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On $\varphi_{1,2}$-countable compactness and filters

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Abstract. In this work the author investigates some relations between $\varphi_{1,2}$-countable compactness, filters, sequences and $\varphi_{1,2}$-closure operators.

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1. Introduction

Many generalizations of the notion of compact space have been defined in the literature, including those of quasi H-closed space, S-closed space, rs-compact space, feebly compact space, countably S-closed space, countably rs-compact space, and many more. Some of these concepts have been characterized in terms of filters and nets, and this has lead to such notions as r-convergence, RC-convergence, SR-convergence, r-accumulation point, RC-accumulation point and SR-accumulation point of filters and filterbases.

The notion of an operation on a topological space is a useful tool when attempting to unify such concepts, and in earlier studies we have defined $\varphi_{1,2}$-countably compact sets, $\varphi_{1,2}$-convergence of a filter and $\varphi_{1,2}$-accumulation points of a filter, and used these to obtain some such unifications.

In the present work we will study the relations between $\varphi_{1,2}$-countable compactness, filters, sequences and $\varphi_{1,2}$-closure operators.

There are several different definitions of an operation in the literature. We have used the one first given in [12] for fuzzy topological spaces.

In a topological space (X, τ), int, cl, scl, pcl etc. will stand for the interior, closure, semi-closure, pre-closure operations, and so on. For a subset A of X, A^o, $\bar{A}$ will also be used to denote the interior and closure of A, respectively.

*Dedicated to the memory of Professor Doğan Çoker.

Definition 1.1. Let (X, τ) be a topological space. A mapping $\varphi : P(X) \to P(X)$ is called an *operation on* (X, τ) if $\varphi(\varnothing) = \varnothing$ and $A^o \subseteq \varphi(A)$, $\forall A \in P(X)$.

The class of all operations on a topological space (X, τ) will be denoted by $O(X, \tau)$.

For $\varphi_1, \varphi_2 \in O(X, \tau)$ we set $\varphi_1 \leq \varphi_2 \iff \varphi_1(A) \subseteq \varphi_2(A),\ \forall\, A \in P(X)$.

The operations φ, $\tilde{\varphi}$ are *dual* if $\tilde{\varphi}(A) = X \setminus \varphi(X \setminus A),\ \forall\, A \in P(X)$.

An operation $\varphi \in O(X, \tau)$ is called *monotonous* if $\varphi(A) \subseteq \varphi(B)$ whenever $A \subseteq B$ $(A, B \in P(X))$.

Definition 1.2. Let $\varphi \in O(X, \tau)$. Then $A \subseteq X$ is called φ*-open* if $A \subseteq \varphi(A)$. Dually, $B \subseteq X$ is called φ*-closed* if $X \setminus B$ is φ-open.

Clearly, X and $\varnothing$ are both φ-open and φ-closed, while each open set is a φ-open set for any $\varphi \in O(X, \tau)$.

If (X, τ) is a topological space, $\varphi \in O(X, \tau)$, then $\varphi O(X)$, $\varphi C(X)$ will denote respectively the set of φ-open, φ-closed subsets of X. For $x \in X$ we set $\varphi O(X, x) = \{U \in \varphi O(X) \mid x \in U\}$.

For $\varphi_2, \varphi_1 \in \varphi O(X)$ sufficient, generally not necessarily, conditions for $\varphi_1 O(X) \subseteq \varphi_2 O(X)$ are $\varphi_2 \geq \varphi_1$ or $\varphi_2 \geq \imath$ [21]. Here $\imath$ is the identity operation.

Definition 1.3. For the operations φ_1, $\varphi_2 \in O(X, \tau)$, φ_2 is called *regular with respect to* $\varphi_1 O(X)$ if for each $x \in X$ and $U, V \in \varphi_1 O(X, x)$, there exists a $W \in \varphi_1 O(X, x)$ such that $\varphi_2(W) \subseteq \varphi_2(U) \cap \varphi_2(V)$.

Clearly, if $\varphi_1 O(X)$ is closed under finite intersection and φ_2 is monotonous, then φ_2 is regular w.r.t. $\varphi_1 O(X)$.

Definition 1.4. Let $\varphi_1, \varphi_2 \in O(X, \tau)$, $A \subseteq X$, $x \in X$. Then:

(a) $x \in \varphi_{1,2}\mathrm{int}\, A$ iff there exists a $U \in \varphi_1 O(X, x)$ such that $\varphi_2(U) \subseteq A$.
(b) $x \in \varphi_{1,2}\mathrm{cl}\, A \iff \varphi_2(U) \cap A \neq \varnothing$ for each $U \in \varphi_1 O(X, x)$.
(c) A is $\varphi_{1,2}$-open $\iff A \subseteq \varphi_{1,2}\mathrm{int}\, A$.
(d) A is $\varphi_{1,2}$-closed $\iff \varphi_{1,2}\mathrm{cl}\, A \subseteq A$.

For any set A we have $X \setminus \varphi_{1,2}\mathrm{int}\, A = \varphi_{1,2}\mathrm{cl}\,(X \setminus A)$ and A is $\varphi_{1,2}$-open iff $X \setminus A$ is $\varphi_{1,2}$-closed.

Definition 1.5. [1] A subfamily $\mathcal{U}$ of the power set of a non-empty set X is called a *supratopology on* X if $\varnothing, X \in \mathcal{U}$ and $\mathcal{U}$ is closed under arbitrary unions.

If $\mathcal{U}$ is a supratopology on X, then the pair $(X, \mathcal{U})$ is called a *supratopological space*.

The notions of *base*, *first* and *second countablility* for a supratopology may be defined as for topological spaces [2].

If the operation $\varphi \in O(X, \tau)$ is monotonous, then $\varphi O(X)$ is a supratopology.

