Applied General Topology
© Universidad Politécnica de Valencia
Volume 14, no. 1, 2013
pp. 33-40

Range-preserving $AE(\mathbf{0})$-spaces

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Abstract

All spaces here are Tychonoff spaces. The class $AE(\mathbf{0})$ consists of those spaces which are absolute extensors for compact zero-dimensional spaces. We define and study here the subclass $AE(\mathbf{0})^{rp}$, consisting of those spaces for which extensions of continuous functions can be chosen to have the same range. We prove these results. If each point of $T \in AE(\mathbf{0})$ is a G_δ-point of T, then $T \in AE(\mathbf{0})^{rp}$. These are equivalent: (a) $T \in AE(\mathbf{0})^{rp}$; (b) every compact subspace of T is metrizable; (c) every compact subspace of T is dyadic; and (d) every subspace of T is $AE(\mathbf{0})$. Thus in particular, every metrizable space is an $AE(\mathbf{0})^{rp}$-space.

2010 MSC: *Primary 54C55. Secondary 06F20, 46E10, 54E18*

Keywords: *absolute extensor; retraction; zero-dimensional space; range-preserving function; Dugundji space; dyadic space; countable chain condition*

1. Preliminaries

All spaces here are assumed Tychonoff.

For spaces X and Y, the symbol $C(X,Y)$ denotes the set of continuous functions from X into Y.

We write $Y \subseteq_h X$ to indicates that X contains a homeomorph of Y.

Let $\mathbf{X}$ be a homeomorphism-closed class of spaces. Then $AE(\mathbf{X})$ [resp., $AE(\mathbf{X})^{rp}$], the class of *absolute extensors* [resp., *range-preserving absolute extensors*] for $\mathbf{X}$, consists of those spaces T for which, whenever $X \in \mathbf{X}$ and F is a closed subset of X, every $f \in C(F,T)$ extends to $\overline{f} \in C(X,T)$ [resp., and with $\overline{f}[X] = f[F]$].

For $\mathbf{X}$ a class of spaces, we write

$$\mathcal{P}\mathbf{X} := \{\Pi_{i\in I} X_i : i \in I \Rightarrow X_i \in \mathbf{X}\}.$$

It is clear for arbitrary $\mathbf{X}$, since $\pi_i \circ f \in C(F, T_i)$ for each space $T = \Pi_{i\in I} T_i$ and $f \in C(X, T)$, that

$$\mathcal{P}AE(\mathbf{X}) = AE(\mathbf{X}) \text{ for every class } X. \tag{1.1}$$

We note below in Theorem 1.5((a) and (d)) that the relation $\mathcal{P}AE(\mathbf{X})^{rp} = AE(\mathbf{X})^{rp}$ can fail—indeed it fails when $\mathbf{X} = \mathbf{0}$, the class of compact zero-dimensional spaces. The class $AE(\mathbf{0})$ has been much studied; see [1] for information and extensive bibliographic citations. In this paper we focus on its subclass $AE(\mathbf{0})^{rp}$, which so far as we know is defined and studied for the first time here.

The class of compact spaces in $AE(\mathbf{0})$ has been intensively studied. According to Haydon [10], it coincides with the class of *Dugundji spaces* as defined by Pełczyński [14], and the subclass $\mathbf{0} \cap AE(\mathbf{0})$ of $AE(\mathbf{0})$ coincides with the class of Stone spaces of projective Boolean algebras ([13]).

Let $\mathbf{2}$ denote the two-point discrete space.

We begin with a simple basic observation.

Theorem 1.1. $\mathbf{2} \in AE(\mathbf{0})^{rp}$.

Proof. Let $f \in C(F, \mathbf{2})$ with F closed in $X \in \mathbf{0}$. If f is a constant function then surely f extends to $\overline{f} \in C(X, \mathbf{2})$ with $\overline{f}[X] = f[F]$, so we assume $F_i := f^{-1}(i) \neq \varnothing$ for $i \in \mathbf{2}$. For $x \in F_0$ there is a clopen neighborhood U_x of $x \in X$ such that $U_x \cap F_1 = \varnothing$, and since F_0 is compact some finitely many of the sets U_x ($x \in F_0$) cover F_0. The union U of those sets covers F_0, is clopen in X, and is disjoint from F_1, and then the function $\overline{f} \in C(X, \mathbf{2})$ defined by

