A response analysis of wheat and barley to nitrogen in Finland

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A nonlinear Mitscherlich function was found to be superior to quadratic and square root functions in estimating yield response to nitrogen based on a Finnish sample of barley. Nonnested hypothesis testing (J-test) indicated the Mitscherlich functional form to fit the data better than the quadratic form based on this sample. In the analysis of the crop response for spring wheat the Mitscherlich functional form could not be proved superior by a J-test. The inferred profit maximizing nitrogen fertilization levels based on the Mitscherlich functional form exceeded the quadratic polynomial forms and were lower than the inferred levels using square root specifications. Implementing 100% nitrogen price increases or 50% producer price reductions lowered the profit maximizing nitrogen application doses by 20-24%, according to the Mitscherlich specification.

Key words: crop response, fertilizers, Mitscherlich, nitrogen fertilizers, nonnested hypothesis test, plant response, polynomial, production functions, yield response.

Introduction

The form of crop response to fertilizers is a controversial issue that has been debated for several decades. Economists have often preferred crop response functions that are smooth, concave and twice differentiable. Polynomial functions, especially quadratic and square root functions, have been commonly used. Quadratic specifications have been frequently used to estimate the crop response to nitrogen fertilizers. In Finland, for instance, IHAMUOTILA (1970), RYYNÄNEN (1970), LUOSTARINEN (1974), HIIVOLA et al. (1974), HEIKKILÄ (1980) and KETTUNEN (1981) have used the quadratic functional form in estimating nitrogen response.

Since the middle of the 1970s, however, several studies have indicated that the use of the polynomial functions (particularly the quadratic function) is not very appropriate for estimating the nitrogen response or the economic optimum of fertilizer use. ANDERSON and NELSON (1975), LANZER and PARIS (1981) and PARIS (1992a,b) have all shown that the quadratic functional form commonly leads to excess estimates of the most economic fertilizer use. Moreover, if more than one plant nutrient is included in the function, polynomial functions assume substitutability between the nutrients. Attempts to estimate the profitability of nitrogen fertilizer use by quadratic or square root functional forms may therefore lead to a biased estimate of the optimal nitrogen fertilizer dose.

In a recent attempt to determine the profitability of nitrogen fertilizers under Finnish conditions LAURILA (1992) found the optimal nitrogen fertilizer application to be 153 kg N/ha. This estimate was based on an updated quadratic function originally estimated by HEIKKILÄ (1980). In the light of previous criticism, it is possible that this figure overestimates the quantity of nitrogen fertilizer corresponding to the economic optimum. If this estimate is biased and used for extension purposes, this may lead to uneconomic fertilizer use. Furthermore, use of an excessive estimate of the optimal fertilization intensity for extension purposes may increase leakages of nitrogen to the waterways.

The primary purpose of this paper is to estimate the functional form of the nitrogen response. The functional form of the nitrogen response is of interest if economic incentives like nitrogen taxes or producer price taxes are applied in order to lower nitrogen appplication doses. RØRSTAD (1992) has pointed out that the form of the production function, not the absolute profit maximizing level, is decisive when one investigates the effects of adjusting application doses. A nitrogen tax, for instance, may lead to quite different losses for different functional forms of the nitrogen response. When a Mitscherlich or a square root specification of the nitrogen response is assumed instead of a quadratic functional form, the decrease in the profit maximizing nitrogen application level is likely to be of different magnitude. Since the quadratic function has been dominant in the analysis of nitrogen response in Finland, this particular form should be tested against other functional forms. Therefore, in this paper it will be examined whether a specification of the nitrogen crop response according to a Mitscherlich form of the production function (also called a Spillman function) leads to more believable estimates of the nitrogen response than those based on the quadratic and the square root functions. A nonnested hypothesis testing will be applied in order to answer this question. Nonnested hypothesis tests concerning the form of the response curve have been carried out by ACKELLO-OGUTU et al. (1985), by GRIMM et al. (1987), by FRANK et al. (1990) and by PARIS (1992a,b).

A secondary purpose of the paper is to estimate whether the optimal nitrogen application estimated by the quadratic form and the square root form substantially differ from an optimum estimated by the Mitscherlich form of the production function. ROMSTAD and RØRSTAD (1993) have suggested another method for estimating ex ante profit maximizing fertilizer doses. Their approach, which identifies possible profit states, may provide a better method of estimating the expected value of perfect information. It should be stressed, however, that no attempt is made here to obtain exact estimates of fertilizer application doses to be used for fertilizer recommendations.

Production functions and conditions for profit maximization

The yield level of a crop is, in general, a function of several economic and biological inputs:

(1) $y = f(X, S, I)$

where y = observed yield
X = a vector of fertilizer nutrients
S = a vector of soil type characteristics
I = a vector of weather factors

Since this form of the production function is too general for estimation, soil characteristics and weather factors are assumed to be given. Omitting all nutrients except one, equation (1) can be specified as:

(2) $y = F(x|X, S, I)$.

The optimal nitrogen application doses (i.e. profit maximizing nitrogen applications) can be derived from the first-order condition for profit maximization. Maximization of profit (net revenue) can be stated as

(3) $\pi(p,w) = \max_{x \geq 0} \{py - wx|y = f(x)\}$.

where π is the profit, p is the product price, w is the input price and f(x) is the functional form. The specification of the response appears as a constraint since the profit maximizing application dose is dependent on the functional form specified.

The optimization problem of the farmer can be written

(4) $\max_{x \geq 0} \pi = pf(x) - wx$

Differentiating with respect to the input x gives the first order condition (FOC) for profit maximization:

(5) $\partial\pi/\partial x = p\partial f(x)/\partial x - w = 0$

(6) $\partial f(x)/\partial x = w/p$

which states that at the profit maximum the marginal product equals the ratio between input and output price. x* can be solved for

(7) $x^* = x(p,w)$

As inputs can be assumed to be nonnegative we can impose the constraint $x \geq 0$. In order to guarantee that this optimum is a local maximum, the second order sufficient condition (SOC) $\partial^2\pi/\partial x\partial x < 0$ must hold.