Theorem 1.6. [22] *Let* $\varphi_1, \varphi_2 \in O(X, \tau)$. *Then:*

(a) $\varphi_{1,2}O(X)$, *the family of all* $\varphi_{1,2}$-*open subsets of* X, *is a supratopology on* X.
(b) *If* φ_2 *is regular w.r.t.* $\varphi_1 O(X)$, *then the operator* $\varphi_{1,2}\mathrm{cl}$ *defines the topology* $\tau_{\varphi_{1,2}} = \{T \mid T \subseteq X,\ \varphi_{1,2}\mathrm{cl}\,(X \setminus T) \subseteq X \setminus T\} = \varphi_{1,2}O(X)$.
(c) *If* φ_2 *is regular w.r.t.* $\varphi_1 O(X)$ *and* $\varphi_1 O(X) \subseteq \varphi_2 O(X)$, *then the operator* $\varphi_{1,2}\mathrm{cl}$ *defines the topology* $\tau_{\varphi_{1,2}} = \{T \mid T \subseteq X, \varphi_{1,2}\mathrm{cl}\,(X \setminus T) = X \setminus T\} = \varphi_{1,2}O(X)$.
(d) *If* φ_2 *is regular w.r.t.* $\varphi_1 O(X)$, $\varphi_1 O(X) \subseteq \varphi_2 O(X)$, *and* $\varphi_2(U) \in \varphi_{1,2}O(X)$ *for each* $U \in \varphi_1 O(X)$, *then the operator* $\varphi_{1,2}\mathrm{cl}$ *is a Kuratowski closure operator and* $\varphi_{1,2}\mathrm{cl}\,A = \tau_{\varphi_{1,2}}\mathrm{cl}\,A,\ \forall\, A \subseteq X$.

Clearly if $\varphi_1 \in O(X,\tau)$ is monotonous and $\varphi_2 = \imath$ then $\varphi_{1,2}O(X) = \varphi_1 O(X)$ and $\varphi_{1,2}C(X) = \varphi_1 C(X)$.

The following example illustrates the wide range of well known concepts covered by the notions defined above.

Example 1.7. For the operations

$\varphi_1 = \mathrm{int}\,,\ \varphi_2 = \mathrm{cl} \circ \mathrm{int}\,,\ \varphi_3 = \mathrm{cl}\,,\ \varphi_4 = \mathrm{scl}\,,\ \varphi_5 = \imath,\ \varphi_6 = \mathrm{int} \circ \mathrm{cl}\,,$

defined on a topological space we have:

- $\varphi_1 \le \varphi_2 \le \varphi_3$ and $\varphi_1 \le \varphi_6 \le \varphi_4 \le \varphi_3$.
- $\varphi_1 O(X) = \tau$,
- $\varphi_2 O(X) = SO(X) =$ the family of semi-open sets.
- $\varphi_3 O(X) = \varphi_5 O(X) = P(X) =$ the power set of X.
- $\varphi_6 = PO(X) =$ the family of pre-open sets.
- $\varphi_{1,3}O(X) = \tau_\theta =$ the topology of all θ-open sets.
- $\varphi_{2,4}O(X) = S\theta O(X) =$ the family of semi-θ-open sets.
- $\varphi_{1,6}O(X) = \tau_s =$ the semi regularization topology of X.
- $\varphi_{2,3}O(X) = \theta SO(X) =$ the family of all θ-semi-open sets.
- The operations φ_1, φ_3 and φ_2, φ_6 are dual to one another.

All these operations are regular w.r.t. $\varphi_1 O(X)$.

2. $\varphi_{1,2}$–Countable Compactness

Definition 2.1. [21] Let $\varphi_1, \varphi_2 \in O(X,\tau)$, $X \in \mathcal{A} \subseteq P(X)$ and $A \in P(X)$. Then:
(a) If each countable $\mathcal{A}$-cover $\mathcal{U}$ of A has a finite subfamily $\mathcal{U}'$ such that $A \subseteq \bigcup\{\varphi_2(U) \mid U \in \mathcal{U}'\}$, then we say that A is $(\mathcal{A}$ - $\varphi_2)$-*countably compact relative to* X (for short, a $(\mathcal{A}$ - $\varphi_2)$-C.C. set).
(b) We call a $(\mathcal{A}$ - $\imath)$-C.C. set a $\mathcal{A}$-C.C. set.
(c) If we take $\mathcal{A} = \varphi_1 O(X)$ in (a) we say that A is a $\varphi_{1,2}$-C.C. set.
If we take $\mathcal{A} = \varphi_{1,2}O(X)$ in (b) we say that A is a $\varphi_{1,2}O(X)$-C.C. set.
If X is $\varphi_{1,2}$-C.C. ($\varphi_{1,2}O(X)$-C.C.) relative to itself, then X will be called a $\varphi_{1,2}$-*C.C.* ($\varphi_{1,2}O(X)$-*C.C.*) *space*.

We remark that the condition $X \in \mathcal{A}$ is added here, and in our earlier papers, to guarantee the existence of an $\mathcal{A}$-cover or of a countable $\mathcal{A}$-cover of a subset of X. However, all the results still hold without this condition.

One may define $\varphi_{1,2}$-*compact*, $\mathcal{A}$-*compact*, $\varphi_{1,2}$-*Lindelöf* and $\mathcal{A}$-*Lindelöf* sets in a similar way [20, 23].

We assume that all the operations φ_i, $i = 1, 2, \dots$ are defined on (X, τ) whenever they are used.

Example 2.2. Let $A \subseteq X$.

(1) If $\varphi_1 = \operatorname{int}$, $\varphi_2 = \imath$, then A is a $\varphi_{1,2}$-C.C. set iff A is countably compact.
(2) If $\varphi_1 = \operatorname{int}$, $\varphi_2 = \operatorname{cl}$, then A is a $\varphi_{1,2}$-C.C. set iff A is feebly compact relative to X [16], and X is $\varphi_{1,2}$-C.C. iff X is feebly compact (or, equivalently, lightly compact). X is $H(1)$-closed [16] iff it is a Hausdorff first countable $\varphi_{1,2}$-C.C. space with respect to these operations.
(3) If $\varphi_1 = \operatorname{cl} \circ \operatorname{int}$, $\varphi_2 = \operatorname{cl}$, then X is $\varphi_{1,2}$-C.C. iff it is countably S-closed [6].
(4) If $\varphi_1 = \operatorname{int}$, $\varphi_2 = \operatorname{int} \circ \operatorname{cl}$, then X is strongly $H(1)$-closed [19] iff it is a Hausdorff first countable $\varphi_{1,2}$-C.C. space.
(5) If $\varphi_1 = \operatorname{cl} \circ \operatorname{int}$, $\varphi_2 = \operatorname{scl}$, then X is $\varphi_{1,2}$-C.C. iff it is countably rs-compact [7].
(6) For $\varphi_1 = \operatorname{int} \circ \operatorname{cl} \circ \operatorname{int}$, $\varphi_2 = \imath$, we have $\varphi_1 O(X) = \varphi_{1,2} O(X) = \tau^\alpha$. Hence, X is countably α-compact [13] iff it is $\varphi_{1,2}$-C.C. iff it is $\varphi_{1,2} O(X)$-C.C. iff it is $\varphi_1 O(X)$-C.C.

