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Pseudo perfectly continuous functions and closedness/compactness of their function spaces

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Abstract

A new class of functions called 'pseudo perfectly continuous' functions is introduced. Their place in the hierarchy of variants of continuity which already exist in the literature is highlighted. The interplay between topological properties and pseudo perfect continuity is investigated. Function spaces of pseudo perfectly continuous functions are considered and sufficient conditions for their closedness and compactness in the topology of pointwise convergence are formulated.

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1. Introduction

The class of pseudo perfectly continuous functions properly contains the class of quasi perfectly continuous functions [36] which in its turn strictly contains the class of δ-perfectly continuous functions [27] and so includes all perfectly continuous functions due to Noiri [48] and hence contains all strongly continuous functions of Levine [37].

It is well known that the set $C(X, Y)$ of all continuous functions from a space X into a space Y is not closed in Y^X in the topology of pointwise convergence. However, Naimpally [46] showed that in contrast to continuous functions the set $S(X, Y)$ of strongly continuous functions is closed in Y^X in the topology of pointwise convergence if X is locally connected and Y is Hausdorff. Naimpally's result is extended in ([25], [27], [36], [55]) for larger classes of functions and spaces. The main purpose of this paper is to further strengthen these results to show that if X is sum connected and Y is a $D_\delta T_0$-space, then several classes of functions are identical and closed in Y^X in the topology of pointwise convergence. Moreover, conditions are formulated for these classes of functions to be compact Hausdorff subspaces of Y^X in the topology of pointwise convergence.

The organization of the paper is as follows: Section 2 is devoted to preliminaries and basic definitions. In Section 3 examples are included to ascertain the distinctiveness of the notion so defined from the existing notions in the mathematical literature. Section 4 is devoted to the study of basic properties of pseudo perfectly continuous functions wherein, in particular, it is shown that (i) pseudo perfect continuity is preserved under compositions and expansion of range (ii) sufficient conditions are formulated for the preservation of pseudo perfect continuity in the passage to the graph function and under restriction of range. The notion of pseudo partition topology is introduced and sufficient conditions are given for its direct and inverse preservation under mappings. A sum theorem is proved showing when the presence of pseudo perfect continuity on parts of a function implies pseudo perfect continuity on the whole space. In Section 5 we discuss the interplay between topological properties and pseudo perfectly continuous functions. Section 6 is devoted to function spaces wherein it is shown that if X is sum connected [16] (e.g. connected or locally connected) and Y is a $D_\delta T_0$-space, then the function space $P_p(X,Y)$ of all pseudo perfectly continuous functions from X to Y is closed in Y^X in the topology of pointwise convergence. Moreover, if Y is a compact $D_\delta T_0$-space, then $P_p(X,Y)$ and several other function spaces are shown to be compact Hausdorff in the topology of pointwise convergence.

2. Preliminaries and Basic Definitions

A collection β of subsets of a space X is called an ***open complementary system*** [12] if β consists of open sets such that for every $B \in \beta$, there exist $B_1, B_2, \ldots \in \beta$ with $B = \cup\{X \setminus B_i : i \in N\}$. A subset A of a space X is called a ***strongly open F_σ-set*** [12] if there exists a countable open complementary system $\beta(A)$ with $A \in \beta(A)$. The complement of a strongly open F_σ-set is called ***strongly closed G_δ-set***. A subset A of a space X is called a ***regular G_δ-set*** [42] if A is an intersection of a sequence of closed sets whose interiors contain A, i.e., if $A = \bigcap_{n=1}^{\infty} F_n = \bigcap_{n=1}^{\infty} F_n^0$, where each F_n is a closed subset of X (here F_n^0 denotes the interior of F_n). The complement of a regular G_δ-set is called a ***regular F_σ-set***. A point $x \in X$ is called a ***θ-adherent point*** [59] of $A \subset X$ if every closed neigbourhood of x intersects A. Let ***$\mathrm{cl}_\theta A$*** denote the set of all θ-adherent points of A. The set A is called ***θ-closed*** if $A = \mathrm{cl}_\theta A$. The complement of a θ-closed set is referred to as a ***θ-open set***. A subset A of a space X is said to be ***regular open*** if it is the interior of its closure, i.e., $A = \overline{A}^0$. The complement of a regular open set is referred to as a ***regular closed set***. Any union of regular open sets is called ***δ-open*** [59]. The complement of a δ-open set is referred to as a ***δ-closed set***. An open subset U of a space X is said to be ***r-open*** [30] if for each $x \in U$ there exists a closed set B such that $x \in B \subset U$ or equivalently, if U is expressible as a union of closed sets.The complement of an r-open set is called ***r-closed set***. A subset A of a space X is said to be ***cl-open*** [54] if for each $x \in A$ there exists a clopen set H such that $x \in H \subset A$; or equivalently A is expressible as a union of clopen sets. The complement of a cl-open set is referred to as a ***cl-closed set***.

Definitions 2.1. A function $f : X \to Y$ from a topological space X into a topological space Y is said to be

(a) **strongly continuous** [37] if $f(\bar{A}) \subset f(A)$ for each subset A of X.
(b) **perfectly continuous** ([32], [48]) if $f^{-1}(V)$ is clopen in X for every open set $V \subset Y$.
(c) **cl-supercontinuous** [54] ($\equiv$ **clopen continuous** [51])if for each $x \in X$ and each open set V containing $f(x)$ there is a clopen set U containing x such that $f(U) \subset V$.
(d) ***z*-supercontinuous** [20] (respectively ***D*$_\delta$-supercontinuous** [21], respectively **supercontinuous** [45]) if for each $x \in X$ and for each open set V containing $f(x)$, there exists a cozero set (respectively regular F_σ-set, respectively regular open set) U containing x such that $f(U) \subset V$.
(e) **strongly θ-continuous** ([39], [47]) if for each $x \in X$ and for each open set V containing $f(x)$, there exists an open set U containing x such that $f(\overline{U}) \subset V$.