The specification of the quadratic, the square root and the Mitscherlich production function as well as the solutions to FOC/SOC for all the functional forms are presented in Table 1.

Table 1. Solutions to FOC/SOC for three functional forms.

Functional form	FOC	SOC
Quadratic	$x^* = \dfrac{\frac{w}{p} - \beta_2}{2\beta_3}$	$2\beta_3 < 0$
Square root	$x^* = \left[\dfrac{\frac{w}{p} - \beta_3}{\frac{1}{2\beta_2}}\right]^{-2}$	$\dfrac{-\beta_2 x^{\frac{-3}{2}}}{4} < 0$
Mitscherlich	$x^* = \dfrac{\ln\left(\frac{pmk\beta}{w}\right)}{\beta}$	$-pmk\beta^2 e^{-\beta x} < 0$

Estimation of production functions

The estimation of production functions depends, besides the particular application purpose, on a number of not so evident factors. ROMSTAD and HEGRENES (1990) point out the following factors: 1. Is time series data or cross sectional data used? 2. The form of the production function 3. Are the signs and magnitude of parameter estimates such that second order conditions are fulfilled? 4. Are the residuals varying systematically? ROMSTAD and RØRSTAD (1993) point out that heteroscedasticity does not cause problems regarding unbiasedness of the estimated parameters but may cause insignificant parmeter estimates. Autocorrelated errors are another form of systematic residual variation connected with time series.

Critique of the polynomial forms for estimating crop response and alternatives suggested

The two most commonly used polynomial forms for estimating the nitrogen response are a quadratic and a square root function. The widespread popularity of these functional forms can be explained by their easy computational properties and the scarcity of computers in earlier decades. It is easy to calculate first and second order derivatives from these functions, which give the necessary and sufficient conditions for profit maximization illustrated in table 1. A quadratic form of the response function is

(8) $y = \beta_1 + \beta_2 x + \beta_3 x^2 + \delta_1 D_1 + \ldots + \delta_n D_n + \delta_t D_t$

where y = yield/ha
β_1 = intercept
$\beta_2, \beta_3, \delta_1, \delta_i, \delta_t$ = parameters, $\beta_2 > 0$, $\beta_3 < 0$
x = nitrogen fertilization
D_i = annual dummies (i = 1...n)
D_t = technology dummy.

Similarly, the square root form of the response function is

$$(9) \quad y = \beta_1 + \beta_2 x^{1/2} + \beta_3 x + \delta_1 D_1 + ... + \delta_n D_n + \delta_t D_t$$

Originally suggested by HEADY and PESEK (1955), the polynomial crop production functions became the dominant forms used in estimating crop response to fertilizers in the 1950s, 1960s and 1970s. These functions provided a good fit for the data as measured by the coefficient of determination. However, concerns have been raised that these polynomial production functions do not illustrate the yield response of nitrogen fertilizers in the best possible way.

Among the first to criticize the use of the quadratic form for estimating crop response were ANDERSON and NELSON (1975). According to these authors, the quadratic production function may result in costly biases in the levels of optimal fertilizer rate, and may also generate a potential pollution problem. Anderson and Nelson proposed "a family of linear-plateau models, consisting of intersecting straight lines, which exhibit a plateau effect." The linear-plateau implies a region of linear response, with possibly several slopes, and a plateau which represents a level where the crop response is evening out and becomes almost flat. An implication of the linear-plateau models is that the estimated production function is kinked. HELLAND and AASTVEIT (1992) show that it is extremely difficult to determine the location of the kink(s).

LANZER and PARIS (1981) showed the Mitscherlich type of response function to outweigh the polynomial specifications in estimating wheat-soybean crop response for nitrogen in Brazil. At the same time they criticized fertilizer recommendations based on polynomial functions and noted that recommendations of nitrogen could be reduced by 10-50%. Graphically a Mitscherlich specification seems to be a hybrid between the kinked linear-plateau models and polynomial specifications. The Mitscherlich functional form as well as the polynomial specifications are presented in Figure 1.

ACKELLO-OGUTU et al. (1985) found that a von Liebig function of the form

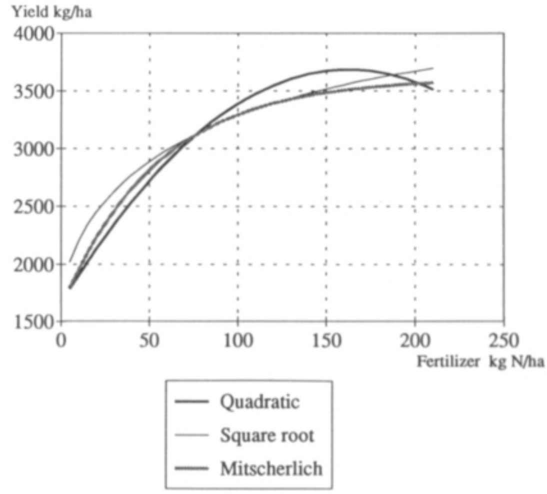

Fig. 1. Quadratic, square root and Mitscherlich functional forms based on nitrogen response for spring wheat on loam clay (estimation results in Table 3).

$$(10) \quad y = A_{SW} \min [f_i(X_{iT})]$$

where A_s = yield plateau for given weather and soil type
i = essential nutrients
$f_i(X_{iT})$ = concave

was to be preferred. The yield level approaches the plateau A_{SW} asymptotically when x increases. In other words, A_{SW} is the maximum attainable yield.