Definition 2.3. Let $\mathcal{F}$ be a filter (or filterbase) on X, (x_n) a sequence in X and $a \in X$. We say that:

(a) $\mathcal{F}$, $\varphi_{1,2}$-*accumulates to* a, if $a \in \bigcap\{\varphi_{1,2}\operatorname{cl} F \mid F \in \mathcal{F}\}$ [20].
(b) $\mathcal{F}$, $\varphi_{1,2}$-*converges to* a, if for each $U \in \varphi_1 O(X, a)$, there exists $F \in \mathcal{F}$ such that $F \subseteq \varphi_2(U)$ [20].
(c) (x_n), $\varphi_{1,2}$-*accumulates to* a, if for each $U \in \varphi_1 O(X, a)$ and for each n, there exists an n_0 such that $n_0 \geq n$ and $x_{n_0} \in \varphi_2(U)$.
(d) (x_n), $\varphi_{1,2}$-*converges to* a, if for each $U \in \varphi_1 O(X, a)$, there exists an n_0 such that for each n $(n \geq n_0)$, $x_n \in \varphi_2(U)$.

Example 2.4. Let $\mathcal{F}$ be a filter (or filterbase) on X and $a \in X$.

(1) If $\varphi_1 = \operatorname{int}$, $\varphi_2 = \imath$, then $\mathcal{F}$, $\varphi_{1,2}$-converges to a iff $\mathcal{F}$ converges to a in (X, τ) and $\mathcal{F}$, $\varphi_{1,2}$-accumulates to a iff $\mathcal{F}$ accumulates to a (or a is an adherent point of $\mathcal{F}$) in (X, τ).
(2) If $\varphi_1 = \operatorname{int}$, $\varphi_2 = \operatorname{cl}$, then $\mathcal{F}$, $\varphi_{1,2}$-converges to a iff $\mathcal{F}$, r-converges [10] (or equivalently Θ-converges [9], almost converges [3]) to a, and $\mathcal{F}$, $\varphi_{1,2}$-accumulates to a iff a is an r-accumulation point [10] (or an almost adherent point [3]) of $\mathcal{F}$.
(3) For $\varphi_1 = \operatorname{cl} \circ \operatorname{int}$, $\varphi_2 = \operatorname{cl}$, it can be seen that, $\mathcal{F}$, $\varphi_{1,2}$-converges ($\varphi_{1,2}$-accumulates) to a iff $\mathcal{F}$, rc-converges (rc-accumulates) to a [9], since $\{\bar{V} \mid V \in \tau,\ a \in \bar{V}\} = \{\bar{U} \mid U \in SO(X),\ a \in U\}$. At the same time, $\mathcal{F}$, $\varphi_{1,2}$-converges ($\varphi_{1,2}$-accumulates) to a iff $\mathcal{F}$, s-converges (s-accumulates) to a [4].
(4) If $\varphi_1 = \operatorname{int} \circ \operatorname{cl} \circ \operatorname{int}$, $\varphi_2 = \imath$, then $\mathcal{F}$, $\varphi_{1,2}$-converges ($\varphi_{1,2}$-accumulates) to a iff $\mathcal{F}$, α-converges (α-accumulates) to a [14].

(5) If $\varphi_1 = \mathrm{cl} \circ \mathrm{int}$, $\varphi_2 = \mathrm{scl}$, it can be easily seen that $\mathcal{F}$, $\varphi_{1,2}$-converges ($\varphi_{1,2}$-accumulates) to a iff $\mathcal{F}$, SR-converges (SR-accumulates) to a [5].
(6) For $\varphi_1 = \mathrm{cl} \circ \mathrm{int}$, $\varphi_2 = \mathrm{int} \circ \mathrm{scl}$, then we see that $\mathcal{F}$, $\varphi_{1,2}$-converges ($\varphi_{1,2}$-accumulates) to a iff $\mathcal{F}$, RS-converges (RS-accumulates) to a [15].
(7) If $\varphi_1 = \mathrm{int}$, $\varphi_2 = \mathrm{int} \circ \mathrm{cl}$ then $\mathcal{F}$, $\varphi_{1,2}$-converges ($\varphi_{1,2}$-accumulates) to a iff $\mathcal{F}$, δ-converges (δ-accumulates) to a [19].

Similar characterizations of the various notions of convergence and accumulation point for sequences and nets given in the literature can be easily given, and we omit the details.

Theorem 2.5. *Let $A \subseteq X$ and $\mathcal{F} = \{F_n \mid n \in \mathbb{N}\}$ be a countable filterbase which meets A. If some sequence satisfying $x_n \in (\bigcap_{i=1}^n F_i) \cap A$ for each n, $\varphi_{1,2}$-accumulates to some point $a \in X$, then the filterbase $\mathcal{F}$, $\varphi_{1,2}$-accumulates to a.*

Conversely if for any sequence (x_n) in A the countable filterbase $\mathcal{F} = \{\{x_m \mid m \geq n\} \mid n \in \mathbb{N}\}$ which consists of the tails of the sequence (x_n), $\varphi_{1,2}$-accumulates to some point $a \in X$, then the sequence (x_n), $\varphi_{1,2}$-accumulates to a.

Proof. Let $\mathcal{F} = \{F_n \mid n \in \mathbb{N}\}$ be a countable filterbase which meets A. Then $\mathcal{F}' = \{\bigcap_{i=1}^n F_i \mid n \in \mathbb{N}\}$ is a decreasing countable filterbase which meets A and generates the same filter as $\mathcal{F}$. Take $x_n \in (\bigcap_{i=1}^n F_i) \cap A$ for each n, and let (x_n), $\varphi_{1,2}$-accumulate to a. Then, for each $U \in \varphi_1 O(X,a)$ and for each n, $\varnothing \neq \varphi_2(U) \cap (\bigcap_{i=1}^n F_i) \cap A \subseteq \varphi_2(U) \cap (\bigcap_{i=1}^n F_i)$, hence $\varphi_2(U) \cap F_n \neq \varnothing$. So, $\mathcal{F}$, $\varphi_{1,2}$-accumulates to a.