$\overline{f} \equiv 0$ on U, $\overline{f} \equiv 1$ on $X \backslash U$

extends f as required. □

From [4](6.2.16) we have for each space X that X is zero-dimensional if and only if there is a cardinal κ such that $X \subseteq_h \mathbf{2}^\kappa$. It follows then quickly from (1.1) that

$$AE(\mathbf{0}) = AE(\mathcal{P}(\{\mathbf{2}\}), \text{ hence } AE(\mathbf{0})^{rp} = AE(\mathcal{P}\{\mathbf{2}\})^{rp}. \tag{1.2}$$

For a qualitative distinction between the class $AE(\mathbf{0})$ and its subclass $AE(\mathbf{0})^{rp}$, one may compare the equivalence (a) ⇔ (b) of the following Theorem with the fact that every (Tychonoff) space Y embeds as a subspace of a (compact) space $T \in AE(\mathbf{0})$. (To see that, recall that $[0,1] \in AE(\mathbf{0})$ by the classical Tietze-Urysohn extension theorem, and that Y embeds into some $T \in \mathcal{P}\{[0,1]\}$; then, use (1.1) with $\mathbf{X} = \{[0,1]\}$.)

Theorem 1.2. *For each space T, these conditions are equivalent.*

(a) $T \in AE(\mathbf{0})^{rp}$;

(b) $S \subseteq T \Rightarrow S \in AE(\mathbf{0})^{rp}$; *and*

(c) $S \subseteq T$, S *compact* $\Rightarrow S \in AE(\mathbf{0})^{rp}$.

Proof. (a) $\Rightarrow$ (b). Given such S and T and $f \in C(F, S)$ with F closed in $X \in \mathbf{0}$, there is $\overline{f} \in C(X, T)$ such that $f \subseteq \overline{f}$ and $\overline{f}[X] = f[F] \subseteq S$.

(b) $\Rightarrow$ (c). This is obvious.

(c) $\Rightarrow$ (a). Given $f \in C(F, T)$ with closed $F \subseteq X \in \mathbf{0}$, the space $S := f[F]$ is compact. Thus there is $\overline{f} \in C(X, T)$ such that $f \subseteq \overline{f}$ and $\overline{f}[F] = f[F] = S$. This shows $T \in AE(\mathbf{0})^{rp}$. □

In Theorem 1.5 we make additional simple observations which highlight differences between the classes $AE(\mathbf{0})$ and $AE(\mathbf{0})^{rp}$. For that, these definitions will be useful.

Definition 1.3. Let T be a space.

(a) T is a *countable chain condition space* (briefly, a *c.c.c. space*) if every family of pairwise disjoint open subsets of T is countable.

(b) T is *dyadic* if for some cardinal κ there is a continuous surjection from $\mathbf{2}^\kappa$ onto T.

Lemma 1.4. (a) *Every compact $T \in AE(\mathbf{0})$ is dyadic.*

(b) *Every dyadic space is a c.c.c. space.*

Proof. (a) As with every compact space, there exist a cardinal κ, a closed subspace F of $\mathbf{2}^\kappa$, and a continuous surjection $f : F \twoheadrightarrow T$ ([4](3.2.2)). Since $\mathbf{2}^\kappa \in \mathbf{0}$ and $T \in AE(\mathbf{0})$ there is $\overline{f} \in C(\mathbf{2}^\kappa, T)$ such that $\overline{f} \supseteq f$. Then $\overline{f}[\mathbf{2}^\kappa] = T$.

(b) It is well known ([4](2.3.18)) that every product of separable spaces—in particular, the space $\mathbf{2}^\kappa$—is a c.c.c. space; and the c.c.c. property is preserved under continuous surjections. □

Here and later we denote by αD the one-point compactification of the discrete space D of cardinality $\aleph_1$.

Theorem 1.5. *Let κ be a cardinal.*

(a) $\mathbf{2} \in AE(\mathbf{0})^{rp}$*;*

(b) $\mathbf{2}^\kappa \in AE(\mathbf{0})$*;*

(c) *if $\kappa > \aleph_0$ then there is compact $T \subseteq \mathbf{2}^\kappa$ such that $T \notin AE(\mathbf{0})$;*

(d) *if $\kappa > \aleph_0$ then $\mathbf{2}^\kappa \notin AE(\mathbf{0})^{rp}$.*

Proof. (a) was noted in Theorem 1.1, and (b) follows from (1.1) since $AE(\mathbf{0})^{rp} \subseteq AE(\mathbf{0})$.