While rejecting the notion of total nonsubstitutability between N and P, FRANK et al. (1990) claimed that a Mitscherlich-Baule model represented by (11) will recognize a plateau:

$$(11) \quad y = \beta_1[1-k^{(-\beta_2(\beta_3 + N_i))}]\,[1-k^{(-\beta_4(\beta_5 + P_i))}]$$

In their analysis they used the same well-known fertilizer/crop yield data sample as HEADY et al. (1955). Applying a nonnested hypothesis testing framework, both the quadratic and the von Liebig model were rejected in favor of the Mitscherlich-Baule model. Their conclusion was that neither a polynomial nor a von Liebig functional form should be applied a priori since the Mitscherlich-Baule model performed better. PARIS (1992b) showed that a von Liebig non-linear production function with Mitscherlich regimes was superior over quadratic, square root, Mitscherlich-Baule and linear von Liebig production functions on the basis of the same Heady-Pesek data analyzed by FRANK et al. (1990) The (non-linear) von Liebig model with Mitscherlich regimes outperformed all other models, based on both a pairwise specification (the P-test) and a collective test (J-test). The optimal value of nitrogen fertilization was 177 lbs per acre (199 kg N/ha) using a quadratic function and 115 lbs per acre (129 kg N/ha) based on the non-linear von Liebig function. Optimal phosphorus (P) fertilization based on the quadratic function was 176 lbs per acre (198 kg P/ha) and only 91 lbs per acre (102 kg P/ha) based on the non-linear von Liebig function. The overestimation of the economic optimum based on polynomial function seemed to be substantial (PARIS 1992b).

Taking into account the criticism raised concerning the use of polynomial production functions for estimating nitrogen fertilization responses, it seems possible that the use of quadratic or square root production functions often leads to overestimated amounts of profit maximizing nitrogen levels. The advancement in computational techniques has facilitated the estimation of functions with a higher degree of sophistication, for instance, non-linear functions for one or several inputs.

If the response to two or more inputs were to be estimated, a nonlinear von Liebig functional form with Mitscherlich regimes may be the most appropriate according to PARIS (1992b). In such a case one fundamental econometric condition needs to be fulfilled: the two inputs cannot be linear combinations of each other. If this requirement is not met, only one of the inputs can be included in the production function.

Unfortunately, some experiments carried out under Finnish conditions have used composite fertilizers, where nitrogen, phosphorus and potassium are perfect linear combination of each other (for instance when N-P-K is 20%, 4%, 8%). Because of this perfect collinearity between the inputs in the experimental data for spring wheat and barley from Tikkurila 1969-1980 only the crop response from one of the nutrients can be included in the production function. This makes the estimation of a non-linear von Liebig function with two nutrients impossible on the basis of the data that has been available to the author. The purpose of this paper is, therefore, to estimate the crop response to one input, nitrogen. Both the Mitscherlich-Baule and the non-linear von Liebig production functions with Mitscherlich regimes are actually extensions of Mitscherlich's specification for one nutrient. Like many agicultural economists have noted, the response curves tend to be quite flat on the top (PERRIN 1976). This can be interpreted as a restatement of "the law of the minimum" formulated by von Liebig. The law states that "the yield of any crop is governed by any chance by its scarcest factor, called the minumum factor, and as the minimum factor is increased the yield will increase in proportion to the supply until another becomes the minimum" (REDMAN and ALLEN 1954). This implies absence of nutrient substitution. A polynomial specification like the quadratic or the square root specification, on the other hand, implies substitution beteween inputs and does not, therefore, comply with the von Liebig principle (LANZER and PARIS 1981).

On the basis of the theory provided by von Liebig, the Mitscherlich form of the production function will lead to more correct estimates than polynomial functions. Testing if the Mitscherlich functional form for the nitrogen crop response leads to more believable estimates of the nitrogen response than those based on the quadratic and the square root functions is therefore equal to a test of von Liebig's hypothesis. In the testing procedure the parameter estimates from the Mitscherlich production function will be compared to parameter estimates from quadratic and square root functional forms by nonnested tests. In order to rigorously test which functional form is more correct, the alternative hypotheses for functional forms will be tested against each other using the J-test, a nonnested test. The J-test seems to have become one of the most commonly used nonnested tests since it was first suggested by DAVIDSON and MACKINNON (1981). Descriptions of the J-test can be found in most of the recently published advanced econometric textbooks, eg. KMENTA (1986) and GREENE (1993).

Mitscherlich functional form and maximum likelihood estimators

Mitscherlich was the first agriculturist to suggest a nonlinear production relation between nutrient input and yields (HEADY and DILLON 1961). The type of crop response function Mitscherlich suggested in 1909 is an exponential nonlinear function of type

$$(12) \quad y = m(1-ke^{-\beta x})\, e^{\delta_1, D_1}\, e^{\delta_2, D_2} \ldots e^{\delta_n, D_n}\, e^{\delta_t, D_t}$$

where y = yield level, kg/ha
x = nitrogen fertilization
m = asymptotic plateau
D_i = annual dummies ($i = 1..n$)
D_t = technology dummy
$k, \beta, \delta_i, \delta_t$ = parameters

The yield level approaches asymptotically a plateau l evel, m, when $x \to \infty$. m is therefore the maximum attainable yield given weather and soil conditions. The parameter k is a parameter describing the rate at which marginal yields decline. Dummies are added to take into account annual variations ($D_1 \ldots D_n$) and technology (D_t). The FOC/SOC conditions for profit maximization are presented in table 1. As no other plant nutrients enter the crop response function, their availability is implicitly assumed to be nonlimiting.

The Mitscherlich form of the production function is nonlinear in the parameters, and it is therefore convenient to estimate by maximum likelihood estimation procedures. The resulting maximum likelihood estimators (MLE) are characterized by some desirable large-sample properties. The estimators are consistent, asymptotically normally distributed and asymptotically efficient.