Definitions 2.2. A function $f : X \to Y$ from a topological space X into a topological space Y is said to be

(a) **D$_\delta$-continuous** [22] (respectively **D-continuous** [18], respectively **z-continuous** [52]) if for each point $x \in X$ and each regular F_σ-set (respectively open F_σ-set, respectively cozero set) V containing $f(x)$ there is an open set U containing x such that $f(U) \subset V$.
(b) **almost continuous** [53] (respectively **faintly continuous** [40], respectively **R-continuous** [35]) if for each $x \in X$ and each regular open set (respectively θ-open set, respectively r-open set) V containing $f(x)$ there is an open set U containing x such that $f(U) \subset V$.
(c) ***d*$_\delta$-map** [23] if for each regular F_σ-set U in Y, $f^{-1}(U)$ is a regular F_σ-set in X.
(d) **θ-continuous** [10] if for each $x \in X$ and each open set V containing $f(x)$ there is an open set U containing x such that $f(\overline{U}) \subset \overline{V}$.
(e) **weakly continuous** [38] if for each $x \in X$ and each open set V containing $f(x)$ there exists an open set U containing x such that $f(U) \subset \overline{V}$.
(f) **quasi θ-continuous function** [50] if for each $x \in X$ and each θ-open set V containing $f(x)$ there exists a θ-open set U containing x such that $f(U) \subset V$.

(g) **slightly continuous**[1][13] if $f^{-1}(V)$ is open in X for every clopen set $V \subset Y$.

Definitions 2.3. A function $f : X \to Y$ from a topological space X into a topological space Y is said to be

(a) **δ-perfectly continuous** [27] if for each δ-open set V in Y, $f^{-1}(V)$ is a clopen set in X.
(b) **almost perfectly continuous** [55] ($\equiv$ **regular set connected** [7]) if $f^{-1}(V)$ is clopen for every regular open set V in Y.
(c) **almost cl-supercontinuous** [26] ($\equiv$ **almost clopen continuous** [9]) if for each $x \in X$ and each regular open set V containing $f(x)$, there is a clopen set U containing x such that $f(U) \subset V$.
(d) **almost z-supercontinuous** [34] (**almost D_δ-supercontinuous**) if for each $x \in X$ and for each regular open set V containing $f(x)$, there exists a cozero set (regular F_σ-set) U containing x such that $f(U) \subset V$.
(e) **almost strongly θ-continuous** [49] if for each $x \in X$ and for each regular open set V containing $f(x)$, there exists an open set U containing x such that $f(\overline{U}) \subset V$.
(f) **quasi perfectly continuous** [36] if $f^{-1}(V)$ is clopen in X for every θ-open set V in Y.
(g) **quasi z-supercontinuous** [33] (**quasi cl-supercontinuos** [19]) if for each $x \in X$ and each θ-open set V containing $f(x)$, there exists a cozero (clopen) set U containing x such that $f(U) \subset V$.
(h) **pseudo z-supercontinuous** [33] (**pseudo cl-supercontinuos** [31]) if for each $x \in X$ and each regular F_σ-set V containing $f(x)$, there exists a cozero (clopen) set U containing x such that $f(U) \subset V$.
(i) **δ-continuous** [47] if for each $x \in X$ and for each regular open set V containing $f(x)$, there exists a regular open set U containing x such that $f(U) \subset V$.

3. Pseudo Perfectly Continuous Functions

We call a function $f : X \to Y$ from a topological space X into a topological space Y **pseudo perfectly continuous** if $f^{-1}(V)$ is clopen in X for every regular F_σ-set V in Y.

The adjoining diagram (Figure 1) well exhibits the interrelations that exist among pseudo perfect continuity and other variants of continuity that already exist in the literature and are related to the theme of the present paper and thus well reflects the place of pseudo perfect continuity in the hierarchy of known variants of continuity.

Examples.

3.1. Let X denote the real line with usual topology and let Y be the real line with cofinite (or cocountable) topology. Then the identity function $f : X \to Y$ is quasi perfectly continuous but not continuous.

[1]Slightly continuous functions have been referred to as cl-continuous in ([22], [35]).

