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Embedding into discretely absolutely star-Lindelöf spaces II

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Abstract. A space X is *discretely absolutely star-Lindelöf* if for every open cover $\mathcal{U}$ of X and every dense subset D of X, there exists a countable subset F of D such that F is discrete closed in X and $St(F,\mathcal{U}) = X$, where $St(F,\mathcal{U}) = \bigcup\{U \in \mathcal{U} : U \cap F \neq \varnothing\}$. We show that every Hausdorff star-Lindelöf space can be represented in a Hausdorff discretely absolutely star-Lindelöf space as a closed G_δ-subspace.

Keywords: star-Lindelöf, absolutely star-Lindelöf, centered-Lindelöf

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1. Introduction

By a space, we mean a topological space. A space X is *absolutely star-Lindelöf* (see [1]) (*discretely absolutely star-Lindelöf*)(see [12, 13]) if for every open cover $\mathcal{U}$ of X and every dense subset D of X, there exists a countable subset F of D such that $St(F,\mathcal{U}) = X$ (F is discrete and closed in X and $St(F,\mathcal{U}) = X$, respectively), where $St(F,\mathcal{U}) = \bigcup\{U \in \mathcal{U} : U \cap F \neq \varnothing\}$.

A space X is *star-Lindelöf* (see [4, 7] under different names) (*discretely star-Lindelöf*)(see [9, 16]) if for every open cover $\mathcal{U}$ of X, there exists a countable subset (a countable discrete closed subset, respectively) F of X such that $St(F,\mathcal{U}) = X$. It is clear that every separable space and every discretely star-Lindelöf space are star-Lindelöf as well as every space of countable extent(in particular, every countably compact space or every Lindelöf space).

A family of subsets is *centered* (*linked*) provided every finite subfamily (every two elements, respectively) has nonempty intersection and a family is called

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σ-centered (*σ-linked*) if it is the union of countably many centered subfamilies(linked subfamilies, respectively). A space X is *centered-Lindelöf* (*linked-Lindelöf*) (see [2, 3]) if for every open cover $\mathcal{U}$ of X has σ-centered (σ-linked) subcover.

From the above definitions, it is not difficult to see that every discretely absolutely star-Lindelöf space is absolutely star-Lindelöf, every discretely absolutely star-Lindelöf space is discretely star-Lindelöf, every absolutely star-Lindelöf space is star-Lindelöf, every star-Lindelöf space is centered-Lindelöf, every centered-Lindelöf space is linked-Lindelöf.

Bonanzinga and Matveev [2] proved that every Hausdorff (regular, Tychonoff) linked-Lindelöf space can be represented as a closed subspace in a Hausdorff (regular, Tychonoff, respectively)star-Lindelöf space. They asked if every Hausdorff (regular, Tychonoff) linked-Lindelöf space can be represented as a closed G_δ-subspace in a Hausdorff (regular, Tychonoff, respectively) star-Lindelöf space. The author [10] gave a positive answer to their question. The author [10] showed that every Hausdorff (regular, Tychonoff) linked-Lindelöf space can be represented as a closed G_δ-subspace in a Hausdorff (regular, Tychonoff, respectively) absolutely star-Lindelöf space. The author [13] showed that every separable Hausdorff (regular, Tychonoff, normal) star-Lindelöf space can be represented in a Hausdorff (regular, Tychonoff, normal, respectively) discretely absolutely star-Lindelöf space as a closed G_δ-subspace. The author [14] showed that every Hausdorff linked-Lindelöf space can be represented in a Hausdorff discretely absolutely star-Lindelöf space as a closed subspace and asked the following question:

Question 1.1. *Is it true that every Hausdorff* (*regular, Tychonoff*) *linked-Lindelöf-space can be represented a closed G_δ-subspace in a Hausdorff* (*regular, Tychonoff, respectively*) *discretely absolutely star-Lindelöf space?*

The purpose of this note is to give a construction showing every Hausdorff linked-Lindelöf space can be represented in a Hausdorff discretely absolutely star-Lindelöf space as a closed G_δ-subspace, which give a positive answer to the above question in the class of Hausdorff spaces.

Throughout this paper, the cardinality of a set A is denoted by $|A|$. Let ω denote the first infinite cardinal. For a cardinal κ, let κ^+ be the smallest cardinal greater than κ. As usual, a cardinal is the initial ordinal and an ordinal is the set of smaller ordinals. When viewed as a space, every cardinal has the usual order topology. For each pair of ordinals α, β with $\alpha < \beta$, we write $[\alpha, \beta] = \{\gamma : \alpha \leq \gamma \leq \beta\}$ and $(\alpha, \beta) = \{\gamma : \alpha < \gamma < \beta\}$. Other terms and symbols that we do not define will be used as in [5].