Taking logarithms of both sides of (12) will yield

$$(13) \quad \ln y = \ln m + \ln((1-ke^{-\beta x}) + \Sigma\delta_i D_i + \delta_t D_t$$

The estimated residuals in the non-logarithmic form will be

$$(14) \quad \hat{u} = y_i - \hat{m}(1-\hat{k}e^{-\hat{\beta}x})\, e^{D_1\delta_1}\, e^{D_2\delta_2} \ldots e^{D_n\delta_n}\, e^{D_t\delta_t}$$

and since $\text{var}(u) = \sigma^2$, the estimator of σ is

$$(15) \quad \hat{\sigma} = \sqrt{\frac{\hat{u}^2}{n-K}}$$

The coefficient of determination according to GREENE (1993) is

$$(16) \quad R^2 = 1 - \frac{\Sigma \hat{u}^2}{\Sigma\,(y_i-\bar{y})^2}$$

and the adjusted coefficient of determination R^2 adj. is

$$(17) \quad \bar{R}^2 = R^2 - \left(\frac{n-1}{n-k}\right)(1-R^2)$$

The Mitscherlich specification was estimated through the MLE procedure. The quadratic and square root production functions were estimated by OLS. All estimates of m, k, β, δ_i, σ^2 and R^2 were calculated using the SHAZAM version 7.0 econometrics computer program.

Data

The sample of experimental data used for the estimation of nitrogen fertilizer crop response consists of pooled cross-sectional and time-series data from fertilizer experiments with spring wheat and barley at the experimental fields of the Agricultural Research Centre in Tikkurila in 1969-1980 (ESALA and LARPES 1984). Five equally spaced treatments were applied to 108 experimental plots: 0, 50, 100, 150 and 200 (kg N/ha). Experiments were carried out at two different types of soils: fine sand clay and loam clay. Two different fertilization technologies were used: top dress fertilization and fertilizer placement. Observations on yield levels were recorded for each year, intensity level, soil type and technology. Thus the pooled data consisted of 108 observations of yield levels for both spring wheat and barley on two different soil types. The average yields were: wheat 3679.4 kg/ha on fine sand clay and 3228.4 kg/ha on loam clay, barley 4160.7 kg/ha on fine sand clay and 3712.2 kg/ha on loam clay.

The annual variations in the data were large. The year 1973 represented low yields because of drought, especially for barley. In some cases yields were extremely low, e.g. only 110 kg/ha. In order to take into account annual differences, eleven annual dummies were introduced. In addition, one dummy for technology was added according to (8), (9) and (12) since a different technology (top dress fertilization respectively fertilizer placement) was used for every second plot. The data were scaled in order to decrease errors in the computational procedure (N-input was scaled to 0-20, yields to 1-99).

Of the four data series, two seem to have been used by Heikkilä (for the period 1969-1978) in estimating the crop response of spring wheat and barley using a quadratic form similar to model (10). This was confirmed by OLS regressions for the period 1969-1978 (cf. HEIKKILÄ 1980, p. 24).

Results

The results for the estimated Mitscherlich function (12) as well as the square root functional form (9) and quadratic functional form (8) for wheat and barley yield on two different soil types are presented in tables 2, 3, 4 and 5.

Initially the specification included no dummies. Since annual variations are known to be important and since they increased the conventional criterion of improving the adjusted coefficient of determination R^2adj., eleven annual dummies were included. Furthermore, a technology dummy was included.

The parameter estimates that determine the yield level (and nitrogen application doses) are significant at $\alpha = 0.005$ for all three functional forms in most cases. In fact, for both the Mitscherlich functional form and the quadratic form, the parameter estimates of m, k, β_1 β_2 and β_3 are all significant at $\alpha = 0.005$ in all cases. For the square root functional form, β_1 and β_2 are also significant at $\alpha = 0.005$ for all four data series, whereas β_3 only is significant at $\alpha = 0.005$ for two of the data series. For the square root form β_3 is insignificant in one case (spring wheat on fine sand clay) and significant only at $\alpha = 0.05$ in another case (barley on fine sand clay).

Table 2. Estimation results for spring wheat on fine sand clay. Standard errors in parenthesis[1)]

	Quadratic	Square root	Mitscherlich
m			53.616***
			(2.529)
k			0.753***
			(0.010)
β_1	13.544***	12.119***	0.100***
	(1.702)	(1.863)	(0.009)
β_2	3.502***	8.439***	
	(0.213)	(0.964)	
β_3	-0.090***	-0.048	
	(0.009)	(0.193)	
δ_1	-3.822*	-3.822*	-0.077
	(1.790)	(1.855)	(0.690)
δ_2	3.578*	3.578*	0.094
	(1.790)	(1.855)	(0.658)
δ_3	-6.067***	-6.0670***	-0.134
	(1.790)	(1.855)	(0.802)
δ_4	-11.456***	-11.456***	-0.325
	(1.790)	(1.855)	(0.818)
δ_5	7.622***	7.622***	0.178
	(1.790)	(1.855)	(0.305)
δ_6	-6.333***	-6.333***	-0.188
	(1.736)	(1.855)	(0.707)
δ_7	0.411	0.411	0.036
	(1.790)	(1.855)	(0.832)
δ_8	-2.144	-2.144	-0.060
	(1.790)	(1.855)	(0.422)
δ_9	-0.833	-0.833	-0.033
	(1.790)	(1.855)	(0.817)
δ_{10}	-13.522***	-13.522***	-0.415
	(1.790)	(1.855)	(0.464)
δ_{11}	-4.033*	-4.033*	-0.152
	(1.790)	(1.855)	(0.670)
δ_t	4.194***	4.480***	0.120***
	(0.765)	(0.802)	(0.019)
df	93	93	93
$\hat{\sigma}^2$	14.419	15.479	1.182
$\hat{\sigma}$	3.797	3.934	1.087
logL	-289.273	-293.104	-154.781
R^2adj.	0.9097	0.9031	0.9926

1) ***: Null hypothesis rejected at 0.5 % level ($t_{.005}$ = 2.58)
**: Null hypothesis rejected at 1 % level ($t_{.01}$ = 2.33)
*: Null hypothesis rejected at 5 % level ($t_{.05}$ = 1.64)

