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Δ-normal spaces and decompositions of normality

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Abstract. Generalizations of normality, called (weakly) (functionally) Δ-normal spaces are introduced and their interrelation with some existing notions of normality is studied. Δ-regular spaces are introduced which is a generalization of seminormal, semiregular and θ-regular space. This leads to decompositions of normality in terms of Δ-regularity, seminormality and variants of Δ-normality.

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1. Introduction and preliminaries

To investigate properly the existing notions of general topology, topologists adopted various techniques. Decomposition of a given topological property in terms of two weaker properties is one of them. None of the existing classical notions of general topology remain untouched of decomposition process. Since normality is an important property, its decomposition is desirable. First step in this direction was initiated by Vigilino [18] and Singal and Arya [13], where a decomposition of normality was given in terms of almost normal spaces and seminormal spaces. Another decomposition of normality was given in [6] in terms of θ-normality and its variants. Mack [10] introduced δ-normal spaces and the same has been utilised in [8] to give a factorization of normality. In an attempt to get another decomposition of normality in terms of seminormal spaces, in this paper we introduce the notion of Δ-normal spaces.

Let X be a topological space and let $A \subset X$. Throughout the present paper the closure of a set A will be denoted by $\overline{A}$ or clA and the interior by $intA$. A set $U \subset X$ is said to be ***regularly open*** [9] if $U = int\overline{U}$. The complement of a regularly open set is called ***regularly closed***. A point $x \in X$ is called a ***θ-limit point*** (respectively ***δ-limit point***) [17] of A if every closed (respectively regularly open) neighbourhood of x intersects A. Let $cl_\theta A$ (respectively $cl_\delta A$) denotes the set of all θ-limit point (respectively δ-limit point) of A. The set A is called ***θ-closed*** (respectively ***δ-closed***) if $A = cl_\theta A$ (respectively $A = cl_\delta A$). The complement of a θ-closed (respectively δ-closed) set will be referred to as a ***θ-open*** (respectively ***δ-open)*** set. The family of θ-open sets as well as the family of δ open sets form topologies on X. The topology formed by the set of δ-open sets is the semiregularization topology whose basis is the family of regularly open sets. A space X is said to be ***almost regular*** [12] if every regularly closed set and a point out side it are contained in disjoint open sets. A space is called ***semi-normal*** [18] if for every closed set F and each open set U containing F, there exists a regular open set V such that $F \subset V \subset U$. A space is called ***almost normal*** [13] if every pair of disjoint closed sets one of which is regularly closed are contained in disjoint open sets and a space X is said to be ***mildly normal*** [15] (or κ-normal [16]) if every pair of disjoint regularly closed sets are contained in disjoint open sets. A space is **almost completely regular** [13] if for every regularly closed set A and a point $x \notin A$, there exists a continuous function $f : X \to [0,1]$ such that $f(x) = 0$ and $f(A) = 1$. A space X is said to be ***nearly compact***[14] if every open covering of X admits a finite subcollection the interiors of the closures of whose members cover X.

A subset G of a space X is called a ***regular G_δ-set*** if it is the intersection of a sequence of closed sets whose interiors contain G, i.e., $G = \bigcap_{n=1}^{\infty} F_n = \bigcap_{n=1}^{\infty} F_n^o$, where each F_n is a closed subset of X. The complement of a regular G_δ-set is called a regular ***F_σ-set***[10].

In a topological space, every zero set is a regular G_δ-set and every regular G_δ-set is θ-closed.

In general the θ-closure operator is a Čech closure operator (see [11]) but not a Kuratowski closure operator, since θ-closure of a set may not be θ-closed (see [4]). However, the following modification yields a Kuratowski closure operator.

Definition 1.1 ([5])**.** *Let X be a topological space and let $A \subset X$. A point $x \in X$ is called a **$u\theta$-limit point** of A if every θ-open set U containing x intersects A. Let $A_{u\theta}$ denote the set of all $u\theta$-limit points of A.*

Lemma 1.2 ([7])**.** *The correspondence $A \to A_{u\theta}$ is a Kuratowski closure operator.*

It turns out that the set $A_{u\theta}$ is the smallest θ-closed set containing A.