It is easily seen, as in [4](6.2.16), that $\alpha D \subseteq_h \mathbf{2}^{\aleph_1}$. Clearly the (compact) space αD is not a c.c.c. space, so $\alpha D \notin AE(\mathbf{0})$ by Lemma 1.4. That shows (c), and (d) follows from Theorem 1.2. □

The gist of Theorem 1.5 is that while the class $AE(\mathbf{0})^{rp}$ is "completely hereditary" (Theorem 1.2), the class $AE(\mathbf{0})$ is not even compact-hereditary; and $AE(\mathbf{0})$, like every class $AE(\mathbf{X})$, is completely productive (1.1), while the class $AE(\mathbf{0})^{rp}$ is not even $\aleph_1$-productive. We will see in Corollary 2.5 below that $AE(\mathbf{0})^{rp}$ is (exactly) countably productive.

2. Characterizing the Spaces in $AE(\mathbf{0})^{rp}$

Our principal results about the class $AE(\mathbf{0})^{rp}$ are given in Theorems 2.1 and 2.2 and its corollaries.

Theorem 2.1. *If $T \in AE(\mathbf{0})$ and each point of T is a G_δ-point, then $T \in AE(\mathbf{0})^{rp}$.*

Proof. Given $f \in C(F,T)$ with F closed in $X \in \mathbf{0}$ and $T \in AE(\mathbf{0})$, we must find $\overline{f} \in C(X,T)$ such that $f \subseteq \overline{f}$ and $\overline{f}[X] = f[F]$. Since $X \in \mathbf{0}$ there is $\kappa \geq \omega$ such that $X \subseteq_h \mathbf{2}^\kappa$, and since $\mathbf{2}^\kappa \in \mathbf{0}$ and $T \in AE(\mathbf{0})$ there is $f^* \in C(\mathbf{2}^\kappa, T)$ such that $f \subseteq f^*$. Then, since $\mathbf{2}$ is a separable space and points of T are G_δ-points, the function f^* factors through a countable subproduct of $\mathbf{2}^\kappa$ in the sense that there exist countable $C \subseteq \kappa$ and $g \in C(\mathbf{2}^C, T)$ such that $f^* = g \circ \pi_C$ (with π_C the usual projection $\pi_C : 2^\kappa \twoheadrightarrow \mathbf{2}^C$). (The theorem just used, due to A. Gleason, is stated and proved in detail by Isbell [12](p. 132).) Since $\mathbf{2}^C$ is compact metrizable, its continuous image $g[\mathbf{2}^C]$ is compact metrizable ([4](3.1.28)). Then since $f[F]$ is closed in the separable, zero-dimensional, metrizable space $g[\mathbf{2}^C]$, there is a (continuous) retraction $r : g[\mathbf{2}^C] \twoheadrightarrow f[F]$ (see [4](6.2.B) for a proof of this assertion, credited by Engelking to Sierpiński [15]). Then $\overline{f} := r \circ f^*|X = r \circ g \circ \pi_C|X$ is as required. In detail:

(1) f^* is defined on $\mathbf{2}^\kappa$, so $\overline{f}$ is well-defined on X;

(2) $x \in X \Rightarrow \overline{f}(x) = (r \circ g \circ \pi_C)(x) \in r[g[\mathbf{2}^C]] \in f[X]$; and

(3) $x \in F \Rightarrow f(x) \in f[F]$, so $\overline{f}(x) = (r \circ g \circ \pi_C)(x) = r(f(x)) = f(x)$. □

Theorem 2.2. *For each space T, these conditions are equivalent.*

(a) *$T \in AE(\mathbf{0})^{rp}$;*

(b) *each compact subspace of T is dyadic;*

(c) *each compact subspace of T is metrizable.*

Proof. (a) $\Rightarrow$ (b). If compact $S \subseteq T$, then $S \in AE(\mathbf{0})^{rp} \subseteq AE(\mathbf{0})$ by Theorems 1.2 and 1.5, so (b) holds by Lemma 1.4.

(b) $\Rightarrow$ (c). Suppose that some compact $S \subseteq T$ is nonmetrizable, so that $w(S) = \kappa > \aleph_0$. Then, since S is dyadic, some point of S has local weight (character) κ (by a theorem of Esenin-Vol′pin [5], cited in [4](3.12.12(e))). Then S contains a copy of the one-point compactification of the discrete space of cardinality κ (by a theorem of Engelking [3], cited in [4](3.12.12(i))). Then S contains the (compact, non-c.c.c.) space αD. Since αD is not dyadic (by Lemma 1.4(b)), the assumption $w(S) > \aleph_0$ is false so S is metrizable.