T-ratios for the annual dummies δ_1-δ_{11} and the technology dummy are higher for both polynomial forms than for the Mitscherlich form of the production function. The technology dummy is positive in

Table 3. Estimation results for spring wheat on loam clay. Standard errors in parenthesis[1)]

	Quadratic	Square root	Mitscherlich
m			36.189***
			(1.598)
k			0.551***
			(0.018)
β_1	16.633***	15.292***	0.181***
	(1.863)	(1.994)	(0.023)
β_2	2.490***	7.329***	
	(0.234)	(1.032)	
β_3	-0.077***	-0.566***	
	(0.010)	(0.206)	
δ_1	-6.200***	-6.200***	-0.198
	(1.959)	(1.986)	(0.887)
δ_2	-1.022	-1.022	-0.002
	(1.959)	(1.986)	(0.930)
δ_3	10.233***	10.233***	0.293
	(1.959)	(1.986)	(0.788)
δ_4	-16.533***	-16.533***	-0.701
	(1.959)	(1.986)	(0.689)
δ_5	19.667***	19.667***	0.486
	(1.959)	(1.986)	(0.674)
δ_6	-10.400***	-10.400***	-0.374
	(1.959)	(1.986)	(0.823)
δ_7	9.311***	9.311***	0.269
	(1.959)	(1.986)	(0.736)
δ_8	5.033**	5.033**	0.142
	(1.959)	(1.986)	(0.786)
δ_9	-1.889	-1.889	-0.090
	(1.959)	(1.986)	(0.841)
δ_{10}	-5.800***	-5.800***	-0.194
	(1.959)	(1.986)	(0.753)
δ_{11}	2.344	2.344	0.052
	(1.959)	(1.986)	(0.840)
δ_t	0.636	0.904	0.027
	(0.837)	(0.858)	(0.024)
df	93	93	93
$\hat{\sigma}^2$	17.275	17.748	1.192
$\hat{\sigma}$	4.156	4.213	1.092
logL	-299.030	-300.490	-155.227
R^2adj.	0.8802	0.8770	0.9917

1) ***: Null hypothesis rejected at 0.5 % level ($t_{.005}$ = 2.58)
**: Null hypothesis rejected at 1 % level ($t_{.01}$ = 2.33)
*: Null hypothesis rejected at 5 % level ($t_{.05}$ = 1.64)

Table 4. Estimation results for barley on fine sand clay. Standard errors in parenthesis[1)].

	Quadratic	Square root	Mitscherlich
m			74.597***
			(5.451)
k			0.875***
			(0.010)
β_1	15.333***	12.499***	0.113***
	(2.112)	(2.197)	(0.015)
β_2	4.960***	13.337***	
	(0.265)	(1.137)	
β_3	-0.135***	-0.498*	
	(0.012)	(0.227)	
δ_1	-8.333***	-8.333***	-0.237
	(2.221)	(2.188)	(0.786)
δ_2	0.522	0.522	-0.054
	(2.221)	(2.188)	(0.854)
δ_3	-10.233***	-10.233***	-0.255
	(2.221)	(2.188)	(0.376)
δ_4	-22.744***	-22.744***	-0.802*
	(2.221)	(2.188)	(0.384)
δ_5	-5.133***	-5.133**	-0.125
	(2.221)	(2.188)	(0.823)
δ_6	-19.256***	-19.256***	-0.616
	(2.221)	(2.188)	(0.692)
δ_7	3.422	3.422	0.057
	(2.221)	(2.188)	(0.341)
δ_8	-8.589***	-8.589***	-0.257
	(2.221)	(2.188)	(0.649)
δ_9	-11.956***	-11.956***	-0.357
	(2.221)	(2.188)	(0.583)
δ_{10}	-20.100***	-20.100***	-0.594
	(2.221)	(2.188)	(0.812)
δ_{11}	-18.311***	-18.311***	-0.559
	(2.221)	(2.188)	(0.857)
δ_t	6.588***	7.156***	0.182
	(0.950)	(0.946)	(0.042)
df	93	93	93
$\hat{\sigma}^2$	22.205	21.535	1.253
$\hat{\sigma}$	4.712	4.641	1.112
logL	-312.589	-310.932	-157.938
R^2adj.	0.9242	0.9265	0.9957

1) ***: Null hypothesis rejected at 0.5 % level ($t_{.005}$ = 2.58)
**: Null hypothesis rejected at 1 % level ($t_{.01}$ = 2.33)
*: Null hypothesis rejected at 5 % level ($t_{.05}$ = 1.64)

all cases, which indicates that fertilizer placement leads to higher yields than top dress fertilization. In addition, the dummy is significant in eight of twelve cases. Positive annual dummies indicate good weather conditions and negative dummies indicate bad weather conditions.

The estimate of error variance, $\hat{\sigma}^2$, is clearly lower for the Mitscherlich form of production func-

Table 5. Estimation results for barley on loam clay. Standard errors in parenthesis[1].

	Quadratic	Square root	Mitscherlich
m			47.149***
			(3.442)
k			0.770***
			(0.016)
β_1	14.112***	11.684***	0.154***
	(2.073)	(2.133)	(0.020)
β_2	3.669***	10.725***	
	(0.260)	(1.104)	
β_3	-0.107***	-0.683***	
	(0.012)	(0.220)	
δ_1	-13.004***	-13.044***	-0.375
	(2.181)	(2.124)	(0.293)
δ_2	-1.167	-1.167	-0.051
	(2.181)	(2.124)	(0.401)
δ_3	6.978***	6.978***	0.177
	(2.181)	(2.124)	(0.363)
δ_4	-27.944***	-27.944***	-1.437***
	(2.181)	(2.124)	(0.105)
δ_5	10.878***	10.878***	0.301
	(2.181)	(2.124)	(0.466)
δ_6	-14.389***	-14.389***	-0.473
	(2.181)	(2.124)	(0.329)
δ_7	20.011***	20.011***	0.460
	(2.181)	(2.124)	(0.596)
δ_8	6.022***	6.022***	0.121
	(2.181)	(2.124)	(0.570)
δ_9	1.300	1.300	0.010
	(2.181)	(2.124)	(0.291)
δ_{10}	-2.578	-2.578	-0.114
	(2.181)	(2.124)	(0.610)
δ_{11}	-3.000	-3.000	-0.099
	(2.181)	(2.124)	(0.474)
δ_t	2.539***	3.026***	0.109*
	(0.932)	(0.919)	(0.039)
df	93	93	93
$\hat{\sigma}^2$	21.396	20.303	1.238
$\hat{\sigma}$	4.626	4.506	1.113
logL	-310.584	-307.753	-157.263
R^2adj.	0.9186	0.9228	0.9953

