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PRECISION ANALYSIS OF VIBRATION ENERGY FLUX IN ANGULAR CONNECTION OF PLATES

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Qualitative assessment of energy balance in mechanical systems is possible by the use of energy flow descriptors and observation of energy density distribution in the mechanical structure. The aim of the analysis was quantitative estimation of vibration energy transmitted through the welded connections of ribbed plates. The structural intensity was used as the parameter for the analysis. Obtained results of calculation gave the quantitative information on amount of energy transmitted, reflected, stored and the damped in welded joints of plates. Analyzed cases were intentional to show the utility of intensity method in diagnostics of joints in mechanical constructions. Calculation results were verified experimentally by measurements of stress with application lock-in-thermography.

Keywords: structural intensity, vibration energy flow.

1. Introduction

The quantity of structural intensity has found the exceptional application in investigations of vibration energy flow in deformable elastic bodies. The analysis of spatial distribution of structural intensity vector fields enables determination and location of transmission paths, sources and sinks of energy of vibrations in application to the mechanical systems. It gives the particular information on the streams of energy flow which is much advantageous than the other methods used earlier to such kind of analysis.

The method of intensity evaluation was based on complex modal analysis elaborated with use of finite elements method (FEM). Presented calculation results show distribution of structural intensity (vector field) on the surface of rectangular steel plates connected by the welded joints. The models of plates included the source (linear force excitation) and sink of energy (dissipating damping elements). The changes of finite elements grid density enabled analysis of vibration energy flow in analysed plates through the different kind of joints. If the frequencies of the external forces are close to the natural frequencies of the structures, the permissible vibration levels may be exceeded, which could result in fatigue failure. Plates and beams are most commonly used built-up structural elements in vehicles and their damage will result in disintegration of system. Solved problems were intended to show the usability of intensity method in diagnostics of joints specially those typical for the vehicles. The possibility of computational analysis of structural intensity is very promising due to the prospects of use of already elaborated for other purposes finite element models of mechanical structures.

2. Structural intensity as the measure of energy flow

2.1. Energy density and intensity vector in a thin plate

The flexural displacement of a thin, transversely vibrating plate excited by time harmonic force $Pe^{j\omega t}$ at a point, oscillating with a circular frequency ω, is equal [2]:

$$D\nabla^4 w(x,y) - \rho h\omega^2 w(x,y) = P\delta(x-x_p,\, y-y_p), \tag{1}$$

where $w(x,\ y)$ is the flexural displacement, D is the complex bending stiffness assumed in form of $D_0(1+\mathrm{j}\eta)$, in which D_0 is the bending stiffness and η is loss factor, ρ is the material density, h is the plate thickness, P magnitude of time harmonic force, and $(x_p,\, y_p)$ is the excitation position. Potential energy density $U(x,\ y)$ is given by [1]:

$$U(x,y) = \frac{D_0}{4}\left(\frac{\partial^2 w}{\partial x^2}\left(\frac{\partial^2 w}{\partial x^2}\right)^* + \frac{\partial^2 w}{\partial y^2}\left(\frac{\partial^2 w}{\partial y^2}\right)^* + 2\nu\frac{\partial^2 w}{\partial x^2}\left(\frac{\partial^2 w}{\partial y^2}\right)^* + 2(1-\nu)\frac{\partial^2 w}{\partial x\partial y}\left(\frac{\partial^2 w}{\partial x\partial y}\right)^*\right), \tag{2}$$

and here * denotes the complex conjugate and ν – the Poisson ratio.

Structural intensity represents the averaged in time net mechanical energy flow through the unit area perpendicular to the direction of flow. Instantaneous value of real part of structural intensity is time dependent vector quantity equal to the change of energy density in the infinitively small volume [4]. The structural intensity components for thin-walled two dimensional structures are orthogonal are [3, 5]:

$$\overline{I} = I_x\overline{i} + I_y\overline{j}. \tag{3}$$

Components I_x and I_y are computed from the internal shears and moments, which are proportional to the spatial derivatives of the transverse plate velocity [4, 5]. On assumption for thin plates were derived the dependencies on components [6]:

$$\begin{aligned} I_x &= -\frac{\omega}{2}\mathrm{Im}\left[\widetilde{N}_x\,\widetilde{u}_0^* + \widetilde{N}_{xy}\,\widetilde{v}_0^* + \widetilde{Q}_x\,\widetilde{w}_0^* + \widetilde{M}_x\theta_y^* - \widetilde{M}_{xy}\theta_x^*\right] \\ I_y &= -\frac{\omega}{2}\mathrm{Im}\left[\widetilde{N}_y\,\widetilde{v}_0^* + \widetilde{N}_{yx}\,\widetilde{u}_0^* + \widetilde{Q}_y\,\widetilde{w}_0^* - \widetilde{M}_y\theta_x^* + \widetilde{M}_{yx}\theta_y^*\right] \end{aligned} \tag{4}$$

where $\widetilde{N}_x$, $\widetilde{N}_y$ – in-plane normal forces, $\widetilde{N}_{xy} = \widetilde{N}_{yx}$ – in-plane tangential forces, $\widetilde{Q}_x$, $\widetilde{Q}_y$ – shear forces, $\widetilde{M}_{xy} = \widetilde{M}_{yx}$ – torques, $\widetilde{M}_x$, $\widetilde{M}_y$ – bending moments.