(c) $\Rightarrow$ (a). According to Theorem 1.2((a) $\Rightarrow$ (b)), it suffices to show for each compact $S \subseteq T$ that $S \in AE(\mathbf{0})^{rp}$. Given such S, from (c) we have $S \subseteq_h [0,1]^\omega$ with $[0,1]^\omega \in AE(\mathbf{0})$ by (1.1) (since surely $[0,1] \in AE(\mathbf{0})$), so $S \subseteq [0,1]^\omega \in AE(\mathbf{0})^{rp}$ by Theorem 2.1. Then $S \in AE(\mathbf{0})^{rp}$, as required. □

It is immediate from Theorem 2.2 that a compact space is closed-hereditarily dyadic if and only if it is metrizable. That is a result of Efimov [2], reproved in [3](p. 300).

Corollary 2.3. *Every metrizable space is an $AE(\mathbf{0})^{rp}$-space.*

Corollary 2.4. *For each space T, these conditions are equivalent.*
(a) $T \in AE(\mathbf{0})^{rp}$;
(b) $S \subseteq T \Rightarrow S \in AE(\mathbf{0})$;
(c) $S \subseteq T$, S *closed* $\Rightarrow S \in AE(\mathbf{0})$; *and*
(d) $S \subseteq T$, S *compact* $\Rightarrow S \in AE(\mathbf{0})$.

Proof. That (a) $\Rightarrow$ (b) is clear, since $AE(\mathbf{0})^{rp} \subseteq AE(\mathbf{0})$ and the class $AE(\mathbf{0})^{rp}$ is hereditary.

That (b) $\Rightarrow$ (c) and (c) $\Rightarrow$ (d) are obvious.

If (d) holds then by Lemma 1.4(a) every compact $S \subseteq T$ is dyadic and Theorem 2.2((b) $\Rightarrow$ (a)) gives (a). □

Corollary 2.5. *Let $\{T_i : i \in I\}$ be a set of nonempty spaces and set $T := \Pi_{i\in I} T_i$. Then $T \in AE(\mathbf{0})^{rp}$ if and only if*
(i) *each $T_i \in AE(\mathbf{0})^{rp}$, and*
(ii) $|\{i \in I : |T_i| > 1\}| \leq \aleph_0$.

Proof. "only if". Each $T_i \subseteq_h T$, so Theorem 1.2((a) $\Rightarrow$ (b)) shows (i). If (ii) fails then $\mathbf{2}^{\aleph_1} \subseteq_h T$, and then from $\mathbf{2}^{\aleph_1} \notin AE(\mathbf{0})^{rp}$ (Theorem 1.5(d)) would follow the contradiction $T \notin AE(\mathbf{0})^{rp}$ (from Theorem 1.2).

"if". We assume without loss of generality that $|I| \leq \aleph_0$. By Theorem 2.2, it suffices to show that each compact $S \subseteq T$ is metrizable. Given such S we have for each $i \in I$ that the (compact) space $\pi_i[S]$ is metrizable, so $\Pi_{i\in I} \pi_i[S]$ (and hence its subspace S) is metrizable. □

We continue with additional corollaries of the foregoing theorems. In Corollary 2.7 we note that a number of familiar spaces are in the class $AE(\mathbf{0})^{rp}$, and in Corollary 2.8 we show that spaces which are "locally in $AE(\mathbf{0})^{rp}$" are in fact in $AE(\mathbf{0})^{rp}$. (That result is in parallel with the theorem from [14] that "locally Dugundji" implies Dugundji; the converse to that result is given by Hoffmann [11].)

We first remind the reader of the relevant definitions.

Definition 2.6. Let $T = (T, \mathcal{T})$ be a space.
(a) A *network* in T is a family $\mathcal{N}$ of subsets of T such that if $x \in U \in \mathcal{T}$ then there is $N \in \mathcal{N}$ such that $x \in N \subseteq U$;
(b) T is a *σ-space* if it has a σ-discrete network;
(c) T is a *P-space* if every G_δ-subset of T is open.

We refer the reader to [7], especially (§4), for a useful introduction to σ-spaces. It is noted there, for example, that every Moore space (in particular, every metrizable space and every countable space), is a σ-space; further, every (countably) compact subspace of a σ-space is metrizable ([7](p. 447)).

Every compact subspace of a P-space, being finite ([6](4K)), is metrizable.

Using those facts, or otherwise, we have the following corollary to Theorem 2.2((c) $\Rightarrow$ (a)).

Corollary 2.7. *Every σ-space, and every P-space, and every countable space, is in the class $AE(\mathbf{0})^{rp}$.*

(This shows that the converse to Theorem 2.1 fails: in a P-space, each G_δ-point is isolated.)

Corollary 2.8. *Let T be a space.*

(a) *If each $x \in T$ has a neighborhood $U_x \in AE(\mathbf{0})^{rp}$, then $T \in AE(\mathbf{0})^{rp}$; and*

(b) *if T is the topological sum (the "disjoint union") of spaces in $AE(\mathbf{0})^{rp}$, then $T \in AE(\mathbf{0})^{rp}$.*

Proof. It suffices to prove (a), since (b) is then immediate.