[1] ***: Null hypothesis rejected at 0.5 % level ($t_{.005}$ = 2.58)
**: Null hypothesis rejected at 1 % level ($t_{.01}$ = 2.33)
*: Null hypothesis rejected at 5 % level ($t_{.05}$ = 1.64)

tion than for the polynomial forms. This indicates that the error connected with the Mitscherlich functional form is smaller than the error connected with the quadratic and the square root form. The $\hat{\sigma}^2$ connected with the Mitscherlich form is in all cases lower than one tenth of the $\hat{\sigma}^2$ connected with the quadratic and the square root forms.

The Mitscherlich form of the production function shows the best fit as measured by the R^2 criterion. R^2 adj. is higher than 0.991 for the Mitscherlich form of production function in all four cases. For the quadratic form, R^2 adj. varies between 0.880 and 0.924 and, for the square root form, R^2 adj. varies between 0.877 and 0.927. The high R^2 measured for the Mitscherlich production function is a natural outcome of a small error variance, σ^2.

Hypothesis testing

In order to determine which of the three specifications is the most appropriate model, they were tested against each other using a nonnested hypothesis test. A simple way to test two nonnested alternative, possibly nonlinear models, f(x) and g(x), is the following J-test proposed by DAVIDSON and MACKINNON (1981), where a compound model of $f(x, \delta)$ and $g(x, \phi)$ is tested:

$$(18)\quad y = (1-\alpha)\, f(x,\delta) + \alpha\hat{g}(x, \phi)$$
$$H: \alpha = 0$$

$\hat{g}(x, \phi)$ is simply the estimate of $g(x, \phi)$. $\hat{g}(x, \phi)$ is, in other words, the fitted value of the function $g(x, \phi)$ estimated by OLS for the polynomial functions and by MLE for the Mitscherlich function. In the testing procedure y is regressed on $(1-\alpha)f(x, \delta)$ and $\alpha\hat{g}(x, \phi)$. If H_0: $\alpha = 0$ is rejected by a conventional asymptotic t-test, this implies that f(x) is rejected over g(x). If H_0: $\alpha = 0$ is insignificant, f(x) is not rejected. The order of both functions should be reversed. It is possible for both functions to reject each other.

Therefore, all three rival models, quadratic, square root and Mitscherlich are tested against each other, which implies six different tests for each crop and soil, 24 tests altogether. A description of the J-test can be found in econometrics textbook, e.g. KMENTA (1986) or GREENE (1993).

The nonlinear regression carried out in the estimation of the Mitscherlich functional form as well as

Table 6. Results from nonnested hypothesis testing based on a J-test, J-test statistic.

1. Wheat on fine sand clay and loam clay

Alternative hypothesis	Fine sand clay Null hypothesis: Quadratic	Fine sand clay Null hypothesis: Square-root	Fine sand clay Null hypothesis: Mitscherlich	Loam clay Null hypothesis: Quadratic	Loam clay Null hypothesis: Square-root	Loam clay Null hypothesis: Mitscherlich
Quadratic		3.589***	2.517**		2.193*	0.774
Square-root	2.702***		-1.842*	2.813***		-1.103
Mitscherlich	0.062	-1.562		0.084	-1.109	

2. Barley on fine sand clay and loam clay

Alternative hypothesis	Fine sand clay Null hypothesis: Quadratic	Fine sand clay Null hypothesis: Square-root	Fine sand clay Null hypothesis: Mitscherlich	Loam clay Null hypothesis: Quadratic	Loam clay Null hypothesis: Square-root	Loam clay Null hypothesis: Mitscherlich
Quadratic		2.940***	0.254		11.315***	0.302
Square-root	-4.216***		-0.143	16.208***		-0.250
Mitscherlich	1.846*	2.005*		-1.892*	-1.915*	

[1] ***: Null hypothesis rejected at 0.5 % level ($t_{.005} = 2.58$)
**: Null hypothesis rejected at 1 % level ($t_{.01} = 2.33$)
*: Null hypothesis rejected at 5 % level ($t_{.05} = 1.65$)

in the nonnested hypothesis testing is based on an iterative process which is sensitive to changes in the starting value given to α. The significance of the J-test statistic is in many cases dependent on the initial starting value. The criterion for choosing a correct starting value for α is therefore to choose a value of α which maximizes the log-likelihood function. Several starting values were given in each case to be certain that a maximum of the loglikelihood function was achieved. The J-tests were carried out by the SHAZAM version 7.0 computer program. The results from the J-test are presented in Table 6.

Based on the J-test, the performance of the Mitscherlich functional form seems to be preferred in the barley response analysis, followed by the square root and in the last place by the quadratic form. The analysis of spring wheat response is not as clear. The Mitscherlich functional form is rejected for wheat on fine sand clay (at a 5% and a 1% risk level). Remarkable is that the Mitscherlich functional form does not reject either of the polynomial forms for wheat.

If one considers both crops, the quadratic form is rejected in six out of eighth cases. The square root form is also rejected in six out of eighth cases. The Mitscherlich functional form is only rejected in two out of eighth cases. It must be added that the polynomial forms both reject each other in all cases. The Mitscherlich form, however, rejects the quadratic and the square root form in all barley cases. The Mitscherlich form is not rejected in any case for the barley response.