Definition 1.3. *A topological space X is said to be*

(i) ***θ-normal*** [6] *if every pair of disjoint closed sets one of which is θ-closed are contained in disjoint open sets;*
(ii) ***Weakly θ-normal***[6] *if every pair of disjoint θ-closed sets are contained in disjoint open sets;*

(iii) ***Functionally*** θ***-normal*** [6] *if for every pair of disjoint closed sets A and B one of which is θ-closed there exists a continuous function $f : X \to$[0,1] such that $f(A)$ = 0 and $f(B)$=1;*
(iv) ***Weakly functionally*** θ***-normal (wf*** θ***-normal)***[6] *if for every pair of disjoint θ-closed sets A and B there exists a continuous function $f : X \to$ [0,1] such that $f(A)$ = 0 and $f(B)$= 1; and*
(v) θ***-regular***[6] *if for each closed set F and each open set U containing F, there exists a θ-open set V such that $F \subset V \subset U$.*
(vi) Σ***-normal***[8] *if for each closed set F and each open set U containing F, there exists a regular F_σ set V such that $F \subset V \subset U$.*

2. Δ-normal spaces

Definition 2.1. *A topological space X is said to be*

(i) Δ***-normal*** *if every pair of disjoint closed sets one of which is δ-closed are contained in disjoint open sets;*
(ii) ***Weakly*** Δ***-normal*** *if every pair of disjoint δ-closed sets are contained in disjoint open sets;*
(iii) ***Weakly functionally*** Δ***-normal (wf*** Δ***-normal)*** *if for every pair of disjoint δ-closed sets A and B there exists a continuous function $f : X \to$ [0,1] such that $f(A)$ = 0 and $f(B)$= 1.*

Theorem 2.2. *For a topological space X, the following statements are equivalent.*

(a) *X is Δ-normal.*
(b) *For every closed set A and every δ-open set U containing A there exists an open set V such that $A \subset V \subset \overline{V} \subset U$.*
(c) *For every δ-closed set A and every open set U containing A there exists an open set V such that $A \subset V \subset \overline{V} \subset U$.*
(d) *for every pair of disjoint closed sets A and B one of which is δ-closed there exists a continuous function $f : X \to Y$ such that $f(A) = 0$ and $f(B) = 1$.*
(e) *For every pair of disjoint closed sets one of which is δ-closed are contained in disjoint θ-open sets.*
(f) *For every δ-closed set A and every open set U containing A there exists a θ-open set V such that $A \subset V \subset V_{u\theta} \subset U$.*
(g) *For every closed set A and every δ-open set U containing A there exists a θ-open set V such that $A \subset V \subset V_{u\theta} \subset U$.*
(h) *For every pair of disjoint closed sets A and B, one of which is δ-closed there exist θ-open sets U and V such that $A \subset U$, $B \subset V$ and $U_{u\theta} \cap V_{u\theta} = \phi$.*

Proof. To prove the assertion (a) $\Rightarrow$ (b), let X be a Δ-normal space and let U be an δ-open set containing a closed set A. Now A is closed set which is disjoint from the δ-closed set $X - U$. By Δ-normality of X there are disjoint open sets

V and W containing A and $X-U$, respectively. Then $A \subset V \subset X-W \subset U$. Since $X-W$ is closed, $A \subset V \subset \overline{V} \subset U$.

To prove the implication (b) $\Rightarrow$ (c), let U be a open set containing a δ-closed set A. Then $X-A$ is an δ-open set containing the closed set $X-U$. So by hypothesis there exists an open set W such that $X-U \subset W \subset \overline{W} \subset X-A$. Let $V = X-\overline{W}$. Then $A \subset V \subset X-W \subset U$. Since $X-W$ is closed, $A \subset V \subset \overline{V} \subset U$.