2.2. Method of structural intensity computation

FEM and modal superposition method are used for calculations of structural intensity. For numerical model complex eigenvalue problem is solved, which takes into account structural damping. Eigenvectors, normalized to unit mass, form a modal matrix $\mathbf{\Phi}$, which allows for transformation of motion equations into modal form. For harmonic excitation $\widetilde{F}e^{j\omega t}$ modal equations of motion obtain the following form:

$$\left(-\omega^2\,\mathbf{I} + \widetilde{\mathbf{\Omega}}^2 + \widetilde{\mathbf{s}}\right)\,\widetilde{\mathbf{q}} = \widetilde{\mathbf{f}}, \tag{5}$$

where $\widetilde{\mathbf{\Omega}}^2$ takes into account influence of structural damping, $\widetilde{\mathbf{s}}$ contains discrete external damping. Complex quantities are denoted by a tilde.

Modal superposition of displacements and generalized stresses in the middle of finite elements is equal as follows:

$$\widetilde{\mathbf{X}} = \boldsymbol{\varphi}\,\widetilde{\mathbf{q}}, \tag{6}$$

$$\widetilde{\boldsymbol{\sigma}} = \widetilde{\mathbf{\Psi}}\,\widetilde{\mathbf{q}}, \tag{7}$$

where $\widetilde{\mathbf{X}}$ is vector of displacements in physical components. Matrix $\boldsymbol{\varphi}$ is created from the modal matrix $\mathbf{\Phi}$ in such a way which allows calculating displacements in the middle of finite elements in their local coordinate systems. Matrix $\widetilde{\mathbf{\Psi}}$ contains generalized stresses in the middle of finite elements in their local coordinate systems, corresponding to eigenvectors from the modal matrix. The complexity of this matrix is connected with structural damping. Calculation of structural intensity values requires displacements and generalized stresses derived at the same points and in the same coordinate systems. Vectors of structural intensities calculated in local coordinate systems of finite elements are transformed into global coordinate system.

3. Model of connected plates

A structure composed of two rectangular plates with central rib stiffener was chosen as a test model for the coupled structure. Two plates were welded at one common boundary and the material properties and dimensions of each plate were the same. All boundaries of plates were free (except common edge). The objective of the analysis was the testing the structural intensity field. Each intensity component was computed at first 100 resonance frequencies of two thin rectangular plates connected at the shorter side, supported on the bolt. The plate's dimensions were $0.2 \times 0.07 \times 0.001$ m. Model of line welding included the stiff connection of plates along the one row of finite elements. The plates were of the constructional steel with the material properties: Young's modulus, $E = 2.11 \cdot 10^{11}$ Pa, Poissons ratio, $\nu = 0.3$ and density, $\rho = 7860$ kg/m^3. The harmonic excitation force 10 N in the direction perpendicular to the horizontal plate at the bolt mounting (point marked as asterisk) was applied. The frequency of excitation was changed from 5 to 100 Hz. The distributed small viscous damping forces were applied to the plate in the purpose of vibration energy absorption. The FEM model was arranged using the NASTRAN software and consisted of square elements of QUAD4 type. Consecutively the Mathlab was used to visualise the results. The results are shown in Fig. 1.