Conversely let (x_n) be a sequence in A, and let $\mathcal{F} = \{T_n \mid n \in \mathbb{N}\}$ be the countable filterbase consisting of the tails of (x_n), which $\varphi_{1,2}$-accumulate to some point a and meets A. Then for each $U \in \varphi_1 O(X,a)$ and for each n, $\varphi_2(U) \cap T_n \neq \varnothing$. This means that a is a $\varphi_{1,2}$-accumulation point of (x_n). □

Corollary 2.6. *Let $A \subseteq X$. Each countable filterbase which meets A, $\varphi_{1,2}$-accumulates to some point of A iff each sequence in A, $\varphi_{1,2}$-accumulates to some point of A.*

Theorem 2.7. *Let $A \subseteq X$. If each countable filterbase which meets A, $\varphi_{1,2}$-accumulates to some point of A, then A is a $\varphi_{1,2}$-C.C. set.*

Proof. Let $A \subseteq \bigcup \mathcal{U}$, $\mathcal{U} = \{U_n \mid n \in I\}$, I countable and $U_n \in \varphi_1 O(X)$. Assume that for each finite subset J of I we have $A \nsubseteq \bigcup_{i \in J} \varphi_2(U_i)$. Then $A \cap (X \setminus \bigcup_{i \in J} \varphi_2(U_i)) \neq \varnothing$. The family $\mathcal{F} = \{X \setminus \bigcup_{i \in J} \varphi_2(U_i) \mid J \subseteq I,\ J \text{ finite}\}$ is a countable filterbase which meets A. So, $A \cap (\bigcap \{\varphi_{1,2}\mathrm{cl}\, F \mid F \in \mathcal{F}\}) \neq \varnothing$. Let $\mathcal{F}$, $\varphi_{1,2}$-accumulate to $a \in A$. There exists an $i_0 \in I$ such that $a \in U_{i_0}$. Now $X \setminus \varphi_2(U_{i_0}) \in \mathcal{F}$, $\varphi_2(U_{i_0}) \cap (X \setminus \varphi_2(U_{i_0})) \neq \varnothing$. This contradiction completes the proof. □

However, the converse of the above theorem need not hold. For operations $\varphi_1 = \mathrm{int}$, $\varphi_2 = \mathrm{cl}$ in (X,τ), each countable filterbase $\varphi_{1,2}$-accumulates in

(X,τ) iff (X,τ) is SQ-closed [18]. Also, (X,τ) is $\varphi_{1,2}$-C.C. iff it is a feebly compact space. Herrington [11] gave an example, occurring in [8], of a regular, feebly compact but not countably compact space. Since this space is regular, a $\varphi_{1,2}$-accumulation point is the same as an accumulation point of a sequence (filterbase) in (X,τ), so there is a sequence (countable filterbase) which does not $\varphi_{1,2}$-accumulate to any point in X.

Clearly any $\varphi_{1,2}$-compact set is a $\varphi_{1,2}$-Lindelöf set and a $\varphi_{1,2}$-C.C. set. A set is a $\varphi_{1,2}O(X)$-compact set iff it is a $\varphi_{1,2}O(X)$-Lindelöf set and a $\varphi_{1,2}O(X)$-C.C. set. If $\varphi_{1,2}O(X)$ has a countable base then each $\varphi_{1,2}O(X)$-C.C. set is a $\varphi_{1,2}O(X)$-compact set.

We will define conditions $(*)$ and $(**)$ on the operations φ_1 and φ_2 in the following way:

$(*)$ $\varphi_2 \geq \varphi_1$ or $\varphi_2 \geq \imath$,
$(**)$ $\varphi_2(U) \in \varphi_1 O(X)$ and $\varphi_2(\varphi_2(U)) \subseteq \varphi_2(U)$, for each $U \in \varphi_1 O(X)$.

Example 2.8.

(1) If $\varphi_1 = \operatorname{int}$, $\varphi_2 = \operatorname{cl}$, then the condition $(*)$ is satisfied.
(2) If $\varphi_1 = \operatorname{cl}\circ\operatorname{int}$, $\varphi_2 = \operatorname{scl}$, then the conditions $(*)$ and $(**)$ are satisfied.
(3) If $\varphi_1 = \operatorname{int}$, $\varphi_2 = \operatorname{int}\circ\operatorname{cl}$, then the conditions $(*)$ and $(**)$ are satisfied.
(4) If $\varphi_1 = \operatorname{cl}\circ\operatorname{int}$, $\varphi_2 = \operatorname{cl}$, then the conditions $(*)$ and $(**)$ are satisfied.

If the condition $(**)$ is satisfied then a set is $\varphi_{1,2}$-compact set iff it is both a $\varphi_{1,2}$-Lindelöf set and a $\varphi_{1,2}$-C.C. set.

Theorem 2.9. *Let φ_1 be monotonous, $(X, \varphi_1 O(X))$ be a second countable supratopological space and $A \subseteq X$. If A is a $\varphi_{1,2}$-C.C. set then each filterbase which meets A, $\varphi_{1,2}$-accumulates to some point of A.*

Proof. Let the supratopology $\varphi_1 O(X)$ have a countable base, A be a $\varphi_{1,2}$-C.C. set and $\mathcal{F}$ a filterbase which meets A.