To prove the implication (c) $\Rightarrow$ (d), let A be a δ-closed set disjoint from a closed set B. Then $A \subset X-B = U_1$ (say). Since U_1 is open, there exists a open set $U_{1/2}$ such that $A \subset U_{1/2} \subset \overline{U}_{1/2}) \subset U_1$. Again, since closure of an open set is δ-closed, $\overline{U}_{1/2}$ is a δ-closed set, so there exist open sets $U_{1/4}$ and $U_{3/4}$ such that $A \subset U_{1/4} \subset \overline{U}_{1/4} \subset U_{1/2}$ and $\overline{U}_{1/2} \subset U_{3/4} \subset \overline{U}_{3/4} \subset U_1$. Continuing the above process, we obtain for each dyadic rational r, a δ-open set U_r satisfying $r < s$ implies $\overline{U}_r \subset U_s$. Let us define a mapping $f : X \to$[0,1] by

$$f(x) = \begin{cases} \inf\{\ x : x \in U_r\ \} & \text{if } x \text{ belongs to some } U_r, \\ 1 & \text{if } x \text{ does not belongs to any } U_r. \end{cases}$$

Clearly f is well defined and $f(A) = 0$, $f(B) = 1$. Now it remains to prove that f is continuous. To this end we first observe that if $x \in U_r$, then $f(x) \le r$. Similarly $f(x) \ge r$ if $x \notin \overline{U}_r$. To prove continuity, let $x \in X$ and $(a,\ b)$ be an open interval containing $f(x)$. Now choose two dyadic rationals p and q such that $a < p < f(x) < q < b$. Let $U = U_q - \overline{U}_p$. Then U is an open set containing x. Now for $y \in U$, $y \in U_q$. So $f(y) \le q$. Also as $y \in U$, $y \notin \overline{U}_p$. Thus $f(y) \ge q$. And so $f(y) \in [p,q]$. Therefore $f(U) \subset [p,q] \subset (a,b)$. Hence f is continuous.

To prove the assertion (d) $\Rightarrow$ (e), Let A, B be disjoint closed sets in X, where B is δ-closed. By the hypothesis there exists a continuous function f: $X \to$[0,1] such that $f(A) = 0$ and $f(B) = 1$. Since every continuous function lifts back every θ-open set to θ-open set, the set $f^{-1}[0,\ 1/2)$ and $f^{-1}\ (1/2,\ 1]$ are disjoint θ-open sets containing A and B respectively.

To prove (e) $\Rightarrow$ (f), let A be a δ-closed set in X and let U be an open set containing A. Since A and $X-U$ are disjoint, by hypothesis there exist disjoint θ-open sets V and W such that $A \subset V$ and $X-U \subset W$. So $A \subset V \subset X-W \subset U$. Since $X-W$ is θ-closed and $V_{u\theta}$ is the smallest θ-closed set containing V, $A \subset V \subset V_{u\theta} \subset U$.

To prove (f) $\Rightarrow$ (g), let A be a closed set contained in a δ-open set U. Then $X-U$ is a δ-closed set contained in the open set $X-A$. By hypothesis, there exists a θ-open set W such that $X-U \subset W \subset W_{u\theta} \subset X-A$. Let $V = X-W_{u\theta}$. Then $A \subset V \subset X-W \subset U$. Since $X-W$ is θ-closed and $V_{u\theta}$ is the smallest θ-closed set containing V, $A \subset V \subset V_{u\theta} \subset U$.

To prove (g) $\Rightarrow$ (h), let A be a closed set disjoint from a δ-closed set B. Then $X-B$ is a δ-open set containing A. So there exists a θ-open set W such that $A \subset W \subset W_{u\theta} \subset X-B$. Again by hypothesis there exists a θ-open set U such that $A \subset U \subset U_{u\theta} \subset W \subset W_{u\theta} \subset X-B$. Let $V = X-W_{u\theta}$, then U and V are θ-open sets containing A and B respectively and $U_{u\theta} \cap V_{u\theta} = \varnothing$.

The assertion (h) $\Rightarrow$ (a) is obvious. $\square$

Theorem 2.3. *A topological space X is weakly Δ-normal if and only if for every δ-closed set A and a δ-open set U containing A there is an open set V such that $A \subset V \subset \overline{V} \subset U$.*

Proof. Let X be a weakly Δ-normal space and U be a δ-open set containing a δ-closed set A. Then A and $X - U$ are disjoint δ-closed sets in X. Thus by weak Δ-normality of X there are disjoint open sets V and W containing A and $X - U$, respectively. Then $A \subset V \subset X - W \subset U$. Since $X - W$ is closed, $A \subset V \subset \overline{V} \subset U$.