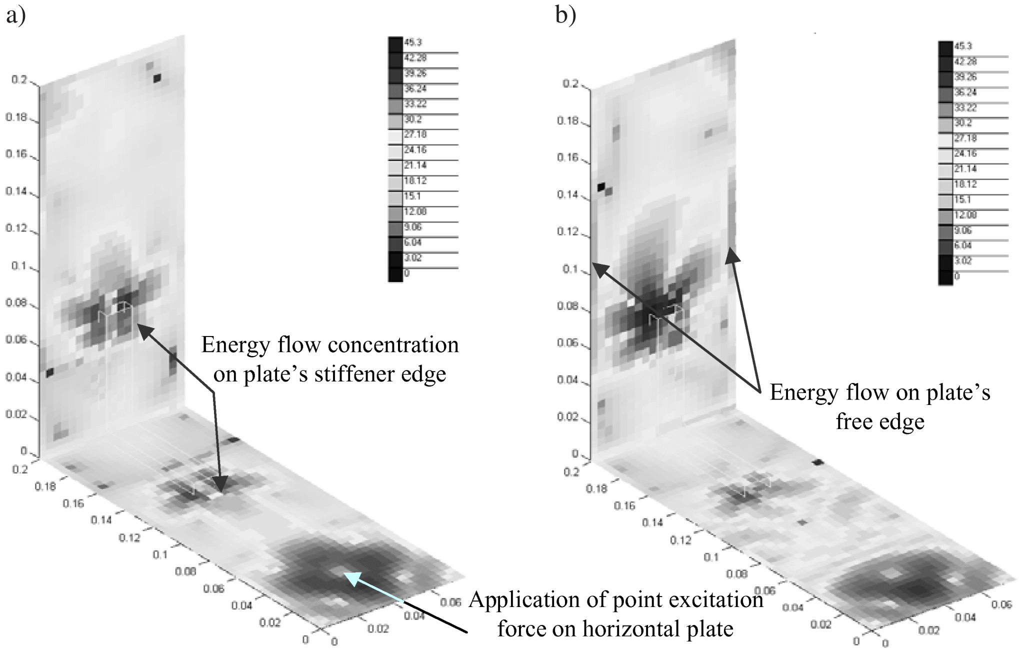

Fig. 1. Structural intensity field for two welded rectangular plates. Excitation frequency: a) 80.78 Hz, b) 100.0 Hz. Logarithmic scale (dB).

4. Experimental verification

4.1. Non-contact measurements of stresses – lock-in thermography

Infrared thermography was used for non-contact measure of the distribution of stress. Infrared thermographic camera measures the fine variations in temperature, applying the lock-in thermography method according to the principle of thermoelasticity. When a solid material is rapidly stressed by load and adiabatically deformed, the variation in temperature occurs in the same way – thermoelastic effect.

Based upon a high performance focal plane array camera and digital image processing software the Altair LI system produces high quality images of stress field. It provides images in real time by using the thermoelastic effect which is a linear relationship between the temperature changes induced by loading and the stress at the material surface. The required thermal resolution to achieve a resolution of 1 MPa depends on the material properties. It is typically equal to 1 mK for steel.

Fig. 2. Infrared camera system for non-contact imaging of stresses in structures. Thermography based measurement of stress for two connected plates.

a) b)

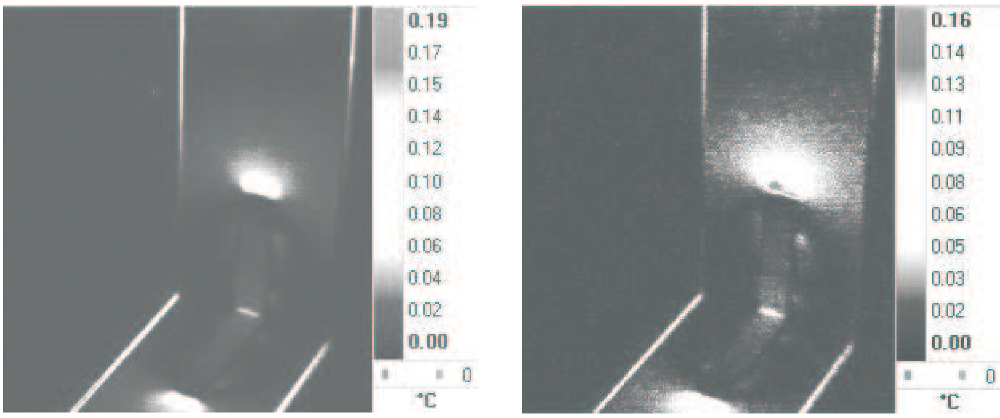

Fig. 3. Temperature field for two rectangular welded plates. Excitation frequency: a) 54.72 Hz, b) 65.70 Hz.

5. Conclusions

The results of detailed analysis of vibration energy transportation obtained from calculations structural intensity vectors values distributions are presented in graphical form. There are clearly seen the very specific disturbances of distribution in places of attachment of stiffening element and application of exciting force.

The distribution of structural intensity vectors gives the qualitative characteristic of vibration energy transportation in mechanical systems. The structural intensity distribution enables the investigation in the regions of high concentration of vibration energy which consequently is exposed to the risk of damage. It can be considered as the identification of the regions for application of additional damping in purpose of lowering of vibration level and resulting noise radiation. Experimental verification of calculation with application of non-contact measurements of stresses (lock-in thermography) have shown the concentration of stress in the place of high concentration of vibration energy flow represented by the maximum structural intensity values.

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