Assume that $A \cap (\bigcap\{\varphi_{1,2}\operatorname{cl} F \mid F \in \mathcal{F}\}) = \varnothing$. For any $x \in A$, there exists a $U_x \in \varphi_1 O(X,x)$ and an $F_x \in \mathcal{F}$ such that $\varphi_2(U_x) \cap F_x = \varnothing$. Now, $\mathcal{U} = \{U_x \mid x \in A\}$ is a φ_1-open open cover of A. Since $\varphi_1 O(X)$ has a countable base, $\mathcal{U}$ has a countable subfamily which covers A. Since A is a $\varphi_{1,2}$-C.C. set, there exists a finite subfamily $\{U_{x_1}, U_{x_2}, \ldots, U_{x_n}\}$ of $\mathcal{U}$ such that $A \subseteq \bigcup_{i=1}^n \varphi_2(U_{x_i})$. Now $(\bigcup_{i=1}^n \varphi_2(U_{x_i})) \cap (\bigcap_{i=1}^n F_{x_i}) = \varnothing$, so $A \cap (\bigcap_{i=1}^n F_{x_i}) = \varnothing$. This contradiction completes the proof. □

Corollary 2.10. *Under the assumptions of Theorem 2.9., the following are equivalent.*

(a) *A is a $\varphi_{1,2}$-C.C. set.*
(b) *A is a $\varphi_{1,2}$-compact set.*
(c) *Each countable filterbase which meets A, $\varphi_{1,2}$-accumulates to some point of A.*

Proof. In [20], it is shown that A is a $\varphi_{1,2}$-compact set iff each filterbase which meets A, $\varphi_{1,2}$-accumulates to some point of A. Since each $\varphi_{1,2}$-compact set is a $\varphi_{1,2}$-C.C. set, the proof is now clear from Theorem 2.7. □

Theorem 2.11. *Let φ_1, φ_2 be monotonous and suppose that the conditions $(*)$ and $(**)$ hold. If the supratopology $\varphi_1 O(X)$ has a countable base $\mathcal{B}(\varphi_1 O(X))$, then $\mathcal{B}' = \{\varphi_2(U) \mid U \in \mathcal{B}(\varphi_1 O(X))\}$ is a countable base for the supratopology $\varphi_{1,2}O(X)$.*

Proof. Under the given conditions, $\mathcal{B} = \{\varphi_2(U) \mid U \in \varphi_1 O(X)\}$ is a base for the supratopology $\varphi_{1,2}O(X)$ and $\mathcal{B}' \subseteq \mathcal{B} \subseteq \varphi_{1,2}O(X)$. Let $V \in \varphi_{1,2}O(X)$ and $x \in V$. There exists a $U \in \varphi_1 O(X, x)$ such that $\varphi_2(U) \subseteq V$. Hence, $x \in U \subseteq \varphi_2(U) \subseteq V$. There exists a $U^{'} \in \mathcal{B}(\varphi_1 O(X))$ such that $x \in U^{'} \subseteq U$. Hence, we have $x \in \varphi_2(U') \subseteq \varphi_2(U) \subseteq V$ and $\varphi_2(U') \in \mathcal{B}'$. □

Theorem 2.12. *Let $(*)$ and $(**)$ hold and let $\mathcal{B} = \{\varphi_2(U) \mid U \in \varphi_1 O(X)\}$. Then the following are equivalent for any subset A of X.*

(a) *A is a $\varphi_{1,2}$-compact set.*
(b) *A is a $\mathcal{B}$-compact set.*
(c) *A is both a $\varphi_{1,2}$-Lindelöf set and a $\varphi_{1,2}$-C.C. set.*
(d) *A is both a $\mathcal{B}$-Lindelöf set and a $\mathcal{B}$-C.C. set.*

Proof. Under the given conditions, A is a $\varphi_{1,2}$-compact set iff it is $\mathcal{B}$-compact set [20], A is a $\varphi_{1,2}$-Lindelöf set iff it is a $\mathcal{B}$-Lindelöf set [23], A is a $\varphi_{1,2}$-C.C. set iff it is a $\mathcal{B}$-C.C. set [22]. Hence (b) $\iff$ (d) is now clear, as are the others. □

Theorem 2.13. *Let φ_1, φ_2 be monotonous and suppose that the conditions $(*)$ and $(**)$ hold. If the supratopology $\varphi_1 O(X)$ has a countable base $\mathcal{B}(\varphi_1 O(X))$, or if $\mathcal{B} = \{\varphi_2(U) \mid U \in \varphi_1 O(X)\}$ is countable, then the following are equivalent.*

(a) *A is a $\varphi_{1,2}$-C.C. set.*
(b) *A is a $\varphi_{1,2}O(X)$-C.C. set.*
(c) *A is a $\mathcal{B}$-C.C. set.*
(d) *A is a $\varphi_{1,2}$-compact set.*
(e) *A is a $\varphi_{1,2}O(X)$-compact set.*
(f) *A is a $\mathcal{B}$-compact set.*

Proof. Under the conditions $(*)$ and $(**)$, (a) $\iff$ (c), (b) $\Longrightarrow$ (c) and (d) $\iff$ (e) $\iff$ (f) are given in [22] and [20] respectively. If $\mathcal{B}$ is a countable base of $\varphi_{1,2}O(X)$, then (c) $\Longrightarrow$ (b) is clear. In the other case, $\mathcal{B}^{'} = \{\varphi_2(U) \mid U \in \mathcal{B}(\varphi_1 O(X))\}$ is a countable base of $\varphi_{1,2}O(X)$ and $\mathcal{B}^{'} \subseteq \mathcal{B} \subseteq \varphi_{1,2}O(X)$. Hence, a $\mathcal{B}$-C.C. set will be a $\mathcal{B}^{'}$-C.C. set and a $\mathcal{B}^{'}$-C.C. set will be a $\varphi_{1,2}O(X)$-C.C. set, so we have again (c) $\Longrightarrow$ (b). In each case (b) $\iff$ (e) is clear. □

Theorem 2.14. *Let φ_1 be monotonous and let $a \in X$ have a countable local base $C_{\varphi_1}(a)$ in the supratopological space $(X, \varphi_1 O(X))$.*

(1) *If φ_2 is monotonous and regular w.r.t. $\varphi_1 O(X)$, then the family $\mathcal{F} = \{\varphi_2(U) \mid U \in C_{\varphi_1}(a)\}$ is a countable filterbase and $\varphi_{1,2}$-converges to a.*
(2) *If $\varphi_1 O(X)$ is a topology and $\varphi_1 O(X) \subseteq \varphi_2 O(X)$, then $C_{\varphi_1}(a)$ is a countable filterbase which $\varphi_{1,2}$-converges to a.*

Proof. (1) For $U, U' \in C_{\varphi_1}(a)$, $a \in U \cap U'$ and $U, U' \in \varphi_1 O(X)$. Since φ_2 is regular w.r.t. $\varphi_1 O(X)$, there exists a $V \in \varphi_1 O(X, a)$ such that $\varphi_2(V) \subseteq \varphi_2(U) \cap \varphi_2(U')$. There exists a $V_c \in C_{\varphi_1}(a)$ such that $V_c \subseteq V$. Since φ_2 is monotonous, we have $\varphi_2(V_c) \subseteq \varphi_2(V) \subseteq \varphi_2(U) \cap \varphi_2(U')$. Hence $\mathcal{F}$ is a countable filterbase. Let $U \in \varphi_1 O(X, a)$. There exists a $U_c \in C_{\varphi_1}(a)$ such that $U_c \subseteq U$. $\varphi_2(U_c) \in \mathcal{F}$ and, since φ_2 is monotonous $\varphi_2(U_c) \subseteq \varphi_2(U)$. So, $\mathcal{F}$ is $\varphi_{1,2}$-convergent to a.