Conversely, let A and B be two disjoint δ-closed sets in X. Then $U = X - B$ is a δ-open set containing the δ-closed set A. Thus by the hypothesis there exists an open set V such that $A \subset V \subset \overline{V} \subset U$. Then V and $X - \overline{V}$ are disjoint open sets containing A and B, respectively. Hence X is weakly Δ-normal. $\square$

The following diagram is immediate from the definitions.

$$
\begin{tikzcd}
\text{normal} \arrow[d] & \text{almost normal} \arrow[ddrr] & & \\
\Delta\text{-normal} \arrow[r] \arrow[d] \arrow[ur] & \text{wf}\Delta\text{-normal} \arrow[d] \arrow[dr] & & \\
\text{f}\theta\text{-normal} \arrow[r] \arrow[d] & \text{wf}\theta\text{-normal} \arrow[d] & \text{w}\Delta\text{-normal} \arrow[r] \arrow[dl] & \kappa\text{-normal} \\
\theta\text{-normal} \arrow[r] & \text{w}\theta\text{-normal} & &
\end{tikzcd}
$$

None of the above implication is reversible (See Examples 2.4 - 2.8 below, [6, Example 3.6 - 3.8] and [5, Example 3.4]).

Example 2.4. A functionally θ-normal and weakly functionally Δ-normal space which is not Δ-normal.

Let $X = \{a, b, c, d\}$ and $\tau = \{\{a, b\}, \{b\}, \{b, c\}, \{c\}, \{b, c, d\}, \{a, b, c\}, X, \phi\}$. Here the δ-closed set { c, d } and closed set { a } can not be separated by disjoint open sets. Thus the space is not Δ-normal but the space is functionally θ-normal and weakly functionally Δ-normal.

Example 2.5. A functionally θ-normal space which is not weakly Δ-normal.

Let X be the set of positive integers. Define a topology on X by taking every odd integer to be open and a set $U \subset X$ is open if for every even integer $p \in U$, the predecessor and successor of p are also in U. The space is not weakly Δ-normal as disjoint δ-closed sets { 2, 3, 4 } and { 6 } cannot be separated by disjoint open sets. But the space is functionally θ-normal.

Example 2.6. A weakly Δ-normal space which is not weakly functionally Δ-normal

Let X denote the interior of the unit square S in the plane together with the points (0, 0) and (1, 0), i.e. $X = \text{int}S \cup \{(0,\ 0),\ (1,\ 0)\ \}$. Every point in $intS$ has the usual Euclidean neighourhoods. The points (0, 0) and (1, 0) have neighbourhoods of the form U_n and V_n respectively, where

$U_n = \{(0, 0)\} \cup \{(x, y) : 0 < x < 1/2, 0 < y < 1/n\}$ and

$V_n = \{(1, 0)\} \cup \{(x, y) : 1/2 < x < 1, 0 < y < 1/n\}$.

The space X is weakly Δ-normal, since every pair of disjoint δ-closed sets are separated by disjoint open sets. However, the δ-closed sets $\{(0, 0)\}$ and $\{(1, 0)\}$ do not have disjoint closed neighbourhoods and hence cannot be functionally separated.

Example 2.7. A Δ-normal space which is not normal.

Let $X = \{a, b, c, d\}$ and $\tau = \{\{a, b\}, \{b, c\}, \{b\}\ ,\{d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, X, \phi\ \}$.

Example 2.8. An almost normal space which is not weakly Δ-normal.

Let $X = \{a, b, c\}$ and $\tau = \{\{b, c\}, \{a, c\}, \{c\}, X, \phi\ \}$.

Theorem 2.9. *For a Hausdorff space X, the following statements are equivalent.*

(a) X is normal.

(b) X is Δ-normal.

(c) X is functionally θ-normal.

(d) X is θ-normal.

Proof. The implications (a) $\Rightarrow$ (b) $\Rightarrow$ (c) $\Rightarrow$ (d) are immediate from definitions and Theorem 2.2. The implication (d) $\Rightarrow$ (a) is shown in [6, Theorem 3.5]. $\square$

Lemma 2.10. *In an almost regular space every δ-closed set is θ-closed.*

Theorem 2.11. *In an almost regular space the following statements are equivalent.*

(a) X is Δ-normal.