2. Embedding into discretely absolutely star-Lindelöf spaces as a closed G_δ-subspaces

First, we show that every Hausdorff star-Lindelöf space can be represented in a Hausdorff discretely absolutely star-Lindelöf space as a closed G_δ-subspace.

Recall the Alexandorff duplicate $A(X)$ of a space X. The underlying set of $A(X)$ is $X \times \{0,1\}$; each point of $X \times \{1\}$ is isolated and a basic neighborhood of a point $\langle x, 0\rangle \in X \times \{0\}$ is of the from $(U \times \{0\}) \cup ((U \times \{1\}) \setminus \{\langle x, 1\rangle\})$, where U is a neighborhood of x in X. It is well-known that $A(X)$ is Hausdorff(regular, Tychonoff, normal) iff X is, $A(X)$ is compact iff X is and $A(X)$ is Lindelöf iff X is.

Recall from [6] that a space X is *absolutely countably compact* (=acc) if for every open cover $\mathcal{U}$ of X and every dense subset D of X, there exists a finite subset F of D such that $St(F,\mathcal{U}) = X$. It is not difficult to show that every Hausdorff acc space is countably compact (see [6]). In our construction, we use the following lemma.

Lemma 2.1 ([8, 15])**.** *If X is countably compact, then $A(X)$ is acc. Moreover, for any open cover $\mathcal{U}$ of $A(X)$, there exists a finite subset F of $X \times \{1\}$ such that $A(X) \setminus St(F,\mathcal{U}) \subseteq X \times \{0\}$ is a finite subset consisting of isolated points of $X \times \{0\}$.*

Theorem 2.2. *Every Hausdorff star-Lindelöf space can be represented in a Hausdorff discretely absolutely star-Lindelöf space as a closed G_δ-subspace.*

Proof. If $|X| \leq \omega$, then X is separable. The author [13] showed that every separable Hausdorff (regular, Tychonoff, normal) space can be represented in Hausdorff (regular, Tychonoff, normal, respectively) discretely absolutely star-Lindelöf space as a closed G_δ-subspace.

Let X be a star-Lindelöf space with $|X| > \omega$ and let T be X with the discrete topology and let

$$Y = T \cup \{\infty\}, \text{ where } \infty \notin T$$

be the one-point Lindelöfication of T. Pick a cardinal κ with $\kappa \geq |X|$. Define

$$S(X,\kappa) = X \cup (Y \times \kappa^+).$$

We topologize $S(X,\kappa)$ as follows: $Y \times \kappa^+$ has the usual product topology and is an open subspace of $S(X,\kappa)$, and a basic neighborhood of a point x of X takes the form

$$G(U,\alpha) = U \cup (U \times (\alpha, \kappa^+)),$$

where U is a neighborhood of x in X and $\alpha < \kappa^+$. Then, it is easy to see that X is a closed subset of $S(X,\kappa)$ and $S(X,\kappa)$ is Hausdorff if X is Hausdorff.

Let

$$\mathcal{R}(X) = A(S(X,\kappa)) \setminus (X \times \{1\}).$$

Then, $\mathcal{R}(X)$ is Hausdorff if X is Hausdorff.

Let

$$\mathcal{P}(\mathcal{R}(X)) = ((X \times \{0\}) \times \{\omega\}) \cup (\mathcal{R}(X) \times \omega)$$

be the subspace of the product of $\mathcal{R}(X) \times (\omega + 1)$. For each $n \in \omega$, let

$$X_\omega = (X \times \{0\}) \times \{\omega\} \text{ and } X_n = \mathcal{R}(X) \times \{n\} \text{ for each } n \in \omega.$$

Then,

$$\mathcal{P}(\mathcal{R}(X)) = X_\omega \cup \cup_{n\in\omega} X_n.$$

From the construction of the topology of $\mathcal{P}(\mathcal{R}(X))$, it is not difficult to see that X can be represented in $\mathcal{P}(\mathcal{R}(X))$ as a closed G_δ-subspace, since X is homeomorphic to X_ω, and $\mathcal{P}(\mathcal{R}(X))$ is Hausdorff if X is Hausdorff.