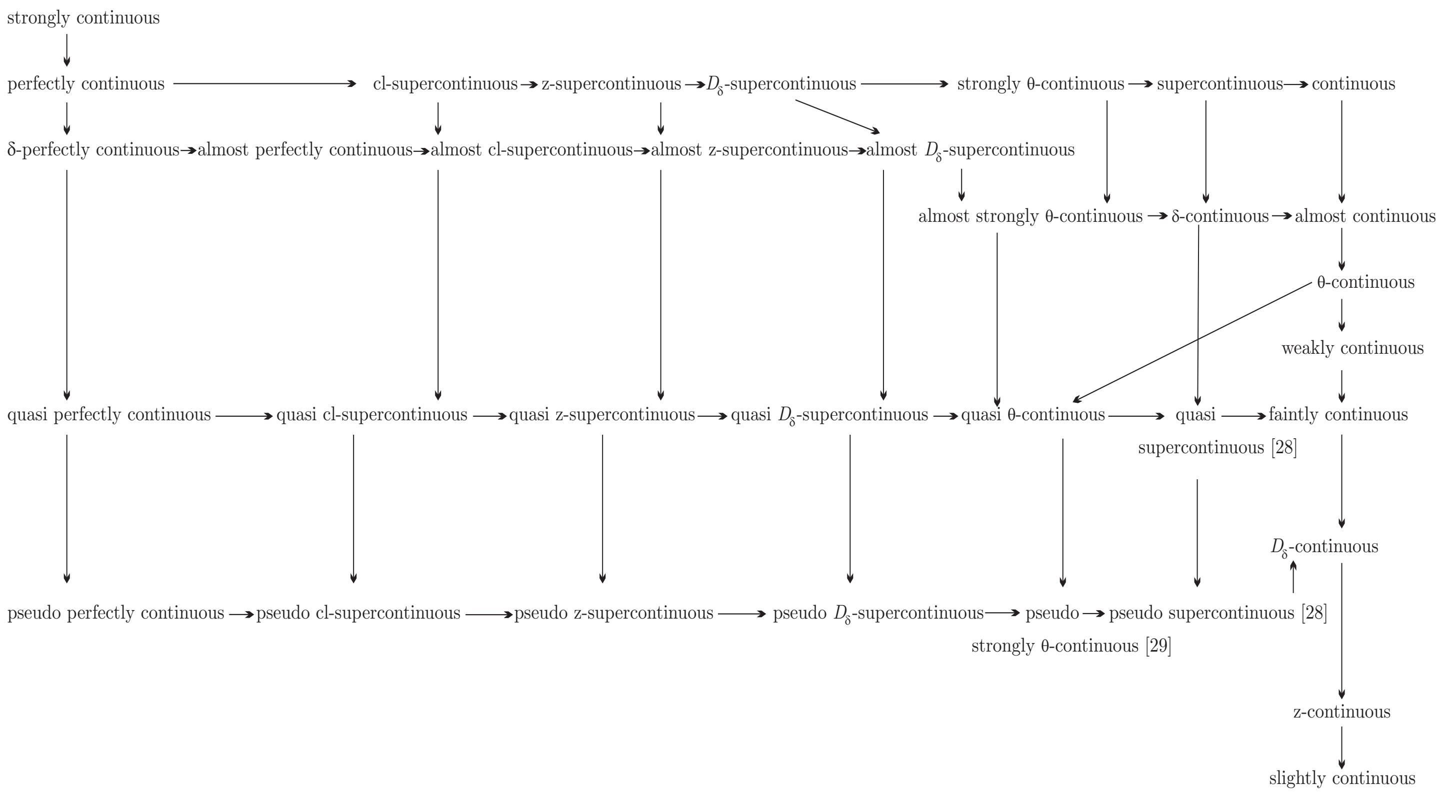
strongly continuous
perfectly continuous
cl-supercontinuous
z-supercontinuous
D_δ-supercontinuous
strongly θ-continuous
supercontinuous
continuous
δ-perfectly continuous
almost perfectly continuous
almost cl-supercontinuous
almost z-supercontinuous
almost D_δ-supercontinuous
almost strongly θ-continuous
δ-continuous
almost continuous
θ-continuous
weakly continuous
quasi perfectly continuous
quasi cl-supercontinuous
quasi z-supercontinuous
quasi D_δ-supercontinuous
quasi θ-continuous
quasi supercontinuous [28]
faintly continuous
D_δ-continuous
pseudo perfectly continuous
pseudo cl-supercontinuous
pseudo z-supercontinuous
pseudo D_δ-supercontinuous
pseudo strongly θ-continuous [29]
pseudo supercontinuous [28]
z-continuous
slightly continuous

Figure 1.

3.2. Let X be the real line endowed with usual topology. Then the identity function defined on X is continuous as well as z-supercontinuous but not pseudo perfectly continuous.

The space E_0 in the following example is due to Misra [44, Example 3.1, p. 352].

3.3. Let w_1 be the first uncountable ordinal. Let the space E_0 be the union of disjoint sets $\{a, b\}$, $\{a_{\alpha\beta} : 0 \leq \alpha, \beta < w_1\}$, $\{b_{\alpha\beta} : 0 \leq \alpha, \beta < w_1\}$ and $\{c_\gamma : 0 \leq \gamma < w_1\}$. The basic neighbourhoods of various points be as follows: all the points $a_{\alpha\beta}$ and $b_{\alpha\beta}$, $0 \leq \alpha$, $\beta < w_1$ are isolated; for each fixed γ, a typical basic neighbourhood of the point c_γ contains the points $a_{\gamma\beta}$ and $b_{\gamma\beta}$ for all but countably many indices β, $0 \leq \beta < w_1$; a typical basic neighbourhood of a (respectively b) contains for every α greater than some ordinal $\delta < w_1$, all but countably many points $a_{\alpha\beta}$ (respectively $b_{\alpha\beta}$). Then E_0 is a Hausdorff, non Urysohn P-space and every real valued continuous function defined on E_0 takes the same value at the points a and b. It is easily verified that in the space E_0 every F_σ-set and hence every regular F_σ-set is clopen. Thus the identity mapping $I_{E_0} : E_0 \to E_0$ is pseudo perfectly continuous. However, it is not a quasi perfectly continuous function, since the inverse image of θ-closed set $\{a\}$ is not clopen.

4. Basic Properties of Pseudo Perfectly Continuous Functions

Proposition 4.1. *If $f : X \to Y$ is a pseudo perfectly continuous function and $g : Y \to Z$ is a d_δ-map, then $g \circ f$ is a pseudo perfectly continuous function. In particular, composition of two pseudo perfectly continuous functions is pseudo perfectly continuous.*

Corollary 4.2. *If $f : X \to Y$ is a pseudo perfectly continuous function and $g : Y \to Z$ is a continuous function, then $g \circ f$ is a pseudo perfectly continuous function.*

Proof. Every continuous map is a d_δ-map. □

Proposition 4.3. *Let $f : X \to Y$ be a slightly continuous function and let $g : Y \to Z$ be a pseudo perfectly continuous function. Then $g \circ f$ is pseudo perfectly continuous.*

Remark 4.4. The hypothesis of 'slightly continuity' in Proposition 4.3 can be traded of by any one of the weak variants of continuity in the following diagram, since each one of them implies slight continuity.

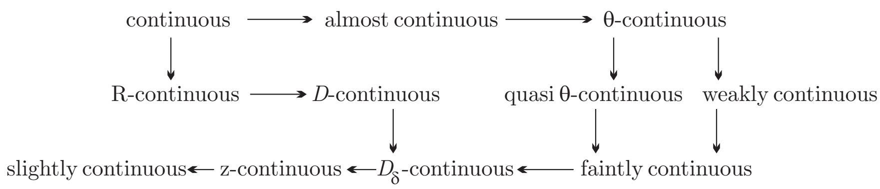

FIGURE 2.

Definition 4.5. A space X is said to be endowed with a

(a) **pseudo partition topology** if every regular F_σ-set in X is closed; or equivalently every regular G_δ-set in X is open.
(b) **partition topology** [58] if every open set in X is closed.
(c) **δ-partition topology** [27] if every δ-open set in X is closed.
(d) **almost partition topology** [55] (≡**extremally disconnected topology** [58]) if every regular open set in X is closed.
(e) **quasi partition topology** [36] if every θ-open set in X is closed.

The following implications are immediate from definitions.