(b) X is functionally θ-normal.

(c) X is θ-normal.

Proof. By Lemma 2.10, every δ-closed set is θ-closed. Thus in an almost regular space every θ-normal space is Δ-normal. $\square$

Theorem 2.12. *In an almost regular space the following statements hold.*

(a) Every weakly functionally θ-normal space is weakly functionally Δ-normal.

(b) Every weakly θ-normal space is weakly Δ-normal.

Recall that a space X is an R_o-space [2] if for every open set U in X, $x \in U$ implies $\overline{\{x\}} \subset U$. R_o-spaces are called S_1-spaces in [1].

Theorem 2.13. *A Δ-normal R_o-space is almost completely regular.*

Proof. Let X be a Δ-normal R_o-space. Let A be a regular closed set and $x \notin A$. Then $x \in X - A$. Since X is R_o, $\overline{\{x\}} \in X - A$. So $\overline{\{x\}}$ is a closed set disjoint from the δ-closed set A. Thus by Theorem 2.2, there exists a continuous function $f : X \to [0,1]$ such that $f(\overline{\{x\}}) = 0$ and $f(A) = 1$. Hence X is almost completely regular. □

Theorem 2.14. *A Hausdorff weakly functionally Δ-normal space is almost completely regular.*

Proof. Let X be a Hausdorff weakly functionally Δ-normal space. Let $x \in X$ and A be a regularly closed set in X such that $x \notin A$. As X is Hausdorff, { x } and A are disjoint δ-closed sets. Thus by weak functional Δ-normality of X there exists a continuous function $f : X \to [0,1]$ such that $f(x) = 0$ and $f(A) = 1$. So f is almost completely regular. □

Theorem 2.15. *A Hausdorff weakly Δ-normal space is almost regular.*

Proof. Let X be a Hausdorff weakly Δ-normal space. Let A be a regularly closed set not containing x. By [3, 2.3], every singleton in X is θ-closed. So { x } and A are disjoint δ-closed sets which can be separated by disjoint open sets by weak Δ-normality. □

Corollary 2.16. *A Hausdorff weakly Δ-normal space is weakly functionally θ-normal.*

Proof. It is immediate in view of Theorem 2.15 and the fact that an almost regular weakly θ-normal space is weakly functionally θ-normal (see [5, Theorem 5.18]). □

3. Decompositions of normality

Definition 3.1. *A topological space x is said to be **Δ-regular** if for every closed set F and each open set U containing F, there exists a δ-open set V such that $F \subset V \subset U$.*

Clearly, every θ-regular space as well as every semi-normal space is Δ-regular.

Theorem 3.2. *Every semiregular space is Δ-regular.*

Proof. Let F be a closed set contained in an open set U. For every $x \in F$ there exists a regular open set U_x such that $x \in U_x \subset U$. Let $\bigcup\limits_{x \in F} U_x = V$. Thus $F \subset V \subset U$, where V need not be regular open but δ-open. Hence X is Δ-regular. □

The following diagram well illustrates the interrelations that exist among variants of regularity and normality.

$$\begin{tikzcd}
\text{normal} \arrow[r] \arrow[d] & \text{Z-normal} \arrow[r] & \Sigma\text{-normal} \arrow[dl] \\
\text{seminormal} \arrow[d] & \theta\text{-regular} \arrow[dl] & \\
\Delta\text{-regular} & \text{semiregular} \arrow[l] & \text{regular} \arrow[l] \arrow[ul]
\end{tikzcd}$$

However, none of the above implications is reversible as is well exhibited by the following examples and Example 3.8 in [8].

Example 3.3. A Δ-regular space which is not θ-regular. Every open set in Example 2.5 is regular open and the only θ-open set in X is X itself. So the space is Δ-regular but not θ-regular.

Example 3.4. A Δ-regular space which is not semiregular. Let $X = \{a, b, c\}$ and let $\tau = \{\{a\}, \{a, b\}, X, \phi\}$. This space is Δ-regular but not semiregular.