We show that $\mathcal{P}(\mathcal{R}(X))$ is discretely absolutely star-Lindelöf. To this end, let $\mathcal{U}$ be an open cover of $\mathcal{P}(\mathcal{R}(X))$. Without loss of generality, we assume that $\mathcal{U}$ consists of basic open sets of $\mathcal{P}(\mathcal{R}(X))$. Let S be the set of all isolated points of κ^+ and let

$$D_{n1} = (((T \times S) \times \{0\}) \times \{n\}) \cup (((T \times \kappa^+) \times \{1\}) \times \{n\}),$$

$$D_{n2} = ((\{\infty\} \times \kappa^+) \times \{1\}) \times \{n\} \text{ and } D_n = D_{n1} \cup D_{n2} \text{ for each } n \in \omega.$$

If we put $D = \cup_{n\in\omega} D_n$. Then, every element of D is isolated in $\mathcal{P}(\mathcal{R}(X))$, and every dense subset of $\mathcal{P}(\mathcal{R}(X))$ contains D. Thus, it is sufficient to show that there exists a countable subset F of D such that

$$F \text{ is discrete closed in } \mathcal{P}(\mathcal{R}(X)) \text{ and } St(F, \mathcal{U}) = \mathcal{P}(\mathcal{R}(X)).$$

For each $x \in X$, there exists a $U_x \in \mathcal{U}$ such that $\langle\langle x, 0\rangle, \omega\rangle \in U_x$, Hence there exist $\alpha_x < \kappa^+$, $n_x \in \omega$ and an open neighborhood V_x of x in X such that

$$((V_x \times \{0\}) \times [n_x, \omega]) \cup (A(V_x \times (\alpha_x, \kappa^+)) \times [n_x, \omega)) \subseteq U_x.$$

If we put $\mathcal{V} = \{V_x : x \in X\}$, then $\mathcal{V}$ is an open cover of X. For each $n \in \omega$, let $X'_n = \cup\{x : n_x = n\}$, then $X = \cup_{n\in\omega} X'_n$. For each $x' \in X \setminus X'_n$, there exists a $U_{x'} \in \mathcal{U}$ such that

$$\langle\langle x', 0\rangle, n\rangle \in U_{x'}.$$

Hence, there exist $\alpha_{x'} < \kappa^+$ and an open neighborhood $V_{x'}$ of x' in X such that

$$((V_{x'} \times \{0\}) \times \{n\}) \cup (A(V_{x'} \times (\alpha_{x'}, \kappa^+)) \times \{n\}) \subseteq U_{x'}.$$

If we put

$$\mathcal{V}_n = \{V_x : x \in X'_n\} \cup \{V_{x'} : x' \in X \setminus X'_n\}.$$

Then, $\mathcal{V}_n$ is an open cover of X. Hence, there exists a countable subset F'_n of X such that $X = St(F'_n, \mathcal{U})$, since X is star-Lindelöf. If we pick

$$\alpha_{n0} > \max\{\sup\{\alpha_x : x \in X'_n\}, \sup\{\alpha_{x'} : x' \in X \setminus X'_n\}\}.$$

Then, $\alpha_{n0} < \kappa^+$, since $|X| \le \kappa$.

Let

$$X_{n1} = ((X \times \{0\}) \times \{n\}) \cup (A(T \times [\alpha_{n0}, \kappa^+)) \times \{n\});$$

$$X_{n2} = A(T \times [0, \alpha_{n0}]) \times \{n\} \text{ and } X_{n3} = A(\{\infty\} \times \kappa^+) \times \{n\}).$$

Then,

$$X_n = X_{n1} \cup X_{n2} \cup X_{n3}.$$

Let

$$F_{n1} = ((F'_n \times \{\alpha_{n0}\}) \times \{1\}) \times \{n\}.$$

Then, F_{n1} is a countable subset of D_{n1} and

$$((X'_n \times \{0\}) \times \{\omega\}) \cup X_{n1} \subseteq St(F_{n1}, \mathcal{U}),$$

since $U_x \cap F_{n1} \neq \varnothing$ for each $x \in X'_n$ and $U_{x'} \cap F_{n1} \neq \varnothing$ for each $x' \in X \setminus X'_n$. Since $F_{n1} \subseteq D_{n1}$ and F_{n1} is countable. Then, F_{n1} is closed in X_n by the construction of the topology of X_n. Hence, F_{n1} is closed in $\mathcal{P}(\mathcal{R}(X))$, since X_n is open and closed in $\mathcal{P}(\mathcal{R}(X))$.