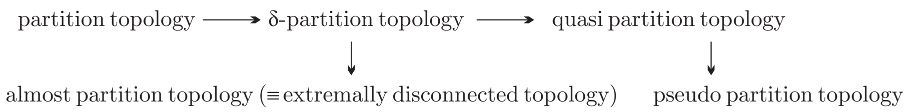

FIGURE 3.

However, none of the above implications is reversible as shown in ([27], [55]) and the following examples.

Example 4.6 ([43, Example 3.20])**.** Consider the space $\mathbb{R}$ of reals with countable complement extension topology τ [58, Example 63, p. 85]. Let $\mathbb{R}\setminus\mathbb{Q}$ be the quotient space obtained from $(\mathbb{R},\tau)$ by identifying the set $\mathbb{Q}$ of rationals to a point. Since $\mathbb{Q}$ is closed in $(\mathbb{R},\tau)$, the space $\mathbb{R}\setminus\mathbb{Q}$ is T_1 and the quotient map $p:(\mathbb{R},\tau)\to\mathbb{R}\setminus\mathbb{Q}$ is a closed map. Let z be an irrational and let a and b be any rationals such that $a<z<b$. Then $G=\{[x]: x\in(a,b), x \text{ irrational}\}$ is a regular open set containing $[z]$ in $\mathbb{R}\setminus\mathbb{Q}$. The quotient topology on $\mathbb{R}\setminus\mathbb{Q}$ is not an almost partition topology and hence not a δ-partition topology since G is a regular open set but not closed in $\mathbb{R}\setminus\mathbb{Q}$. On the other hand $\mathbb{R}\setminus\mathbb{Q}$ is the only θ-open set in $\mathbb{R}\setminus\mathbb{Q}$ so it is equipped with a quasi partition topology.

Example 4.7. The Hausdorff space E_0 also discussed in Example 3.3 which is due to Misra [44, Example 3.1] has a pseudo partition topology since every regular F_σ-set is clopen in E_0 but it is not endowed with a quasi partition topology.

Theorem 4.8. *Let $f : X \to Y$ be a function and $g : X \to X \times Y$, defined by $g(x) = (x, f(x))$ for each $x \in X$, be the graph function. If g is pseudo perfectly continuous, then so is f and the space X is endowed with a pseudo partition topology. Further, if X has a pseudo partition topology and f is pseudo perfectly continuous, then g is pseudo cl-supercontinuous.*

Proof. Suppose that the graph function $g : X \to X \times Y$ is pseudo perfectly continuous. Consider the projection map $p_y : X \times Y \to Y$. Since it is continuous, it is a d_δ-map. Hence in view of Proposition 4.1, the function $f = p_y \circ g$ is pseudo perfectly continuous. To prove that the space X possesses a pseudo partition topology, let U be a regular F_σ-set in X. Then $U \times Y$ is a regular F_σ-set in $X \times Y$. Since g is pseudo perfectly continuous, $g^{-1}(U \times Y) = U$ is clopen in X and so the topology of X is a pseudo partition topology.

Finally, suppose that X has pseudo partition topology and f is a pseudo perfectly continuous function. To show that g is pseudo cl-supercontinuous, let $U \times V$ be a basic regular F_σ-set in $X \times Y$. Then $g^{-1}(U \times V) = U \cap f^{-1}(V)$ is a clopen set in X and so g is pseudo cl-supercontinuous. □

The following result gives sufficient conditions on mappings for domain or range of the mapping to be endowed with pseudo partition topology.

Theorem 4.9. *Let $f : X \to Y$ be a pseudo perfectly continuous surjection which maps clopen sets to closed (open) sets. Then Y is endowed with a pseudo partition topology. Moreover, if f is a bijection which maps regular F_σ-sets (regular G_δ-sets) to regular F_σ-sets (regular G_δ-sets), then X is also equipped with a pseudo partition topology.*

Proof. Suppose f maps clopen sets to closed (open) sets. Let V be a regular F_σ-set (regular G_δ-set) in Y. In view of pseudo perfect continuity of f, $f^{-1}(V)$ is a clopen set in X. Again, since f is a surjection which maps clopen sets to closed (open) sets, the set $f(f^{-1}(V)) = V$ is closed (open) in Y and hence clopen in Y. Thus Y is endowed with a pseudo partition topology.

To prove the last part of the theorem assume that f is a bijection which maps regular F_σ-sets (regular G_δ-sets) to regular F_σ-sets (regular G_δ-sets) and let U be a regular F_σ-set (regular G_δ-set) in X. Then $f(U)$ is a regular F_σ-set (regular G_δ-set) in Y. Since f is a pseudo perfectly continuous bijection, $f^{-1}(f(U)) = U$ is a clopen set in X and so X is endowed with a pseudo partition topology. □

Remark 4.10. A space X is endowed with a pseudo partition topology if and only if every d_δ-map $f : X \to Y$ is pseudo perfectly continuous. Necessity is obvious in view of definitions. To prove sufficiency, assume contrapositive and let V be a regular F_σ-set in X which is not clopen. Then the identity mapping defined on X is a d_δ-map but not pseudo perfectly continuous.

Proposition 4.11. *If $f : X \to Y$ is a surjection which maps clopen sets to open sets and $g : Y \to Z$ is a function such that $g \circ f$ is pseudo perfectly continuous, then g is a D_δ-continuous function. Moreover, if f maps clopen sets to clopen sets, then g is a pseudo perfectly continuous function.*

Proof. Let V be a regular F_σ-set in Z. Since $g \circ f$ is pseudo perfectly continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen set in X. Again, since f is a surjection which maps clopen sets to open sets, $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$ is open in Y and so g is a D_δ-continuous function. The last assertion is immediate, since in this case $g^{-1}(V)$ is a clopen set in Y. □

Proposition 4.12. *If $f : X \to Y$ is a pseudo perfectly continuous function and $g : Y \to Z$ is a D_δ-supercontinuous function, then their composition is cl-supercontinuous.*