Theorem 3.5. *An R_o Δ-regular space is semiregular.*

Proof. Let X be an R_o Δ-regular space. Let $x \in X$ and let U be an open set containing x. Since X is an R_o-space, $\overline{\{x\}} \subset U$. So by θ-regularity of X, there exists a δ-open set V such that $\overline{\{x\}} \subset V \subset U$. Since V is the union of regular open sets, there exists a regular open set W such that $x \in W \subset V \subset U$. So X is semiregular. $\square$

Theorem 3.6. *Every nearly compact θ-regular space is Δ-normal.*

Proof. Let A and B be two disjoint closed sets where A is δ-closed. Since every δ-closed subset of a nearly compact space is N-closed relative to X, A is N-closed relative to X. Now $A \subset X - B$. Thus by θ-regularity of X, there exists a θ-open set V such that $A \subset V \subset X - B$. Now for every $x \in A$, there exists an open set U_x such that $x \in U_x \subset \overline{U_x} \subset V$. Thus $\mathcal{U} = \{int\overline{U_x} : x \in A\}$ is a regular open cover of A. Since A is N-closed relative to X, there exist a finite subcollection $\{int\overline{U_{x_i}} : x \in A\}$ which covers A. Then $P = \bigcup_{i=1}^{n} \overline{intU_{x_i}}$ and $Q = \bigcap_{i=1}^{n} (X - \overline{U_{x_i}})$ are disjoint open sets containing A and B respectively. Hence X is Δ-normal. $\square$

Remark 3.7. Even a compact Δ-regular space need not be weakly Δ-normal. e.g.; Let $X = \{a, b, c\}$ and $\tau = \{\{b, c\}, \{a, c\}, \{c\}, \phi, X\}$. Here the space is compact and Δ-regular but not θ-regular.

The following Theorem and the corollary provides factorizations of normality in terms of Δ-regularity, seminormality and variants of Δ-normality.

Theorem 3.8. *In a Δ-regular space the following statements are equivalent.*
(a) X is normal.
(b) X is Δ-normal.
(c) X is weakly functionally Δ-normal.
(d) X is weakly Δ-normal.

Proof. The implications (a) $\Rightarrow$ (b) $\Rightarrow$ (c) $\Rightarrow$ (d) are immediate. To prove (d) $\Rightarrow$ (b), let X be Δ-regular, wΔ-normal space. Let A and B be two disjoint closed subsets of X, where one of them is δ-closed say A. Then $A \subset X - B$. Thus by Δ-regularity of X, there exists a δ-open set U such that $A \subset U \subset (X - B)$. So A and $X - U$ are disjoint δ-closed sets which can be separated by disjoint open sets by weak Δ-normality. Hence X is δ-normal. To show that (b) $\Rightarrow$ (a), let A and B be two disjoint closed subsets of X. Since X is Δ-regular, there is a δ-open set W such that $A \subset W \subset X - B$. Then $X - W$ is a δ-closed set containing B. By Δ-normality of X, there exist disjoint open sets U and V containing A and $X - W$, respectively and so A and B respectively. □

Corollary 3.9. *In a seminormal space the following statements are equivalent.*
(a) X is normal.
(b) X is Δ-normal.
(c) X is weakly functionally Δ-normal.
(d) X is weakly Δ-normal.

Corollary 3.10. *In a semiregular space the following statements are equivalent.*
(a) X is normal.
(b) X is Δ-normal.
(c) X is weakly functionally Δ-normal.
(d) X is weakly Δ-normal.

Theorem 3.11. *In a θ-regular space the following statements are equivalent.*
(a) X is normal.
(b) X is Δ-normal.
(c) X is functionally θ-normal.
(d) X is weakly functionally Δ-normal.
(e) X is θ-normal.
(f) X is weakly functionally θ-normal.
(g) X is weakly Δ-normal.
(h) X is weakly θ-normal.

Proof. The implications (a) $\Rightarrow$ (b) $\Rightarrow$ (c) $\Rightarrow$ (e) $\Rightarrow$ (h), (a) $\Rightarrow$ (b) $\Rightarrow$ (d) $\Rightarrow$ (f) $\Rightarrow$ (h) and (a) $\Rightarrow$ (b) $\Rightarrow$ (d) $\Rightarrow$ (g) $\Rightarrow$ (h) are immediate. To prove (h) $\Rightarrow$ (a), let X be a θ-regular weakly θ-normal space by [6, Theorem 3.11], X is normal. □

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