On the other hand, since Y is Lindelöf and $[0, \alpha_{n0}]$ is compact, then $Y \times [0, \alpha_{n0}]$ is Lindelöf, hence $X_{n2} = A(Y \times [0, \alpha_{n0}]) \times \{n\}$ is Lindelöf. For each $\alpha \leq \alpha_{n0}$, there exists a $U_\alpha \in \mathcal{U}$ such that

$$\langle\langle\langle\infty, \alpha\rangle, 0\rangle, n\rangle \in U_\alpha.$$

Hence, there exists an open neighborhood V_α of α in κ^+ and an open neighborhood V'_α of ∞ in Y such that

$$(A(V'_\alpha \times V_\alpha) \times \{n\}) \setminus (\langle\langle\langle\infty, \alpha\rangle, 1\rangle, n\rangle) \subseteq U_\alpha.$$

Let $\mathcal{V}'_n = \{V_\alpha : \alpha \leq \alpha_{n0}\}$. Then, $\mathcal{V}'_n$ is an open cover of $[0, \alpha_{n0}]$. Hence, there exists a finite subcover $V_{\alpha_1}, V_{\alpha_2}, ...V_{\alpha_m}$, since $[0, \alpha_{n0}]$ is compact. Let

$$T_n = \cup\{T \setminus V'_{\alpha_i} : i \leq m\}.$$

Then, T_n is a countable subset of T. For each $i \leq m$, we pick $x_i \in D_n \cap U_{\alpha_i}$. Let $F'_{n2} = \{x_i : i \leq m\}$. Then, F'_{n2} is a finite subset of D_n and

$$(((\{\infty\} \times [0, \alpha_{n0}]) \times \{0\}) \times \{n\}) \cup (A((T \setminus T_n) \times [0, \alpha_{n0}]) \times \{n\}) \subseteq St(F'_{n2}, \mathcal{U}).$$

For each $t \in T_n$, since $\{t\} \times [0, \alpha_{n0}]$ is compact, then $A(\{t\} \times [0, \alpha_{n0}]) \times \{n\}$ is compact, hence there exists a finite subset F_t of D_n such that

$$A(\{t\} \times [0, \alpha_0]) \times \{n\} \subseteq St(F_t, \mathcal{U}).$$

Let $F''_{n2} = \cup\{F_t : t \in T_n\}$. Then, F''_{n2} is countable, since T_n is countable. Since $F''_{n2} \cap (A(Y \times \{\alpha\}) \times \{n\})$ is countable for each $\alpha < \kappa^+$ and $F''_{n2} \cap (A(\{t\} \times \kappa^+) \times \{n\}$ is finite for each $t \in T$, then F''_{n2} is closed in X_n by the construction of the topology of X_n, hence F_{n2} is closed in $\mathcal{P}(\mathcal{R}(X))$, since X_n is open closed in $\mathcal{P}(\mathcal{R}(X))$. By the definition of F''_{n2}, we have

$$A(T_n \times [0, \alpha_{n0}]) \times \{n\} \subseteq St(F''_{n2}, \mathcal{U}).$$

If we put $F_{n2} = F'_{n2} \cup F''_{n2}$. Then, F_{n2} is a countable subset of D_n and F''_{n2} is closed in $\mathcal{P}(\mathcal{R}(X))$, since F'_{n1} is finite and and F''_{n2} is closed in $\mathcal{P}(\mathcal{R}(X))$. By the definition of F_{n2}, we have

$$X_{n2} \cup (((\{\infty\} \times [0, \alpha_{n0}]) \times \{0\}) \times \{n\}) \subseteq St(F_{n2}, \mathcal{U}).$$

Finally, we show that there exists a finite subset F_n of D_n such that $X_{n3} \subseteq St(F_{n3}, \mathcal{U})$. Since $\{\infty\} \times \kappa^+$ is countably compact, then, By Lemma 2.1, $A(\{\infty\} \times \kappa^+) \times \{n\}$ is acc and there exists a finite subset $F'_{n3} \subseteq D_{n2}$ such that

$$E_n = X_{n3} \setminus St(F'_{n3}, \mathcal{U}) \subseteq ((\{\infty\} \times \kappa^+) \times \{0\}) \times \{n\} \text{ is a finite subset}$$

and each point of E_n is an isolated point of $((\{\infty\} \times \kappa^+) \times \{0\}) \times \{n\}$. For each point $x \in E_n$, there exists a $U_x \in \mathcal{U}$ such that $x \in U_x$. For each point $x \in E_n$, pick $d_x \in D_n \cap U_x$. Let $F''_{n3} = \{d_x : x \in E\}$, then F''_{n3} is a finite subset of D_n and $E \subseteq St(F''_{n3}, \mathcal{U})$. If we put $F_{n3} = F'_{n3} \cup F''_{n3}$, then F_{n3} is a finite subset of D_n and

$$X_{n3} \subseteq St(F_{n3}, \mathcal{U}).$$

If we put $F_n = F_{n1} \cup F_{n2} \cup F_{n3}$, then F_n is a countable subset of D_n such that

$$((X'_n \times \{0\}) \times \{\omega\}) \cup X_n \subseteq St(F_n, \mathcal{U}).$$

Since F_{n1} and F_{n2} are closed in $\mathcal{P}(\mathcal{R}(X))$, F_{n3} is finite and each point of F_n is isolated, then F_n is discrete closed in $\mathcal{P}(\mathcal{R}(X))$.