(2) For $U, U' \in C_{\varphi_1}(a)$, $a \in U \cap U' \in \varphi_1 O(X, a)$. There exists a $U_c \in C_{\varphi_1}(a)$ such that $U_c \subseteq U \cap U'$. Hence $C_{\varphi_1}(a)$ is a countable filterbase.

Now, let $V \in \varphi_1 O(X, a)$. There exists a $V_c \in C_{\varphi_1}(a)$ such that $V_c \subseteq V$. Since $\varphi_1 O(X) \subseteq \varphi_2 O(X)$, we have $V_c \subseteq V \subseteq \varphi_2(V)$. Hence $C_{\varphi_1}(a)$, $\varphi_{1,2}$-converges to a. □

Theorem 2.15. *Let φ_1, φ_2 be monotonous, let $a \in X$ have a countable local base $C_{\varphi_1}(a)$ in $(X, \varphi_1 O(X))$ and also let φ_2 be regular w.r.t. $\varphi_1 O(X)$. For $A \subseteq X$, $a \in \varphi_{1,2}\mathrm{cl}\, A$ iff there exists a filter which contains A, has a countable base and $\varphi_{1,2}$-converges to a.*

Proof. Let $a \in \varphi_{1,2}\mathrm{cl}\, A$. Then for each $U \in \varphi_1 O(X, a)$, $\varphi_2(U) \cap A \neq \varnothing$. As in the proof of Theorem 2.14.(1), it is easly seen that $\mathcal{F}_b = \{\varphi_2(V) \cap A \mid V \in C_{\varphi_1}(a)\}$ is a countable filterbase. The filter $\mathcal{F}$ generated by $\mathcal{F}_b$ contains A, and $\{\varphi_2(V) \mid V \in C_{\varphi_1}(a)\} \subseteq \mathcal{F}$. Clearly $\mathcal{F}$ is $\varphi_{1,2}$-convergent to a.

The other part of the proof is clear from Corollary 3.4. in [20]. □

Theorem 2.16. *Let φ_1, φ_2 be monotonous, $(X, \varphi_1 O(X))$ be a first countable supratopological space, and define $\mathrm{cl}^* : P(X) \longrightarrow P(X)$ by $\mathrm{cl}^*(A) = \{ x \mid$ there exists a filter that contains A, has a countable base and $\varphi_{1,2}$-converges to $x\}$, for each $A \in P(X)$.*

1. *If φ_2 is regular w.r.t. $\varphi_1 O(X)$, then $\mathrm{cl}^*(A) = \varphi_{1,2}\mathrm{cl}\, A$ for each $A \in P(X)$, and cl^* defines the topology $\tau^* = \{U \subseteq X \mid (X \setminus U)^* \subseteq X \setminus U\} = \varphi_{1,2} O(X)$.*
2. *If φ_2 is regular w.r.t. $\varphi_1 O(X)$ and $\varphi_1 O(X) \subseteq \varphi_2 O(X)$, then cl^* defines the topology $\tau^* = \{U \subseteq X \mid (X \setminus U)^* = X \setminus U\} = \varphi_{1,2} O(X)$.*
3. *If φ_2 is regular w.r.t. $\varphi_1 O(X)$, $\varphi_1 O(X) \subseteq \varphi_2 O(X)$, and $\varphi_2(U) \in \varphi_{1,2} O(X)$ for each $U \in \varphi_1 O(X)$, then the operator cl^* is a Kuratowski closure operator defining $\tau^* = \{U \subseteq X \mid (X \setminus U)^* = X \setminus U\} = \varphi_{1,2} O(X)$.*

Hence, if φ_1, φ_2 are monotonous and $(X, \varphi_1 O(X))$ is a first countable topological space, then the $\varphi_{1,2}$-closure operator and the topology $\tau_{\varphi_{1,2}} = \{U \subseteq X \mid \varphi_{1,2}\mathrm{cl}\,(X \setminus U) \subseteq X \setminus U\} = \varphi_{1,2} O(X)$ can be defined using filters with countable bases.

Proposition 2.17. *If $\varphi_1 O(X) \subseteq \varphi_2 O(X)$ (hence, if $\varphi_2 \geq \varphi_1$ or $\varphi_2 \geq \imath$), then $A \subseteq \varphi_{1,2}\mathrm{cl}\, A$ for each $A \in P(X)$.*

Proposition 2.18. *If* $(**)$ *holds, then* $\varphi_2(U) \subseteq \varphi_{1,2}\mathrm{int}\,(\varphi_2(U))$ *(i.e.,* $\varphi_2(U) \in \varphi_{1,2}O(X)$*) for each* $U \in \varphi_1 O(X)$*.*

Proof. Let $U \in \varphi_1 O(X)$ and $x \in \varphi_2(U)$. Then $x \in \varphi_2(U) \in \varphi_1 O(X)$ and $\varphi_2(\varphi_2(U)) \subseteq \varphi_2(U)$. So $x \in \varphi_{1,2}\mathrm{int}\,(\varphi_2(U))$. □

Corollary 2.19. (a) *Under the condition* $(**)$*, we have,* $\varphi_{1,2}\mathrm{cl}\,(X \setminus \varphi_2(U)) \subseteq X \setminus \varphi_2(U)$ *for each* $U \in \varphi_1 O(X)$*.*

