**}, z, t)) \geq \psi(M(Az^{**}, z, t)) > M(Az^{**}, z, t),$$

which is a contradiction. Then $z = Az^{**}$. Since S and A are compatible mappings, we have

$$M(Sz, Az, t) = M(SAz^{**}, ASz^{**}, t) = 1 \quad \text{for any } t > 0$$

and so $Sz = Az$. Therefore, $Sz = Tz = Az = Bz = z$ and so z is a common fixed point of A, B, S and T.

Case 4. Suppose that B is continuous. In the same way as in Case 1, we can obtain the result.

Now, we prove the uniqueness of the common fixed point of A, B, S and T. Assume that $x, y \in X$ are two common fixed points of A, B, S and T. If $x \neq y$, then there exists $t > 0$ such that $0 < M(x, y, t) < 1$. By the condition (4), we have

$$\begin{aligned} M(x,y,t) &= M(Ax,By,t) \\ &\geq \psi(m(x,y,t)) \\ &= \psi(\min\{M(Sx,Ty,t), M(Ax,Sx,t), M(By,Ty,t)\}) \\ &= \psi(\min\{M(x,y,t), M(x,x,t), M(y,y,t)\}) \\ &= \psi(\min\{M(x,y,t),1,1\}) \\ &= \psi(M(x,y,t)) \\ &> M(x,y,t), \end{aligned}$$

which is a contradiction. Therefore, $x = y$. This completes the proof. □

If we put $S = T = I_X$ (the identity mapping on X) in Theorem 3.4, then we obtain the result of Vetro [16] as follows:

Corollary 3.5 ([16])**.** *Let $(X, M, *)$ be a complete weak non-Archimedean fuzzy metric space and $A, B : X \to X$ be two mappings. Assume that there exists $\psi \in \Psi$ such that, for all $x, y \in X$ and $t \in (0, \infty)$ with $M(x, y, t) > 0$, the following condition holds:*

$$M(Ax, By, t) \geq \psi(m(x, y, t)),$$

where

$$m(x,y,t) = \min\{M(x,y,t), M(Ax,x,t), M(y,By,t)\}.$$

Suppose that, for any $x, y \in X$ with $x \neq y$, there exists $t > 0$ such that $0 < M(x, y, t) < 1$. If there exists $x_0 \in X$ such that $M(x_0, Sx_0, t) > 0$ for all $t > 0$, then A and B have a unique common fixed point.

If we put $A = B$ and $S = T = I_X$ in Theorem 3.4, then we obtain the following:

Corollary 3.6. *Let $(X, M, *)$ be a complete weak non-Archimedean fuzzy metric space and $A : X \to X$ be a mapping. Assume that there exists $\psi \in \Psi$ such that, for all $x, y \in X$ and $t \in (0, \infty)$ with $M(x, y, t) > 0$, the following condition holds:*

$$M(Ax, Ay, t) \geq \psi(m(x, y, t)),$$

where

$$m(x,y,t) = \min\{M(x,y,t), M(Ax,x,t), M(y,Ay,t)\}.$$

Suppose that, for any $x, y \in X$, with $x \neq y$, there exists $t > 0$ such that $0 < M(x, y, t) < 1$. If there exists $x_0 \in X$ such that $M(x_0, Ax_0, t) > 0$ for all $t > 0$, then S has a unique fixed point.

As a consequence of Corollary 3.6, by Remark 2.4, we obtain the result of Mihet [12] as follows:

Corollary 3.7 ([12])**.** *Let $(X, M, *)$ be a complete non-Archimedean fuzzy metric space and $A : X \to X$ be a mapping. Assume that there exists $\psi \in \Psi$ such* *that, for all $x, y \in X$ and $t \in (0, \infty)$ with $M(x, y, t) > 0$, the following condition holds:*

$$M(Ax, Ay) \geq \psi(M(x, y, t)),$$

Suppose that, for any $x, y \in X$ with $x \neq y$, there exists $t > 0$ such that $0 < M(x, y, t) < 1$. If there exists $x_0 \in X$ such that $M(x_0, Ax_0, t) > 0$ for all $t > 0$, then A has a unique fixed point.

We now give an example to illustrate Theorem 3.4.