Proof. Let V be an open set in Z. In view of D_δ-supercontinuity of g, $g^{-1}(V)$ is a d_δ-open set in Y and so $g^{-1}(V) = \bigcup_\alpha V_\alpha$, where each V_α is a regular F_σ-set. Since f is pseudo perfectly continuous, each $f^{-1}(V_\alpha)$ is a clopen set. Hence $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V)) = f^{-1}(\bigcup_\alpha V_\alpha) = \bigcup_\alpha f^{-1}(V_\alpha)$ is cl-open. So $g \circ f$ is cl-supercontinuous. □

Proposition 4.13. *If $f : X \to Y$ is a pseudo perfectly continuous function and $g : Y \to Z$ is an almost D_δ-supercontinuous function, then their composition $g \circ f$ is almost cl-supercontinuous.*

Proposition 4.14. *If $f : X \to Y$ is a pseudo perfectly continuous function and $g : Y \to Z$ is quasi D_δ-supercontinuous, then their composition $g \circ f$ is quasi cl-supercontinuous.*

Theorem 4.15. *Let $f : X \to Y$ be a function and let $Q = \{X_\alpha : \alpha \in \Lambda\}$ be a locally finite clopen cover of X. For each $\alpha \in \Lambda$, let $f_\alpha = f_{|X_\alpha} : X_\alpha \to Y$ denote the restriction map. Then f is pseudo perfectly continuous if and only if each f_α is pseudo perfectly continuous.*

Proof. Necessity is immediate in view of the fact that quasi perfect continuity is preserved under the restriction of domain. To prove sufficiency, let V be a regular F_σ-set in Y. Then $f^{-1}(V) = \bigcup_{\alpha\in\Lambda} (f_{|X_\alpha})^{-1}(V) = \bigcup_{\alpha\in\Lambda} (f^{-1}(V) \cap X_\alpha)$. Since each $f^{-1}(V) \cap X_\alpha$ is clopen in X_α and hence in X. Thus $f^{-1}(V)$ is open being the union of clopen sets. Moreover, since the collection Q is locally finite, the collection $\{f^{-1}(V) \cap X_\alpha : \alpha \in \Lambda\}$ is a locally finite collection of clopen sets. Since the union of a locally finite collection of closed sets is closed, $f^{-1}(V)$ is also closed and hence clopen. □

Definition 4.16. A subset S of a space X is said to be **regular G_δ-embedded** [6] in X if every regular G_δ-set in S is the intersection of a regular G_δ-set in X with S; or equivalently every regular F_σ-set in S is the intersection of a regular F_σ-set in X with S.

Proposition 4.17. *Let $f : X \to Y$ be a pseudo perfectly continuous function. If $f(X)$ is regular G_δ-embedded in Y, then $f : X \to f(X)$ is pseudo perfectly continuous.*

Proof. Let V_1 be a regular F_σ-set in $f(X)$. Since $f(X)$ is regular G_δ-embedded in Y, there exists a regular F_σ-set V in Y such that $V_1 = V \cap f(X)$. In view of pseudo perfect continuity of f, $f^{-1}(V)$ is clopen in X. Now $f^{-1}(V \cap f(X)) = f^{-1}(V) \cap f^{-1}(f(X)) = f^{-1}(V)$ and hence the result. □

Definition 4.18. A topological space X is called an **Alexandroff space** [2] if any intersection of open sets in X is itself an open in X, or equivalently any union of closed sets in X is closed in X.

Alexandroff spaces have been referred to as **saturated spaces** by Lorrain in [41].

Theorem 4.19. *For each $\alpha \in \Lambda$, let $f_\alpha : X \to X_\alpha$ be a function and let $f : X \to \prod_{\alpha\in\Lambda} X_\alpha$ be defined by $f(x) = (f_\alpha(x))$ for each $x \in X$. If f is pseudo perfectly continuous, then each f_α is pseudo perfectly continuous. Further, if X is an Alexandroff space and each f_α is pseudo perfectly continuous, then f is pseudo perfectly continuous.*

Proof. Let f be pseudo perfectly continuous. Now for each α, $f_\alpha = p_\alpha \circ f$, where $p_\alpha : \prod_{\alpha\in\Lambda} X_\alpha \to X$ denotes the projection map. Since each projection map p_α is continuous and hence a d_δ-map, in view of Proposition 4.1 it follows that each f_α is pseudo perfectly continuous.

Conversely, suppose that X is an Alexandroff space and each f_α is a pseudo perfectly continuous function. Since X is Alexandroff, to show that the function f is pseudo perfectly continuous, it is sufficient to show that $f^{-1}(S)$ is clopen for every subbasic regular F_σ-set S in the product space $\prod_{\alpha\in\Lambda} X_\alpha$. Let $U_\beta \times \prod_{\alpha\neq\beta} X_\alpha$ be a subbasic regular F_σ-set in $\prod_{\alpha\in\Lambda} X_\alpha$, where U_β is a regular F_σ-set in X_β. Then $f^{-1}(U_\beta \times \prod_{\alpha\neq\beta} X_\alpha) = f^{-1}(p_\beta^{-1}(U_\beta)) = f_\beta^{-1}(U_\beta)$ is clopen in X and so f is pseudo perfectly continuous. □

5. Interplay between Topological Properties and Pseudo Perfectly Continuous Functions

Definition 5.1. A space X is called a $\boldsymbol{D_\delta T_0}$**-space** if for each pair of distinct points x, y in X, there is a regular F_σ-set U containing one of the points x and y but not the other.

Definition 5.2. A space X is said to be $\boldsymbol{D_\delta}$**-Hausdorff** [22] (**ultra Hausdorff** [57]) if every pair of distinct points in X are contained in disjoint regular F_σ-sets (clopen sets).

In particular, every $D_\delta T_0$-space is Hausdorff.

The following diagram illustrates the relationships that exist among $D_\delta T_0$-spaces and other strong variants of Hausdorffness.

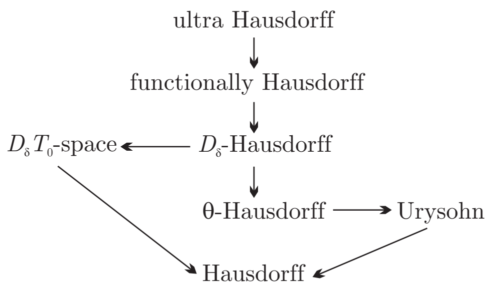

FIGURE 4.

