ARCHIVES OF ACOUSTICS
31, 1, 77–87 (2006)

EFFECT OF ACTIVE VIBRATION CONTROL OF A CIRCULAR PLATE ON SOUND RADIATION

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(*received April 30, 2005; accepted October 23, 2005*)

An active control of the vibration cancellation of a circular fluid-loaded plate is analytically studied. The purpose of this paper is to examine the effect of the active vibration control strategy on the sound radiation generated by a plate. It was assumed that a planar vibrating structure located in a finite baffle and interacting with fluid is driven by a periodic force with constant amplitude. This structure radiates the acoustic waves into a surrounding fluid and a point control force is used to reduce its vibrations. The simulations of the active cancellation of the plate vibrations were made with a Simulink/Matlab computer program. The results demonstrate, that while a control law provided a significant reduction in the plate vibration, it is rather ineffective for noise attenuation.

Key words: vibration control, sound radiation, circular plate, LQR controller.

1. Introduction

In the engineering practice, many components of machines and structures are subject to dynamic effects produced by time-dependent external forces, so that the resulting stresses, deformations and sound fields at its vicinity are time-dependent as well. Each sound or vibration field radiated from such sources has its own characteristics, which depend on several properties of the relevant geometry, material and dynamic loads. Thus, it is purposeful to examine the effect of vibrations and their reduction on the existing sound field.

The problem of suppressing plate vibrations is often solved by the application of *active methods*. Two main strategies for active control are proposed [2, 3, 13]. The first approach is based on controlling the acoustic radiation in a double manner: by sound, or by applying control forces to the structure (Active Structural Acoustic Control – ASAC). The second strategy is a vibration control (AVC) and is considered in this paper. The objective of AVC is to cancel the vibrations of the structure as much as possible, while the accompanying sound field is not included in the control law.

For the active vibration control in linear systems, the structural response of the controlled structure is a superposition of the response caused by external disturbances, which is defined as the "primary" response, and the response caused by the active vibration controller, which is defined as the "secondary" response. It is well known that the sound radiation from a structure is a function of the velocity distribution over the surface of the structure. If an efficient control system tends to attenuate the amplitude of the out-of-plane vibration of the structure (AVC), the structural modes may destructively interfere with one another in the acoustic medium.

For the design of an effective kind of control for suppressing the plate vibration and related acoustic radiation, an accurate modeling of the acoustic structural and coupling components is necessary. The derivation of the models for planar structures with point or surface mounted actuators can be carried out in two ways. The first way is commonly referred to as *system identification* [14]. In this case a model of the system can be inferred from a set of data collected during a practical experiment. Classical linear methods provide good performance over a relatively small range of uncertainty and are extensively used for linear control techniques [7, 8]. The second approach consists of modeling the fluid-acoustic-structural dynamics in the form of partial differential equations derived from physical principles such as forces and moments' balance and this is also considered here. The objective of control is to cancel the vibrations of a structure. There are many control strategies that could be developed on the basis of such a model [1–3]. Modern control theory has been applied also by the author [4–9] to reduce circular plate vibrations by using a linear-quadratic (LQR), PI2D and fuzzy controllers. Those systems have been successfully implemented on an experimental plant [5, 7, 8].

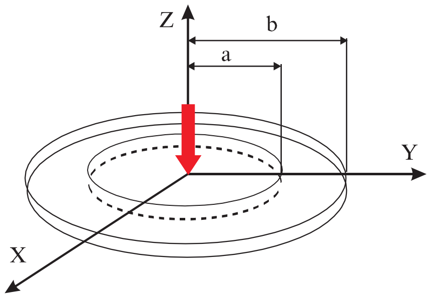
Conventionally, the actuators are driven by a single controller, which is supplied with signals from the mounted sensors. It should be noted that, in practice, the actuators cannot cancel completely the vibrations of the structure, and then, the radiated sound cannot be zero too, or even – in some cases – they may increase [2].