Example 3.8. Let $(X, M, *)$ be a complete weak non-Archimedean fuzzy metric space, where $X = [0, \infty)$ with the t-norm defined by $a * b = ab$ for any $a, b \in [0, 1]$ and the fuzzy set M given by: $M(x, y, 0) = 0$, $M(x, x, t) = 1$ for all $t > 0$, $M(x, y, t) = 0$ for $x \neq y$ and $0 < t \leq 1$, $M(x, y, t) = t^2/4$ for $x \neq y$ and $1 < t \leq 2$, $M(x, y, t) = 1$ for $x \neq y$ and $t > 2$. Define a function $\psi : [0, 1] \to [0, 1]$ by $\psi(t) = \sqrt{t}$ for all $t \in [0, 1]$. Then we see that $\psi \in \Psi$. Define four mappings $A, B, S, T : X \to X$ by

$$Ax = x, \quad Bx = \sqrt{x}, \quad Sx = 2x, \quad Tx = 4\sqrt{x}.$$

Then we have the following:

(1) $AX = BX = SX = TX$;
(2) A, B, S and T are all continuous mappings;
(3) The pair A, S and B, T are compatible;
(4) The four couple $(A, B; S, T)$ is a ψ-contractive mapping;
(5) If we choose $x_0 = x_1 = 0$, then $A0 = T0$ and $M(x_0, x_1, t) = M(0, 0, t) > 0$;
(6) If we choose $t = \frac{3}{2}$, then, for any $x, y \in X$ with $x \neq y$, we have

$$0 < M(x, y, \frac{3}{2}) = \frac{9}{16} < 1.$$

Therefore, all the conditions of Theorem 3.4 are satisfied. Also, we see that $A0 = B0 = S0 = T0$ and so 0 is a unique common fixed point of A, B, S and T.

4. Common fixed points for mappings with the common limit

In this section, we prove some common fixed points for mappings with the common limit with respect to the value of the given mappings in weak non-Archimedean fuzzy metric spaces.

Definition 4.1. Two pairs (A, S) and (B, T) of self-mappings of a weak non-Archimedean fuzzy metric space $(X, M, *)$ are said to have the *common limit* with respect to the value of the mapping S (resp., T) if there exist two sequence $\{x_n\}$ and $\{y_n\}$ of X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = Sz \text{ (resp., } \ Tz)$$

for some $z \in X$.

Theorem 4.2. *Let $(X, M, *)$ be a weak non-Archimedean fuzzy metric space and $A, B, S, T : X \to X$ be four mappings. Suppose that the following conditions holds:*

1. *$A(X) \subset T(X)$ or $B(X) \subset S(X)$;*
2. *The pairs A, S and B, T are also weakly compatible;*
3. *The pairs (A, S) and (B, T) have the common limit with respect to the value of the mapping S (or T);*
4. *There exists $\psi \in \Psi$ such that, for all $x, y \in X$ and $t \in (0, \infty)$ with $M(x, y, t) > 0$, the following condition holds:*

$$M(Ax, By, t) \geq \psi(m(x, y, t)),$$

where

$$m(x, y, t) = \min\{M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t), M(By, Sx, t), M(Ax, Ty, t)\}.$$

5. *For any $x, y \in X$ with $x \neq y$, there exists $t > 0$ such that*

$$0 < M(x, y, t) < 1.$$

Then A, B, S and T have a unique common fixed point.

Proof. Since the pairs (A, S) and (B, T) have the common limit with respect to the value of the mapping S, there exists two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = Sz$$

for some $z \in X$.

First, we show that $Az = Sz$. Suppose that $Az \neq Sz$. Then, by the condition (5), there exists $t > 0$ such that $0 < M(Az, Sz, t) < 1$. By using the condition (4), we obtain

$$M(Az, By_n, t) \geq \psi(m(z, y_n, t)), \tag{4.1}$$

where

$$m(z, y_n, t) = \min\{M(Sz, Ty_n, t), M(Az, Sz, t), M(By_n, Ty_n, t), M(By_n, Sz, t), M(Az, Ty_n, t)\}.$$

By taking the limit as $n \to \infty$ in (4.1), we have

$$M(Az, Sz, t) \geq \psi(M(Az, Sz, t)) > M(Az, Sz, t),$$

which is a contradiction and so $Az = Sz$. Since $A(X) \subset T(X)$, there exists $v \in X$ such that $Az = Tv$.