The following example shows that even a Hausdorff regular space need not be a $D_\delta T_0$-space.

Example 5.3. Let X be the Skyline space due to Heldermann [12, Example 7.7]. The space X is a Hausdorff regular space. It is not a $D_\delta T_0$-space since X is the only regular F_σ-set containing the points p^- and p^+. So there exists no regular F_σ-set containing one of the points p^- and p^+ and missing other.

Proposition 5.4. *Let $f : X \to Y$ be a pseudo perfectly continuous injection into a $D_\delta T_0$-space Y. Then X is an ultra Hausdorff space.*

Proof. Let $x, y \in X$, $x \neq y$. Then $f(x) \neq f(y)$. Since Y is a $D_\delta T_0$-space, there exists a regular F_σ-sets V containing one of the points $f(x)$ and $f(y)$ but not both. To be precise, suppose that $f(x) \in V$. Since f is pseudo perfectly continuous, $f^{-1}(V)$ is a clopen set containing x but not y. Then $f^{-1}(V)$ and $X \setminus f^{-1}(V)$ are disjoint clopen sets containing x and y, respectively. Hence X is an ultra Hausdorff space. □

The following theorem is related to a class of spaces, important in the theories studying the p-adic topologies and the Stone duality for Boolean algebras, namely spaces having large inductive dimension zero or ultranormal spaces [57]. These are precisely the spaces in which each pair of nonempty disjoint closed sets can be separated by disjoint clopen sets.

Theorem 5.5. *Let $f : X \to Y$ be a closed, pseudo perfectly continuous injection into a normal space Y. Then X is an ultranormal space.*

Proof. Let A and B be any two disjoint closed sets in X. Since the function f is closed and injective, $f(A)$ and $f(B)$ are disjoint closed subsets of Y. Again, since Y is normal, by Urysohn's Lemma there exists a continuous function $\varphi : Y \to [0, 1]$ such that $\varphi(f(A)) = 0$ and $\varphi(f(B)) = 1$. Then $V = \varphi^{-1}([0, 1/2))$ and $W = \varphi^{-1}((1/2, 1])$ are disjoint cozero sets in Y containing $f(A)$ and $f(B)$, respectively. Since every cozero set is a regular F_σ-set, $f^{-1}(V)$ and $f^{-1}(W)$ are disjoint clopen sets containing A and B, respectively and so X is an ultranormal space. □

Definition 5.6. A space X is said to be ***D_δ*-compact** [23] (**mildly compact** [57]) if every cover of X by regular F_σ-sets (clopen sets) has a finite subcover.

Proposition 5.7. *Let $f : X \to Y$ be a pseudo perfectly continuous function from a mildly compact space X onto a space Y. Then Y is D_δ-compact.*

Proof. Let $\Omega = \{U_\alpha : \alpha \in \Lambda\}$ be a cover of Y by regular F_σ-sets. Since f is pseudo perfectly continuous, the collection $\beta = \{f^{-1}(U_\alpha) : \alpha \in \Lambda\}$ is a clopen cover of X. Since X is mildly compact, let $\{f^{-1}(U_{\alpha_1}), \ldots, f^{-1}(U_{\alpha_n})\}$ be a finite subcollection of β which covers X. Then $\{U_{\alpha_1}, \ldots, U_{\alpha_n}\}$ is a finite subcollection of Ω which covers Y. Hence Y is D_δ-compact. □

Proposition 5.8. *Let $f : X \to Y$ be a pseudo perfectly continuous function from a space X onto a space Y. If* (i) *f is an open bijection; or* (ii) *f is a closed surjection, then any pair of disjoint regular G_δ-sets in Y are clopen in Y.*

Proof. Let A and B be disjoint regular G_δ-subsets of Y. Since f is pseudo perfectly continuous $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint clopen subsets of X.
(i) In case f is an open bijection, $f(f^{-1}(A)) = A$ and $f(f^{-1}(B)) = B$ are disjoint open sets and hence clopen sets in Y.
(ii) In case f is a closed surjection, the sets $A = Y \setminus f(X \setminus f^{-1}(A))$ and $B = Y \setminus f(X \setminus f^{-1}(B))$ are disjoint clopen sets in Y. □

Definition 5.9. A space X is said to be ***δ*-completely regular space** ([22], [24]) if for each regular G_δ-set F and a point $x \notin F$, there exists a continuous function $f : X \to [0, 1]$ such that $f(x) = 0$ and $f(F) = 1$.

Theorem 5.10. *Let $f : X \to Y$ be an open closed pseudo perfectly continuous surjection. Then Y is a δ-completely regular space.*

Proof. Let $K \subset Y$ be a regular G_δ-set and let $z \notin K$. Since f is pseudo perfectly continuous, $f^{-1}(K)$ is clopen. Let $x_0 \in f^{-1}(z)$. Then $x_0 \notin f^{-1}(K)$. Since $f^{-1}(K)$ is clopen, its characteristic function $\phi : X \to [0, 1]$ is continuous and $\phi(x_0) = 0$ and $\phi(f^{-1}(K)) = 1$. Define $\hat{\varphi} : Y \to [0, 1]$ by taking $\hat{\varphi}(y) = \sup\{\phi(x) : x \in f^{-1}(y)\}$. Then $\hat{\varphi}(z) = 0$, $\hat{\varphi}(K) = 1$ and by [8, Exercise 16] $\hat{\varphi}$ is continuous. Hence Y is a δ-completely regular space. □

Remark 5.11. There exists no open closed pseudo perfectly continuous surjection from a space onto a non δ-completely regular space.