The purpose of this paper is to examine the effect of the active vibration control strategy on the reduction of the sound generated by a plate. The structure under study is a vibrating circular plate of radius a, having a constant thickness h. It was assumed that the plate clamped at the edge is excited on one side by a uniform periodic force with constant amplitude F_0 and it radiates the acoustic waves into a surrounded fluid of density ρ_0. To develop successfully an effective system model for the active vibration control, it is therefore assumed in this analysis that the plate in question is located in a finite baffle and it interacts with a surrounding ideal compressible fluid. The control problem lies in using a point control force to reduce the plate vibrations and the aim is to examine if by canceling the plate vibrations, the control system will achieve a good reduction of the radiated acoustic pressure.

2. Flexural vibrations of a circular plate

The structure under study is a vibrating circular plate of radius a, having a constant thickness h (Fig. 1), surrounded by a lossless medium with static density ρ_0. It is assumed that the plate, clamped in a flat, rigid and finite baffle of radius b, ($b > r > a$, $z = 0$), is made of a homogeneous, isotropic material with density ρ, and has a Kelvin–Voigt internal damping.

a)



b)

Fig. 1. a) Circular plate in a finite baffle of radius b with a shaker located at the origin; b) experimental realization.

In the case being considered, the applied loading and end restraints of the circular plate are independent of the angle φ (axially symmetrical vibrations), thus we can write the governing differential equation of the forced motion of the plate as follows [10, 12, 4]:

$$B\nabla^4 w(r,t) + R\frac{\partial}{\partial t}\left[\nabla^4 w(r,t)\right] + \rho h \frac{\partial^2}{\partial t^2} w(r,t) = f(r,t), \tag{1}$$

where $\nabla^4 = \nabla^2\nabla^2$, $\nabla^2 = \left[\dfrac{\partial^2}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial}{\partial r}\right]$ is the Laplace operator, $B = Eh^3/12(1-\nu^2)$ is the bending stiffness of the plate, E, ρ, ν, and R are the Young's modulus, density, Poisson's ratio and Kelvin–Voigt damping coefficient for the plate, respectively. The displacement $w(r,t)$ and its derivative $\partial w(r,t)/\partial r$ satisfy the boundary condition for a clamped plate: they are both equal zero at the edge of the plate.

Equation (1) is the governing equation of the forced linear vibrations of plates and, as such, it must obey the rules of a linear superposition. For the analytical development being undertaken here, the right side of Eq. (1) will be expressed as follows [4, 5]:

$$f(r,t) = f_w(r,t) + f_p(r,t) + f_s(r,t). \tag{2}$$

Let us assume, that the structure under study will be controlled by the use of an electrodynamic shaker attached to the plate in question in its middle.

Thus, the control force $f_s(r,t)$, which will minimise the plate vibration and the related radiated sound field, is a point force located at the origin:

$$f_s(r,t) = u(t)\delta(r - r_s)\big|_{r_s=0}, \tag{3}$$

where r_s is the location of the point force input on the plate surface.

It is also assumed that the plate is excited on one side by a uniform periodic force with a constant amplitude F_0 generated by a loudspeaker

$$f_w(r,t) = F_0 e^{-i\omega t} \qquad \text{for} \quad 0 \leq r \leq a \tag{4}$$

and it radiates into free space filled with fluid of density ρ_0. The system model is formulated when the coupling effect between the structure and the acoustic medium is taken into account, so the second component of the right hand side of Eq. (2), $f_p(r,t)$, represents the acoustic fluid-loading acting on the plate as an additional force. The value of this force exerted by the fluid on the plate surface can be calculated as follows

$$f_p(r,t) = -p(r,z,t)\big|_{z=0}, \tag{5}$$

where $p(r, z = 0, t)$ is the acoustic pressure at the point on the surface of the plate. The acoustic waves propagating through the fluid must satisfy the wave equation [10]:

$$\nabla^2 p(r,z,t) = \frac{1}{c^2}\,\frac{\partial^2 p(r,z,t)}{\partial t^2}\,, \tag{6}$$

where ∇^2 is the two-dimensional Laplace operator, and c is the sound velocity in the fluid. At the fluid-structure interface, the pressure must satisfy the boundary condition [11]:

$$\left.\frac{\partial p(r,z,t)}{\partial n}\right|_{z=0} = -\rho_0 \frac{\partial^2}{\partial t^2} w(r,t) = -\rho_0 \ddot{w}(r,t), \tag{7}$$

with n denoting the normal to the structure.