Consequently, the hypothesis of the Mitscherlich functional form being superior to the quadratic functional form only seems to be confirmed in the barley crop response by the nonnested hypothesis testing. However, the results from the spring wheat nitrogen response do not lead to the same conclusion. The nonnested hypothesis testing does not establish the Mitscherlich functional form as superior to the polynomial form on the basis of the spring wheat analysis since the Mitscherlich function was not able to reject the polynomial forms. The square root form is, on the other hand, rejected by the quadratic form and vice versa.

One reason for the different results concerning wheat and barley may be that the stability of the J-statistic and of the log-likelihood function seems to be affected by some outlier years. That 1973 is an outlier year is confirmed by looking upon the original data. Therefore the whole estimation procedure was repeated leaving out this particular year. When the nonnested hypothesis testing was repeated, the J-test statistics proved to be more stable with regard to starting value of α. For instance, the Mitscherlich function for wheat on fine sand clay was not rejected by the quadratic function when this year was left out.

Table 7. Optimal N fertilizer level, kg N/ha.

Soil: fine sand clay	Wheat	Barley
Mitscherlich	291.1	309.0
Quadratic	183.0	176.0
Square root	2462.3	859.8
Soil: loam clay	**Wheat**	**Barley**
Mitscherlich:	154.6	210.0
Quadratic	148.2	161.9
Square root	217.0	352.0

Optimal fertilizer level

The optimal fertilizer levels for profit maximization stipulated by the first order conditions of profit maximization for each of the functional forms are summarized in Table 7. The prices of wheat and barley used in the calculation were the average realized producer prices of 1991 (FIM/kg 2.22 resp. FIM/kg 1.58). The price of nitrogen FIM/kg 4.90) was calculated as a weighted average of the monthly prices and purchases of ammonium nitrate (27.5% N). The nitrogen price included a tax on nitrogen (FIM 0.28/kg N). Second order conditions for a maximum were satisfied in all cases.

Contrary to the assumptions and findings of other scholars, the optimal nitrogen application doses estimated with a Mitscherlich specification were higher than with a quadratic polynomial specification in all of the cases. The optimal fertilization application doses estimated on the basis of the quadratic polynomial form were between 57% and 96% lower than when estimated on the basis of the Mitscherlich form. The initial assumption concerning an excessive bias by the use of the quadratic form was therefore not confirmed. Thus the profit maximizing nitrogen application doses estimated by Heikkilä and updated by Laurila do not seem overestimated.

The optimal application doses were the highest for the square root form. In the two cases in which all parameter estimates determining nitrogen application doses were significant at a level of $\alpha = 0.005$ the optimum application doses estimated on the basis of the square root form were between 146% and 217% higher than the nitrogen application doses estimated by a quadratic form.

The high optimal fertilizer doses estimated by the square root form seem to be an outcome of negative and small β_3 coefficients, which are very sensitive to changes in the data. The extremely high estimate of optimal nitrogen application doses for wheat on fine sand clay soils (2462.3 kg N/ha) is an outcome of a very low, insignificant parameter estimate of β_3. The corresponding parameter estimate of β_3 for barley on fine sand clay is accepted at $\alpha = 0.05$, which implies a 5% risk. The optimal nitrogen application dose estimated by the square root form remains very high (859.8 kg N/ha) in this case. On loam clay soils both β_3 are accepted at a significance level $\alpha = 0.005$, and profit maximizing nitrogen doses estimated by the square root form are, while being high, closer to the other estimates of the other specifications.

Increasing the nitrogen fertilizer price by 100% or decreasing the producer price by 50% will result in the same optimal nitrogen doses since the profit function is linearly homogenous, π (tp, tw) = tπ(p, w). The reductions in optimal nitrogen application doses as a result of a 100% increase in the price of the nitrogen fertilizer, a 50% decrease of the producer price or both measures is presented in Table 8.

According to the Mitscherlich form of the production function, a 100% nitrogen price increase or a 50% reduction of producer prices will lower opti-

Table 8. Reductions in optimal N fertilizer level as a result of 100% increased input prices (w), 50% decreased producer prices (p) or both a 100% input price increase and a 50% producer price decrease.

Soil: fine sand clay

Specification	Wheat				Barley			
	Increase of w 100% or decrease of p 50%		Both increase of w 100 % and decrease of p 50%		Increase of w 100% or decrease of p 50%		Both increase of w 100 % and decrease of p 50%	
	kgN/ha	%	kgN/ha	%	kgN/ha	%	kgN/ha	%
Mitscherlich	-69.5	24	-138.9	48	-61.2	20	-122.4	40
Quadratic	-12.3	7	-36.9	20	-8.2	5	-24.6	14
Square root	-1719.6	70	-2256.9	92	-356.4	41	-626.7	73

Soil: loam clay

Specification	Wheat				Barley			
	Increase of w 100% or decrease of p 50%		Both increase of w 100 % and decrease of p 50%		Increase of w 100% or decrease of p 50%		Both increase of w 100 % decrease of of p 50%	
	kgN/ha	%	kgN/ha	%	kgN/ha	%	kgN/ha	%
Mitscherlich	-38.4	25	-76.7	50	-45.1	21	90.1	43
Quadratic	-14.4	10	-43.2	29	10.4	6	31.1	19
Square root	-84.7	39	-153	71	124.7	35	235.0	67

mal fertilizer application doses 38.4-69.5 kg N/ha or 20-24%. Implementing both measures will lower nitrogen application doses by 76.7 - 138.9 kg N/ha or 40-50%. The decrease in the optimal nitrogen application is in most cases lower when estimated by the quadratic function. Increasing fertilizer prices by 100% or reducing producer prices by 50% lowers fertilizer doses only 5-10%, according to the quadratic form. Applying both measures will lower nitrogen application rates by 14 - 29%, according to the quadratic form. In the cases where the yield determining square root parameter estimates are significant at a level $\alpha = 0.005$ the profit maximizing nitrogen application doses are reduced 35 - 39% when one or the other of the measures is applied.