Proposition 5.12. *Let $f, g : X \to Y$ be pseudo perfectly continuous functions from a space X into a D_δ-Hausdorff space Y. Then the set $A = \{x : f(x) = g(x)\}$ is cl-closed in X.*

Proof. Let $x \in X \setminus A$. Then $f(x) \neq g(x)$, and so by hypothesis on Y, there are disjoint regular F_σ-sets U and V containing $f(x)$ and $g(x)$, respectively. Since f and g are pseudo perfectly continuous, the sets $f^{-1}(U)$ and $g^{-1}(V)$ are clopen and containing the point x. Let $G = f^{-1}(U) \cap g^{-1}(V)$. Then G is a clopen set contain x and $G \cap A = \varnothing$. Thus A is cl-closed in X. □

Proposition 5.13. *Let $f : X \to Y$ be a pseudo perfectly continuous function from a space X into a D_δ-Hausdorff space Y. Then the set $A = \{(x_1, x_2) \in X \times X : f(x_1) = f(x_2)\}$ is cl-closed in $X \times X$.*

Proof. Let $(x_1, x_2) \in X \times X \setminus A$. Then $f(x_1) \neq f(x_2)$. Since Y is a D_δ-Hausdorff space, there exist disjoint regular F_σ-sets U and V containing $f(x_1)$ and $f(x_2)$, respectively. Since f is pseudo perfectly continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint clopen sets in X containing x_1 and x_2, respectively. Let $G = f^{-1}(U) \times f^{-1}(V)$. Then G is a clopen subset of $X \times X$ containing (x_1, x_2) and $G \cap A = \varnothing$. Thus A is cl-closed in $X \times X$. □

Definition 5.14. A space X is said to be

- (i) **pseudo hyperconnected** if there exists no nonempty proper regular G_δ-set in X or equivalently there exists no nonempty proper regular F_σ-set in X ($\equiv X$ is the only regular F_σ-set in X).
- (ii) **hyperconnected** ([1], [58]) if every nonempty open subset of X is dense in X ($\equiv X$ is the only regular open set in X).
- (iii) **quasi hyperconnected** [19] if there exists no nonempty proper θ-open set in X or equivalently there exists no nonempty proper θ-closed set in X ($\equiv X$ is the only θ-open set in X).

Following implications are immediate from definitions.

hyperconnected ⟶ quasi hyperconnected ⟶ pseudo hyperconnected

Figure 5.

Example 5.15. The space $\mathbb{R} \setminus \mathbb{Q}$ given by Mancuso [43, Example 3.20] and also discussed in Example 4.6 is quasi hyperconnected but not hyperconnected.

Proposition 5.16. *Let $f : X \to Y$ be a pseudo perfectly continuous surjection from a connected space X onto Y. Then Y is pseudo hyperconnected.*

Proof. Suppose Y is not pseudo hyperconnected and let V be a nonempty proper regular F_σ-set in Y. Since f is pseudo perfectly continuous, $f^{-1}(V)$ is a nonempty proper clopen subset of X contradicting the fact that X is connected. □

Remark 5.17. There exists no pseudo perfectly continuous surjection from a connected space onto a non pseudo hyperconnected space.

Definition 5.18. The graph $G(f)$ of a function $f : X \to Y$ is said to be

(i) **clopen θ-closed** [19] if for each $(x, y) \notin G(f)$ there exists a clopen set U containing x and a θ-open set V containing y such that $(U \times V) \cap G(f) = \varnothing$.
(ii) **clopen D_δ-closed** if for each $(x, y) \notin G(f)$ there exists a clopen set U containing x and a regular F_σ-set V containing y such that $(U \times V) \cap G(f) = \varnothing$.

Proposition 5.19. *Let $f : X \to Y$ be a pseudo perfectly continuous function into a D_δ-Hausdorff space Y. Then the graph $G(f)$ of f is a clopen D_δ-closed set in $X \times Y$.*

Proof. Suppose $(x, y) \notin G(f)$. Then $f(x) \neq y$. Since Y is D_δ-Hausdorff, there exist disjoint regular F_σ-sets V and W containing $f(x)$ and y, respectively. Since f is pseudo perfectly continuous, $f^{-1}(V)$ is a clopen set containing x. Clearly $(f^{-1}(V) \times W) \cap G(f) = \varnothing$ and so the graph $G(f)$ of f is clopen D_δ-closed in $X \times Y$. □

Corollary 5.20. *If $f : X \to Y$ is a pseudo perfectly continuous function into a D_δ-Hausdorff space Y, then the graph $G(f)$ of f is clopen θ-closed in $X \times Y$.*

6. Function Spaces and Pseudo Perfectly Continuous Functions

It is of fundamental importance in topology, analysis and other branches of mathematics to know whether a given function space is closed/compact in Y^X in the topology of pointwise convergence. So it is of considerable significance both from intrinsic interest as well as from applications viewpoint to formulate conditions on the spaces X, Y and subsets of $C(X, Y)$ or Y^X to be closed/compact in the topology of pointwise convergence. Results of this type and Ascoli type theorems abound in the literature (see [3], [14]). Naimpally's result [46] that in contrast to continuous functions, the set $S(X, Y)$ of strongly continuous functions is closed in Y^X in the topology of pointwise convergence if X is locally connected and Y is Hausdorff; is extended to a larger framework by Kohli and Singh [25] wherein it is shown that if X is sum connected and Y is Hausdorff, then the function space $P(X, Y)$ of all perfectly continuous functions as well as the function space $L(X, Y)$ of all cl-supercontinuous functions is closed in Y^X in the topology of pointwise convergence. This result is further extended in ([27], [55], [36]) for the set $P_\Delta(X, Y)$ of all δ-perfectly continuous functions as well as for the set $P_\delta(X, Y)$ of all almost perfectly continuous ($\equiv$ regular set connected) functions and the set $P_q(X, Y)$ of all quasi perfectly continuous functions under the same hypotheses on X and Y. Herein we further strengthen these results to show that if X is a sum connected space and Y is a $D_\delta T_0$-space, then all the seven classes of functions are identical, i.e. $S(X, Y) = P(X, Y) = L(X, Y) = P_\Delta(X, Y) = P_\delta(X, Y) = P_q(X, Y) = P_p(X, Y)$ and are closed in Y^X in the topology of pointwise convergence.