(b) *If* $\varphi_1 O(X) \subseteq \varphi_2 O(X)$ *and* $(**)$ *holds, then* $\varphi_{1,2}\mathrm{cl}\,(X \setminus \varphi_2(U)) = X \setminus \varphi_2(U)$ *for each* $U \in \varphi_1 O(X)$*.*

Remark 2.20. a) If $\tilde{\varphi_2}$ is the dual operation of φ_2, then $\{X \setminus \varphi_2(U) \mid U \in \varphi_1 O(X)\} = \{\tilde{\varphi_2}(X \setminus U) \mid U \in \varphi_1 O(X)\} = \{\tilde{\varphi_2}(K) \mid K \in \varphi_1 C(X)\}$.

b) If φ_1 is monotonous (in which case $\varphi_1 O(X)$ is a supratopology), and $\varphi_2(U \cup V) = \varphi_2(U) \cup \varphi_2(V)$ for each $U, V \in \varphi_1 O(X)$, then for each finite subfamily $\{U_1, U_2, \ldots, U_n\}$ of $\varphi_1 O(X)$, $\bigcup_{i=1}^n U_i \in \varphi_1 O(X)$ and $\varphi_2(\bigcup_{i=1}^n U_i) = \bigcup_{i=1}^n \varphi_2(U_i)$.

Theorem 2.21. *Consider the following statements:*

(i) φ_1 *is monotonous.*
(ii) φ_2 *is monotonous.*
(iii) $\varphi_2 \geq \imath$ *or* $\varphi_2 \geq \varphi_1$ *(i.e.* $(*)$*),*
(iv) $\forall\, U \in \varphi_1 O(X),\ \varphi_2(U) \in \varphi_1 O(X)$ *and* $\varphi_2(\varphi_2(U)) \subseteq \varphi_2(U)$ *(i.e.* $(**)$*).*
(v) *For each* $U, V \in \varphi_1 O(X)$, $\varphi_2(U \cup V) = \varphi_2(U) \cup \varphi_2(V)$*,*
(vi) $\tilde{\varphi_2}$ *is the dual of* φ_2*.*

and

(a) A *is a* $\varphi_{1,2}$*-C.C. set.*
(b) *Each countable filterbase* $\mathcal{F} \subseteq \{X \setminus \varphi_2(U) \mid U \in \varphi_1 O(X)\}$ *which meets* A*,* $\varphi_{1,2}$*-accumulates to some point of* A*.*
(c) *For each countable filterbase* $\mathcal{F} \subseteq \{X \setminus \varphi_2(U) \mid U \in \varphi_1 O(X)\}$ *which meets* A*, we have* $A \cap (\bigcap \mathcal{F}) \neq \varnothing$*.*
(d) *For each decreasing countable filterbase* $\mathcal{F} \subseteq \{X \setminus \varphi_2(U) \mid U \in \varphi_1 O(X)\}$ *which meets* A*, we have* $A \cap (\bigcap \{\varphi_{1,2}clF \mid F \in \mathcal{F}\}) \neq \varnothing$*.*
(e) *For each decreasing countable filterbase* $\mathcal{F} \subseteq \{X \setminus \varphi_2(U) \mid U \in \varphi_1 O(X)\}$ *which meets* A*, we have* $A \cap (\bigcap \mathcal{F}) \neq \varnothing$*.*
(f) *If* Φ *is any decreasing sequence of countable non-empty* φ_1*-closed sets such that for each* $F \in \Phi$*,* $A \cap \tilde{\varphi_2}(F) \neq \varnothing$*, then* $A \cap (\bigcap \Phi) \neq \varnothing$*.*

Then,

(1) $(b) \Longrightarrow (d)$ *and* $(c) \Longrightarrow (e)$*.*
(2) *If* (iii) *holds, then* $(c) \Longrightarrow (b)$ *and* $(e) \Longrightarrow (d)$*.*
(3) *If* (iii) *and* (iv) *hold, then* $(c) \iff (b)$ *and* $(e) \iff (d)$*.*
(4) *If* (iv) *holds, then* $(a) \Longrightarrow (c)$*.*
(5) *If* (i) *and* (v) *hold, then* $(d) \Longrightarrow (b)$ *and* $(b) \Longrightarrow (a)$*.*
(6) *If* (ii) *and* (vi) *hold, then* $(a) \Longrightarrow (f)$*.*
(7) *If* (i)*,* (iii)*,* (v) *and* (vi) *hold, then* $(f) \Longrightarrow (a)$*.*

Proof. (1) Immediate.

2) Clear from Proposition 2.17.

(3) Clear from Corollary 2.19.

(4) Let A be a $\varphi_{1,2}$-C.C. set, and $\mathcal{F} = \{X \setminus \varphi_2(U_i) \mid i \in I\}$, $U_i \in \varphi_1 O(X)$, be a countable filterbase which meets A. Assume that $A \cap (\bigcap \mathcal{F}) = \varnothing$ and $A \subseteq \bigcup_{i\in I} \varphi_2(U_i)$. Since, $\varphi_2(U) \in \varphi_1 O(X)$, $\varphi_2(\varphi_2(U)) \subseteq \varphi_2(U)$, for each $U \in \varphi_1 O(X)$, and A is a $\varphi_{1,2}$-C.C. set, there exists a finite subset J of I such that, $A \subseteq \bigcup_{i\in J} \varphi_2(\varphi_2(U_i)) \subseteq \bigcup_{i\in J} \varphi_2(U_i)$. We have $A \cap (\bigcap_{i\in J}(X \setminus \varphi_2(U_i))) = \varnothing$. This contradiction completes the proof.

(5) Let $\mathcal{F} \subseteq \{X \setminus \varphi_2(U) \mid U \in \varphi_1 O(X)\}$ be a countable filterbase which meets A. Then $\mathcal{F} = \{F_n \mid n \in \mathbb{N}\}$, where $F_n = X \setminus \varphi_2(U_n)$, $n \in \mathbb{N}$ and $U_n \in \varphi_1 O(X)$. Let $F'_n = \bigcap_{i=1}^n F_i$ for each n. Then $\mathcal{F}' = \{F'_n \mid n \in \mathbb{N}\}$ is a decreasing countable filterbase, and $F'_n = \bigcap_{i=1}^n F_i = \bigcap_{i=1}^n (X \setminus \varphi_2(U_i)) = X \setminus \bigcup_{i=1}^n \varphi_2(U_i) = X \setminus \varphi_2(\bigcup_{i=1}^n U_i)$. Hence, $\mathcal{F}' \subseteq \{X \setminus \varphi_2(U) \mid U \in \varphi_1 O(X)\}$.