Next, we show that $Bv = Tv$. Suppose that $Bv \neq Tv$. Then, by the condition (5), there exists $t > 0$ such that $0 < M(Bv, Tv, t) < 1$. By using the condition (4), we obtain

$$M(Tv, Bv, t) = M(Az, Bv, t) \geq \psi(m(z, v, t)), \tag{4.2}$$

where

$$\begin{aligned} m(z,v,t) &= \min\{M(Sz,Tv,t), M(Az,Sz,t), M(Bv,Tv,t),\\ &\qquad M(Bv,Sz,t), M(Az,Tv,t)\}.\\ &= \min\{M(Tv,Tv,t), M(Az,Sz,t), M(Bv,Tv,t),\\ &\qquad M(Bv,Tv,t), M(Tv,Tv,t)\}\\ &= \min\{1, M(Bv,Tv)\}. \end{aligned}$$

Hence, in (4.2), we obtain

$$M(Tv,Bv,t) \geq \psi(M(Tv,Bv,t)) > M(Tv,Bv,t),$$

which is a contradiction and so $Tv = Bv$. Therefore, we have $u = Az = Sz = Bv = Tv$. Since the pairs A, S and B, T are weakly compatible, $Az = Sz$ and $Bv = Tv$, we have

$$Au = AAz = ASz = SAz = Su, \quad Bu = BTv = TBv = Tu. \tag{4.3}$$

Next, we show that $Au = u$. Suppose that $Au \neq u$. Then there exists $t > 0$ such that $0 < M(Au,u,t) < 1$. By the condition (4), we obtain

$$M(Au,u,t) = M(Au,Bv,t) \geq \psi(m(u,v,t)), \tag{4.4}$$

where

$$\begin{aligned} m(u,v,t) &= \min\{M(Su,Tv,t), M(Au,Su,t), M(Bv,Tv,t)\\ &\qquad M(Bv,Su,t), M(Au,Tv,t)\}\\ &= \min\{M(Au,Bv,t), M(Au,Au,t), M(Bv,Bv,t)\\ &\qquad M(Bv,Au,t), M(Au,Bv,t)\}\\ &= \min\{1, M(Au,u,t)\}. \end{aligned}$$

Hence, in (4.4), we obtain

$$M(Au,u,t) \geq \psi(M(Au,u,t)) > M(Au,u,t),$$

which is a contradiction and so $Au = u$.

Next, we show that $Bu = u$. Suppose $Bu \neq u$. Then there exists $t > 0$ such that $0 < M(Bu,u,t) < 1$. By the condition (4), we obtain

$$M(u,Bu,t) = M(Au,Bu,t) \geq \psi(m(u,u,t)), \tag{4.5}$$

where

$$\begin{aligned} m(u,u,t) &= \min\{M(Su,Tu,t), M(Au,Su,t), M(Bu,Tu,t)\\ &\qquad M(Bu,Su,t), M(Au,Tu,t)\}\\ &= \min\{M(Au,Bu,t), M(Au,Au,t), M(Bu,Bu,t)\\ &\qquad M(Bu,Au,t), M(Au,Bu,t)\}\\ &= \min\{1, M(Au,Bu,t)\}. \end{aligned}$$

Hence, in (4.5), we obtain

$$M(Bu,u,t) \geq \psi(M(Bu,u,t)) > M(Bu,u,t),$$

which is a contradiction and so $Bu = u$. Therefore, we have

$$u = Au = Bu = Su = Tu,$$

that is, u is a common fixed point of A, B, S and T. The uniqueness of the common fixed point follows the proof of Theorem 3.4. This completes the proof. □

Remark 4.3. We don't need the completeness of a weak non-Archimedean fuzzy metric space $(X, M, *)$ in the proof of Theorem 4.2.

As a consequence of Theorem 4.2, by putting $S = T = I_X$, we obtain the following:

Corollary 4.4. *Let $(X, M, *)$ be a weak non-Archimedean fuzzy metric space and $A, B : X \to X$ be two mappings. Suppose that the following conditions holds:*

(1) *The pairs (A, I_X) and (B, I_X) have the common limit with respect to the value of the mapping I_X;*
(2) *There exists $\psi \in \Psi$ such that, for all $x, y \in X$ and $t \in (0, \infty)$ with $M(x, y, t) > 0$, the following condition holds:*

$$M(Ax, By, t) \geq \psi(m(x, y, t)),$$

where

$$m(x, y, t) = \min\{M(x, y, t), M(Ax, x, t), M(By, y, t), M(By, x, t), M(Ax, y, t)\}.$$

(3) *For any $x, y \in X$ with $x \neq y$, there exists $t > 0$ such that*

$$0 < M(x, y, t) < 1.$$

Then A and B have a unique common fixed point.