The yield, production value, cost and profit per ha measured at profit maximizing nitrogen application levels using different functional forms are reported in Table 9.

The potential profit maximizing yield level and the corresponding cost and profit levels estimated by the Mitscherlich functional form are approximately the same as estimated by the quadratic functional form. On loam clay, where all yield determining coefficients of the square root form were significant at $\alpha = 0.005$, the square root form yields estimates of the same range as the other functional forms. On fine sand clay the low negative values of β_3 lead to excessive estimates of the yield level and profits.

Conclusion

This article has shown that in estimating the form of the nitrogen response the Mitscherlich specification proved superior in the crop response for barley. With respect to the crop response for wheat no specification could be established superior. The results can be summarized as follows: the quadratic

Table 9. Estimated yield level, kg/ha, production value, FIM/ha, cost, FIM/ha and profit, FIM/ha, 1969-1980 by different functional forms.

		Spring wheat		Barley	
		fine sand clay	loam clay	fine sand clay	loam clay
Mitscherlich	Yield	5140	3497	7205	4571
	Production value	11412	7762	15995	10148
	Cost	1427	757	1515	1029
	Profit	9985	7005	14480	9119
Quadratic	Yield	4761	3671	6092	4559
	Production value	10570	8151	13525	10120
	Cost	897	726	862	793
	Profit	9673	7425	12663	9327
Square root	Yield	13267	3715	9331	5129
	Production value	29453	8247	20714	11385
	Cost	12065	1063	4213	1726
	Profit	17388	7184	16501	9659

function, the square root function and the Mitscherlich function all produced highly significant parameter estimates. The estimate of the error variance was lower for the Mitscherlich functional form in all cases. Consequently, the coefficient of determination was also higher. In order to establish which functional form is the most appropriate, a nonnested hypothesis test was carried out. As a result of the 24 tests, the quadratic and square root functions were both rejected in six out of eight cases. The Mitscherlich function was only rejected in two out of eighth cases. In the barley response analysis the Mitscherlich functional form was superior to both other functional forms by all central criteria. The law of the minimum proposed by von Liebig, which implies absence of nutrient substitution, was therefore confirmed by the barley response analysis. In the spring wheat response analysis the Mitscherlich function did not, however, reject the polynomial forms in spring wheat response analysis and could not be established as superior to polynomial forms. Efforts to determine the appropriate form of the crop response based on cross-sectional data is likely to avoid the problems connected with outlier years encountered in this study, which was based on time series.

A secondary purpose of this paper was to evaluate whether the profit maximizing nitrogen application doses differ substantially between the specifications. The results show that, in spite of significant parameter estimates for all three specifications, in most of the cases the profit maximizing doses differed substantially. Contrary to initial assumptions, fertilizer recommendations based on quadratic functional forms were not found to lead to excessive fertilizer recommendations relative to the other two functional forms. In the estimation of the most economic fertilizer doses the Mitscherlich specification lead to higher fertilizer recommendations than the quadratic specification. The square root functional form lead to still higher optimal nitrogen application doses than both the quadratic and the Mitscherlich specification. Small variations in parameter estimates produced large variations in estimated profit maximizing fertilizer doses so the high absolute profit maxizing level may have been due to the particular data sets. The large variation in the absolute levels of profit maximizing nitrogen application confirmed that a different approach is needed for the estimation of optimal nitrogen application levels to be used for extension purposes.

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SELOSTUS

Typpilannoituksen vaikutus vehnän ja ohran satoon Suomessa

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Maatalouden taloudellinen tutkimuslaitos

Typpilannoituksen vaikutusta ohran ja vehnän satotasoon tutkittiin vertailemalla kolmea eri tuotantofunktiomuotoa: Mitscherlichin tuotantofunktiota, kvadraattifunktiota ja neliöjuurifunktiota. Aineisto perustui Maatalouden tutkimuskeskuksen Tikkurilassa vuosina 1969-1980 tekemiin typpilannoituskokeisiin. Tutkimuksen ensisijaisena tarkoituksena oli löytää paras funktiomuoto kuvaamaan vehnän ja ohran tuotantofunktiota l. satotason muuttumista lannoitustason muutoksen seurauksena. Tutkimuksen toissijaisena tarkoituksena oli tutkia, eroaako voittoa maksimoiva lannoitepanos eri tuotantofunktiomuotoja käytettäessä.

Ohran typpilannoituksen analyysissa Mitscherlichin funktiomuoto (joka tunnetaan myös Spillmanin funktion nimellä) osoittautui parhaimmaksi funktiomuodoksi J-testin perusteella. Vehnän typpilannoituksen analyysissa mitään funktiomuotoa ei pystytty osoittamaan muita paremmaksi J-testin perusteella. Selitysasteen R^2, virhetermin varianssiestimaatin $\hat{\sigma}^2$ ja lannoitustasoa määrittävien parametriestimaattien merkitsevyyden perusteella Mitscherlichin funktiomuoto kuvasi typpilannoituksen vaikutusta kvadraattifunktiota ja neliöjuurifunktiota paremmin sekä ohran että vehnän osalta. Mitscherlichin tuotantofunktion mukaan kaksinkertaisella typen hinnalla voittoa maksimoiva typpilannoitus väheni 20-24 % ja kvadraattifunktion mukaan vain 5-10 %.

Tutkimuksen perusteella Suomessa yleisimmin käytetty funktiomuoto, kvadraattifunktio, aliarvioi voittoa maksimoivan lannoitustason alenemista lannoitteen hinnaa noustessa tai tuottajahinnan alentuessa. Mitscherlichin tuotantofunktio johti vastoin ennakkooletuksia korkeampaan voittoa maksimoivaan lannoitepanokseen kuin kvadraattifunktio.