If we assume that (d) holds then $A \cap (\bigcap\{\varphi_{1,2}\mathrm{cl}\, F'_n \mid F'_n \in \mathcal{F}'\}) \neq \varnothing$. Since $F'_n \subseteq F_n$ for each n, we have $\varphi_{1,2}\mathrm{cl}\, F'_n \subseteq \varphi_{1,2}\mathrm{cl}\, F_n$. So $A \cap (\bigcap\{\varphi_{1,2}\mathrm{cl}\, F_n \mid F_n \in \mathcal{F}\}) \neq \varnothing$.

Now, let us verify that $(b) \Longrightarrow (a)$. Let $A \subseteq \bigcup \mathcal{U}$, $\mathcal{U} \subseteq \varphi_1 O(X)$ and $\mathcal{U} = \{U_i \mid i \in I\}$ be countable. Assume that for each finite subset J of I, $A \not\subseteq \bigcup_{i\in J} \varphi_2(U_i)$. Then, $A \cap (X \setminus \bigcup_{i\in J} \varphi_2(U_i)) \neq \varnothing$. From our hypotheses, $\bigcup_{i\in J} U_i \in \varphi_1 O(X)$ and $\varphi_2(\bigcup_{i\in J} U_i) = \bigcup_{i\in J} \varphi_2(U_i)$. So, for each finite subset J of I, we have $A \cap (X \setminus \varphi_2(\bigcup_{i\in J} U_i)) \neq \varnothing$. Let $\mathcal{F} = \{X \setminus \varphi_2(\bigcup_{i\in J} U_i) \mid J \subseteq I, J \text{ finite}\}$. Then $\mathcal{F} \subseteq \{X \setminus \varphi_2(U) \mid U \in \varphi_1 O(X)\}$ and $\mathcal{F}$ is a countable filterbase which meets A. There exists an $a \in A$ such that $a \in \bigcap\{\varphi_{1,2}\mathrm{cl}\, F \mid F \in \mathcal{F}\}$ and a $U_a \in \mathcal{U}$ such that $a \in U_a$. Now, $X \setminus \varphi_2(U_a) \in \mathcal{F}$ and $\varphi_2(U_a) \cap (X \setminus \varphi_2(U_a)) = \varnothing$. This contradiction completes the proof.

(6) Let Φ be a countable decreasing sequence of nonempty φ_1-closed sets such that for each $F \in \Phi$, $A \cap \tilde{\varphi}_2(F) \neq \varnothing$. Assume that $A \cap (\bigcap \Phi) = \varnothing$. Then, $A \subseteq \bigcup\{X \setminus F \mid F \in \Phi\}$. Since for each $F \in \Phi$, $X \setminus F \in \varphi_1 O(X)$, and A is a $\varphi_{1,2}$-C.C. set, there exists a finite subfamily Φ' of Φ such that $A \subseteq \bigcup\{\varphi_2(X \setminus F) \mid F \in \Phi'\}$. Since φ_2 is monotonous, $A \subseteq \varphi_2(\bigcup_{F\in\Phi'}(X \setminus F))$. There exists an $F' \in \Phi'$ such that $\bigcup_{F\in\Phi'}(X \setminus F) = X \setminus F'$. Then $A \subseteq \varphi_2(X \setminus F') = X \setminus \tilde{\varphi}_2(F')$, so $A \cap \tilde{\varphi}_2(F') = \varnothing$. This contradiction completes the proof.

(7) Let $\mathcal{U} = \{U_n \mid n \in \mathbb{N}\}$ be a countable φ_1-open cover of A. Assume that for each finite subset J of $\mathbb{N}$, $A \not\subseteq \bigcup_{i\in J} \varphi_2(U_i)$. In this case, for each finite subset J of $\mathbb{N}$, $X \neq \bigcup_{i\in J} U_i$ since, otherwise, we would have $A \subseteq \bigcup_{i\in J} U_i \subseteq \bigcup_{i\in J} \varphi_2(U_i)$ for a finite subset J of $\mathbb{N}$.

Let $F_n = X \setminus \bigcup_{i=1}^n U_i$ for each n. For each n, $F_n \neq \varnothing$, $F_n \in \varphi_1 C(X)$ and $A \cap (X \setminus \bigcup_{i=1}^n \varphi_2(U_i)) \neq \varnothing$. Now

$$A \cap (X \setminus \bigcup_{i=1}^{n} \varphi_2(U_i)) = A \cap (X \setminus \varphi_2(\bigcup_{i=1}^{n} U_i))$$
$$= A \cap (\tilde{\varphi_2}(X \setminus \bigcup_{i=1}^{n} U_i))$$
$$= A \cap \tilde{\varphi_2}(F_n)$$
$$\neq \varnothing.$$

Hence, $A \cap (\bigcap_{n=1}^{\infty} F_n) \neq \varnothing$. But $\bigcap_{n=1}^{\infty} F_n = X \setminus (\bigcup_{n=1}^{\infty} U_n)$ and we obtain that $A \cap (X \setminus \bigcup_{n=1}^{\infty} U_n) = \varnothing$. This contradiction completes the proof. □

Example 2.22.

(1) If $\varphi_1 = \mathrm{int}$, $\varphi_2 = \mathrm{cl}$, then $\tilde{\varphi_2} = \mathrm{int}$ and the conditions (i), (ii), (iii), (v) and (vi) are satisfied.

(2) If $\varphi_1 = \mathrm{cl} \circ \mathrm{int}$, $\varphi_2 = \mathrm{scl}$, then conditions (i), (ii), (iii), (iv) and (vi) are satisfied, and $\tilde{\varphi_2} =$ semi-interior is the dual of φ_2.

(3) If $\varphi_1 = \mathrm{cl} \circ \mathrm{int}$, $\varphi_2 = \mathrm{cl}$, then $\tilde{\varphi_2} = \mathrm{int}$ and all the conditions are satisfied.

Many known results, see for example [6,11,17,18,19], and also many new results, may now be obtained by choosing particular operations and combining the above results with the unifications obtained in [20-23].

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