Let $F = \cup_{n\in\omega} F_n$. Then, F is a countable subset of D and

$$St(F, \mathcal{U}) = \cup_{n\in\omega} St(F_n, \mathcal{U}) \supseteq \cup_{n\in\omega}(((X'_n \times \{0\}) \times \{\omega\}) \cup X_n) = \mathcal{P}(\mathcal{R}(X)).$$

Since each point of F is isolated, then F is discrete in $\mathcal{P}(\mathcal{R}(X))$. Since F_n is discrete closed in X_n and X_n is open closed in $\mathcal{P}(\mathcal{R}(X))$ for each $n \in \omega$, then F has not accumulation points in $\mathcal{R}(X) \times \omega$. On the other hand, since F is countable and $\kappa \geq |X| > \omega$, then every point of X_ω is not accumulation point of F by the construction of the topology of $\mathcal{P}(\mathcal{R}(X))$. This shows that F is closed in $\mathcal{P}(\mathcal{R}(X))$, which completes the proof. □

Since every discretely absolutely star-Lindelöf space is discretely star-Lindelöf, the next corollary follows from Theorem 2.2.

Corollary 2.3. *Every Hausdorff star-Lindelöf space can be represented in a Hausdorff discretely star-Lindelöf space as a closed G_δ-subspace.*

Since every discretely absolutely star-Lindelöf space is absolutely star-Lindelöf, the next corollary follows from Theorem 2.2.

Corollary 2.4. *Every Hausdorff star-Lindelöf space can be represented in a Hausdorff absolutely star-Lindelöf space as a closed G_δ-subspace.*

The author [10] proved that every Hausdorff (regular, Tychonoff) linked-Lindelöf space can be represented a closed G_δ-subspace in Hausdorff (regular, Tychonoff, respectively) star-Lindelöf space. Thus, we have the next corollary.

Corollary 2.5. *Every Hausdorff linked-Lindelöf space can be represented in a Hausdorff discretely absolutely star-Lindelöf space as a closed G_δ-subspace.*

On the separation of Theorem 2.2, Song [14] showed that $\mathcal{R}(X)$ is Tychonoff if X is a locally-countable (ie., each point of X has a neighborhood U with $|U| \leq \omega$) Tychonoff space. Thus, we have the following proposition by the construction of the topology of $\mathcal{P}(\mathcal{R}(X))$.

Proposition 2.6. *If X is a locally countable Tychonoff space, then $\mathcal{P}(\mathcal{R}(X))$ is Tychonoff.*

By Theorem 2.2 and Proposition 2.6, we have the next corollary.

Corollary 2.7. *Every locally-countable, star-Lindelöf Tychonoff space can be represented in a discretely absolutely star-Lindelöf Tychonoff space as a closed G_δ-subspace.*

The author [10] proved that every Hausdorff (regular, Tychonoff) linked-Lindelöf space can be represented a closed G_δ-subspace in Hausdorff (regular, Tychonoff, respectively) star-Lindelöf space. Thus, we have the following corollary by Corollary 2.7.

Corollary 2.8. *Every locally-countable, linked-Lindelöf Tychonoff space can be represented in a discretely absolutely star-Lindelöf Tychonoff space as a closed G_δ-subspace.*

Remark 2.9. In Theorem 2.2, even if X is locally-countable normal, $\mathcal{R}(X)$ need not be normal (hence, $\mathcal{P}(\mathcal{R}(X))$ need not be normal). Indeed, $X \times \{0\}$ and $A(\{\infty\} \times \kappa^+)$ are disjoint closed subsets of $\mathcal{R}(X)$ that can not be separated by disjoint open subsets of $\mathcal{R}(X)$. Thus, the author does not know if every locally countable, normal star-Lindelöf space can be represented in a normal discretely absolutely star-Lindelöf space as a closed G_δ-subspace.

Remark 2.10. The author does not know if every regular (Tychonoff, normal) star-Lindelöf space can be represented in a regular (Tychonoff, normal, respectively) discretely absolutely star-Lindelöf space as a closed subspace or as a closed G_δ-subspace.

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