By putting $A = B$ and $S = T$ in Theorem 4.2, we obtain the following:

Corollary 4.5. *Let $(X, M, *)$ be a weak non-Archimedean fuzzy metric space and $A, S : X \to X$ be a mapping. Suppose that the following conditions holds:*

(1) *$A(X) \subset S(X)$;*
(2) *A pair A, S is weakly compatible;*
(3) *There exists a sequence $\{x_n\}$ in X such that*

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = Sz$$

for some $z \in X$;
(4) *There exists $\psi \in \Psi$ such that, for all $x, y \in X$ and $t \in (0, \infty)$ with $M(x, y, t) > 0$, the following condition holds:*

$$M(Ax, Ay, t) \geq \psi(m(x, y, t)),$$

where

$$m(x, y, t) = \min\{M(Sx, Sy, t), M(Ax, Sx, t), M(Ay, Sy, t), M(Ay, Sx, t), M(Ax, Sy, t)\}.$$

(5) *For any* $x, y \in X$ *with* $x \neq y$, *there exists* $t > 0$ *such that*

$$0 < M(x, y, t) < 1.$$

Then A *and* S *have a unique common fixed point.*

By putting $A = B$ and $S = T = I_X$ in Theorem 4.2, we obtain the following:

Corollary 4.6. *Let* $(X, M, *)$ *be a weak non-Archimedean fuzzy metric space and* $A, S : X \to X$ *be two mappings. Suppose that the following conditions holds:*

(1) *There exists a sequence* $\{x_n\}$ *in* X *such that*

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} x_n = z$$

for some $z \in X$*;*

(2) *There exists* $\psi \in \Psi$ *such that, for all* $x, y \in X$ *and* $t \in (0, \infty)$ *with* $M(x, y, t) > 0$, *the following condition holds:*

$$M(Ax, Ay, t) \geq \psi(m(x, y, t)),$$

where

$$m(x, y, t) = \min\{M(x, y, t), M(Ax, x, t), M(Ay, y, t), M(Ay, x, t), M(Ax, y, t)\}.$$

(3) *For any* $x, y \in X$ *with* $x \neq y$, *there exists* $t > 0$ *such that*

$$0 < M(x, y, t) < 1.$$

Then A *has a unique fixed point.*

Now, we give an example to illustrate Theorem 4.2.

Example 4.7. Let $(X, M, *)$ be a complete weak non-Archimedean fuzzy metric space, where $X = [0, 30)$ with the t-norm defined by $a * b = ab$ for any $a, b \in [0, 1]$ and the fuzzy set M given by: $M(x, y, 0) = 0$, $M(x, x, t) = 1$ for all $t > 0$, $M(x, y, t) = 0$ for $x \neq y$ and $0 < t \leq 1$, $M(x, y, t) = t^2/4$ for $x \neq y$ and $1 < t \leq 2$, $M(x, y, t) = 1$ for $x \neq y$ and $t > 2$. Then, for any $x, y \in X$ with $x \neq y$,

$$0 < M(x, t, \frac{3}{2}) = \frac{9}{16} < 1.$$

Define a function $\psi : [0, 1] \to [0, 1]$ by $\psi(t) = \sqrt{t}$ for all $t \in [0, 1]$. Then we see that $\psi \in \Psi$. Define four mappings $A, B, S, T : X \to X$ by

$$Ax = \begin{cases} 1, & \text{if } x \in \{1\} \cup (5, 30), \\ x + 7, & \text{if } x \in (1, 5], \end{cases} \qquad Bx = \begin{cases} 1, & \text{if } x \in \{1\} \cup (5, 30), \\ x + 6, & \text{if } x \in (1, 5], \end{cases}$$

$$Sx = \begin{cases} 1, & \text{if } x = 1, \\ 7, & \text{if } x \in (1, 5], \\ \frac{x+1}{6} & \text{if } x \in (5, 30) \end{cases} \qquad Tx = \begin{cases} 1, & \text{if } x = 1, \\ 9, & \text{if } x \in (1, 5], \\ x - 4, & \text{if } x \in (5, 30). \end{cases}$$

If we choose two sequences $\{x_n\}$ and $\{y_n\}$ defined by $x_n = y_n = 5 + \frac{1}{n}$ for each $n \geq 1$, then

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = S1 = 1 \in X.$$

This implies that the pairs (A, S) and (B, T) have the common limit with respect to the value of the mapping S. We see that

$$A(X) = \{1\} \cup (8, 12], \quad B(X) = \{1\} \cup (7, 11],$$
$$S(X) = [1, 5) \cup \{7\}, \quad T(X) = [1, 26)$$

and so $A(X) \subset T(X)$, but $B(X) \not\subset S(X)$. It is easy to show that the mappings A, B, S, T satisfy the condition (4) in Theorem 4.2. Therefore, all the conditions of Theorem 4.2 are satisfied and, also, we see that 1 is a unique common fixed point of A, B, S and T.

Remark 4.8. Example 3.8 and Example 4.7 show how significant of our main results (Themrem 3.4 and Theorem 4.2). These two theorems can guarantee existence of a common fixed point of four nonlinear mappings satisfying new type of contractive conditions while the previous known results cannot be applied.

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