By Theorem 2.1, it suffices to show that every compact $S \subseteq T$ is metrizable. Let $\{U_x : x \in T\}$ be a cover of T as indicated (with each $U_x \in AE(\mathbf{0})^{rp}$), and for $x \in T$ choose open V_x such that $x \in V_x \subseteq \overline{V_x} \subseteq U_x$. There is finite $F \subseteq S$ such that

$S \subseteq \bigcup_{x \in F} V_x \subseteq \bigcup_{x \in F} \overline{V_x} \subseteq \bigcup_{x \in F} U_x$

and hence $S = \bigcup_{x \in F} (S \cap \overline{V_x})$. Each space $S \cap \overline{V_x}$ is compact (being closed in S) and is in $AE(\mathbf{0})^{rp}$ (being a subset of $U_x \in AE(\mathbf{0})^{rp}$). So by Theorem 2.2, each space $S \cap \overline{V_x}$ is metrizable. Thus S, the union of finitely many of its closed, metrizable subspaces, is itself metrizable ([4](4.19)). □

3. An Application to Lattice-Ordered Groups

We consider the category $\mathcal{W}^*$ of archimedean lattice-ordered groups G with distinguished strong order unit e_G (that means: for each $g \in G$ there is $n \in \mathbb{N}$ such that $|g| \leq ne_G$), together with group- and lattice-homomorphisms which preserve unit. The notation $G \leq H$ indicates that $G \in \mathcal{W}^*$ is a subobject of $H \in \mathcal{W}^*$. The Yosida representation theorem, as exposed in [9], tells us that each $G \in \mathcal{W}^*$ has an essentially unique representation $G \simeq \widehat{G} \leq C(YG, \mathbb{R})$ with YG compact (Hausdorff) and with $\widehat{G}$ separating points of YG; and, for each $\phi : G \to H \in \mathcal{W}^*$ there corresponds a unique continuous $\tau : YH \twoheadrightarrow YG$ such that $\widehat{\phi}(g) = \widehat{g} \circ \tau$ for each $g \in G$, and with τ an injection (hence an embedding) if ϕ is a surjection. We identity each $G \in \mathcal{W}^*$ with its $\widehat{G}$. Thus, a surjection $\phi : G \twoheadrightarrow H$ becomes the restriction to YH of the functions in G.

Now let $E \leq \mathbb{R}$ (that is, E is a subgroup of $\mathbb{R}$, and $1 \in E$), and set

$\mathcal{C}_E := \{C(X, E) : X \text{ is compact}\} \subseteq \mathcal{W}^*$.

Theorem 3.1. *$\mathcal{C}_E$ is closed under surjections in $\mathcal{W}^*$.*

Proof. [We sketch.] First consider the case $E = \mathbb{R}$. Then $YC(X, \mathbb{R}) = X$, and each surjection $\phi : C(X, \mathbb{R}) \twoheadrightarrow H$ is induced by the restriction $g \to g|YH$ to the subspace $YH \subseteq X$. Each $f \in C(YH, \mathbb{R})$ has an extension $g \in C(X, \mathbb{R})$ (Tietze-Urysohn), so $f = g|YH$ and $H = C(YH, \mathbb{R})$.

Now if $E \neq \mathbb{R}$ then E is zero-dimensional, and $Y := YC(X, E)$ is the zero-dimensional reflection of X: $Y \in \mathbf{0}$. So for a surjection $\phi : C(X, E) \twoheadrightarrow H$ the "dual" topological inclusion $YH \subseteq Y$ lives in $\mathbf{0}$. Then, each $f \in C(YH, E)$ has an extension $g \in C(Y, E)$, because $E \in AE(\mathbf{0})^{rp}$ (*e.g.*, by Theorem 2.2), so $f = g|YH$ and again $H = C(YH, E)$, as required. □

We note that when $E \neq \mathbb{R}$ in the preceding theorem, either E is cyclic (and thus discrete) or E is dense in $\mathbb{R}$. In the former case, an extension g of $f \in C(YH, E)$ is easily manufactured, using the fact that $|f[YH]| < \omega$, by extending the resulting finite clopen partition of YH to one of Y (much as in the proof of Theorem 1.1). In the (proof of the) dense case, however, the relation $E \in AE(\mathbf{0})^{rp}$ is crucial; the proof of that appears to require much of the argumentation we have given above in Theorem 2.2.

More issues of the sort addressed in this section are considered in the work [9].

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(Received November 2011 – Accepted November 2012)

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