Proposition 6.1. *Let $f : X \to Y$ be a pseudo perfectly continuous function into a $D_\delta T_0$-space Y. Then f is constant on each connected subset of X. In particular, if X is connected, then f is constant on X and hence strongly continuous.*

Proof. Assume contrapositive and let C be the connected subset of X such that $f(C)$ is not a singleton. Let $f(x)$, $f(y) \in f(C)$, $f(x) \neq f(y)$. Since Y is a $D_\delta T_0$-space, there exists a regular F_σ-set V containing one of the points $f(x)$ and $f(y)$ but not other. For definiteness assume $f(x) \in V$. Since f is a pseudo perfectly continuous, $f^{-1}(V) \cap C$ is a non empty proper clopen subset of C, contradicting the fact that C is connected. The last part of the theorem is immediate, since every constant function is strongly continuous. □

Remark 6.2. The hypothesis of '$D_\delta T_0$-space' in Proposition 6.1 cannot be omitted. For let X be the real line with usual topology and let Y denote the real line endowed with cofinite topology. Let f denote the identity mapping from X onto Y. Then f is a nonconstant pseudo perfectly continuous function.

Corollary 6.3. *Let $f : X \to Y$ be a pseudo perfectly continuous function from a sum connected space X into a $D_\delta T_0$-space Y. Then f is constant on each component of X and hence strongly continuous.*

Proof. Clearly, in view of Proposition 6.1 f is constant on each component of X. Since X is a sum connected space, each component of X is clopen in X. Hence it follows that any union of components of X and the complement of this union are complementary clopen sets in X. Thus f is constant on each component on X. Therefore, for every subset A of Y, $f^{-1}(A)$ and $X \setminus f^{-1}(A)$ are complementary clopen sets in X being the union of component of X. So f is strongly continuous. □

We may recall that a space X is a $\boldsymbol{\delta T_0}$**-space** [26] if for each pair of distinct points x and y in X there exists a regular open set containing one of the points x and y but not the other. In particular, every Hausdorff space is a δT_0-space.

Next, we quote the following results from ([27], [36], [55]).

Theorem 6.4 ([27, Theorem 5.3])**.** *Let $f : X \to Y$ be a function from a sum connected space X into a δT_0-space Y. Then the following statements are equivalent.*

(a) *f is strongly continuous*
(b) *f is perfectly continuous*
(c) *f is cl-supercontinuous*
(d) *f is δ-perfectly continuous.*

Theorem 6.5 ([55, Theorem 4.5])**.** *Let $f : X \to Y$ be a function from a sum connected space X into a δT_0-space Y. Then the following statements are equivalent.*

(a) *f is strongly continuous*
(b) *f is perfectly continuous*

(c) *f is cl-supercontinuous*
(d) *f is δ-perfectly continuous*
(e) *f is almost perfectly continuous.*

Theorem 6.6 ([36, Theorem 5.6])**.** *Let $f : X \to Y$ be a function from a sum connected space X into a Hausdorff space Y. Then the following statements are equivalent.*

(a) *f is strongly continuous*
(b) *f is perfectly continuous*
(c) *f is cl-supercontinuous*
(d) *f is δ-perfectly continuous*
(e) *f is almost perfectly continuous*
(f) *f is quasi perfectly continuous.*

Theorem 6.7. *Let $f : X \to Y$ be a function from a sum connected space X into a $D_\delta T_0$-space Y. Then the following statements are equivalent.*

(a) *f is strongly continuous*
(b) *f is perfectly continuous*
(c) *f is cl-supercontinuous*
(d) *f is δ-perfectly continuous*
(e) *f is almost perfectly continuous*
(f) *f is quasi perfectly continuous*
(g) *f is pseudo perfectly continuous.*

Proof. Since every $D_\delta T_0$-space is Hausdorff, the equivalence of the assertions (a)-(f) is a consequence of Theorem 6.6. The implications (a)⇒(b)⇒(d)⇒(f) ⇒(g) are trivial and the implication (g)⇒(a) is immediate in view of Corollary 6.3. □

Theorem 6.8. *Let X be a sum connected space and let Y be a $D_\delta T_0$-space. Then $S(X,Y) = P(X,Y) = L(X,Y) = P_\Delta(X,Y) = P_\delta(X,Y) = P_q(X,Y) = P_p(X,Y)$ is closed in Y^X in the topology of pointwise convergence.*

Proof. It is immediate from Theorem 6.7 that the above seven classes of functions are identical and its closedness in Y^X in the topology of pointwise convergence follows either from [27, Theorem 5.4] or [55, Theorem 4.6] or [36, Theorem 5.7].

The above results are important from applications view point since in particular it follows that if X is sum connected (e.g. connected or locally connected) and Y is $D_\delta T_0$-space, then the pointwise limit of a sequence $\{f_n : X \to Y : n \in N\}$ of pseudo perfectly continuous functions is pseudo perfectly continuous. □

We conclude this section with the following result which seems to be of considerable significance from applications view point.

Theorem 6.9. *If X is a sum connected space and Y is a compact $D_\delta T_0$-space, then the spaces $S(X,Y) = P(X,Y) = L(X,Y) = P_\Delta(X,Y) = P_\delta(X,Y) = P_q(X,Y) = P_p(X,Y)$ are compact Hausdorff subspaces of Y^X in the topology of pointwise convergence.*

7. Change of Topology

The technique of change of topology of a space is prevalent all through mathematics and is of considerable significance and widely used in topology, functional analysis and several other branches of mathematics. For example, weak and weak* topology of a Banach space, hull kernel topology and the multitude of other topologies on $Id(A)$ the space of all closed two sided ideals of a Banach algebra A ([4], [5], [56]). Moreover, to taste the flavour of applications of the technique in topology see ([11], [15], [17], [30], [60]).

Here we show that if the range of a pseudo perfectly continuous function is retopologized in an appropriate way, then it is simply a cl-supercontinuous function.

Let (X, τ) be a topological space and let B_{d_δ} denote the collection of all regular F_σ-subsets of (X, τ). Since the intersection of two regular F_σ-sets is a regular F_σ-set, the collection B_{d_δ} is a base for a topology τ_{d_δ} on X which is coarser than τ (see [21], [22]). For interrelations and interplay among various other coarser topologies obtained in this way for a given topology we refer the interested reader to [30]. Finally, we conclude with the following result.

Proposition 7.1. *If $f : (X, \tau) \to (Y, \vartheta)$ is a pseudo perfectly continuous function, then $f : (X, \tau) \to (Y, \vartheta_{d_\delta})$ is cl-supercontinuous.*

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