The goal in the control problem is to determine the control force which, when applied to the plate (realized via a voltage $u(t)$ for the shaker), leads to a reduced level of vibrations. The third component in Eq. (2) represents such a control force, $f_s(r,t)$, which will cancel the plate vibrations. The location of the actuator is assumed to be in the middle of the plate.

3. Development of the state equation

To approximate the plate dynamics, a Fourier–Bessel expansion of the plate displacement is used to discretize the infinite dimensional system (1). The plate displacement can be approximated by

$$w^N(r,t) = \sum_{m}^{N} s_m(t) w_m(r), \tag{8}$$

where N is considered to be a finite number suitably large for accurately modelling the system dynamics and $w_m(r)$ is the $(0,\, m)$ plate mode described as follows [10, 12]:

$$w_m(r) = u_{0m}\left[J_0\left(\gamma_m \frac{r}{a}\right) - \frac{J_0(\gamma_m)}{I_0(\gamma_m)} I_0\left(\gamma_m \frac{r}{a}\right)\right]. \tag{9}$$

$J_0(x)$, $I_0(x)$ designate the cylinder functions, $\gamma_m = k_m a$ is the m-th root of the frequency equation and $s_m(t)$ is the corresponding modal amplitude in time t. In a similar way let us expand the right side of the plate equation of motion (2) into series:

$$f_w^N(r,t) \;=\; \sum_m^N r_m(t) w_m(r), \tag{10}$$

$$f_s^N(r,t) \;=\; \sum_m^N u_m(t) w_m(r), \tag{11}$$

$$f_p(r,r) \;=\; p^N(r,\, z=0,\, t) = \sum_m^N z_m(t) w_m(r). \tag{12}$$

Inserting the above expansions into Eq. (1), multiplying both sides by the orthogonal eigenfunction $w_n(r)$, and integrating over the surface of the structure S, the governing equation of motion can be re-expressed as [4]:

$$\sum_{m=1}^N \left[\ddot{s}_m(t) + 2\mu\omega_m^2 \dot{s}_m(t) + \omega_m^2 s_m(t) = r_m(t) + u_m(t) + z_m(t)\right], \tag{13}$$

where

$$\left.\begin{matrix} r_m(t) \\ u_m(t) \\ z_m(t) \end{matrix}\right\} = \iint_S f_j(r,t) w_m(r)\,\mathrm{d}S, \qquad j = w,\, s,\, p; \quad m = 1,\, 2..\, N \tag{14}$$

mean the modal generalised forces.

4. State-space system model

Equation (13) can be expressed in the state space format as [4]:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{V}\mathbf{r}(t), \tag{15}$$

where the dot denotes differentiation with respect to time, $\mathbf{x}$ is the $(n \times 1)$ state vector, $\mathbf{u}$ is the $(m \times 1)$ control vector, and $\mathbf{A}$ is the $(n \times n)$ state matrix, $\mathbf{B}$ is the $(n \times m)$

control input matrix, $\mathbf{V}$ is the $(1 \times n)$ disturbance matrix described as follows:

$$\begin{aligned} \mathbf{A} &= \begin{bmatrix} \mathbf{0} & \mathbf{1} \\ -(\mathbf{I}+\mathbf{D})^{-1}\mathbf{\Omega}^2 & -2\mu(\mathbf{I}+\mathbf{D})^{-1}\mathbf{\Omega}^2 \end{bmatrix}, \\ \mathbf{B} &= \begin{bmatrix} \mathbf{0} \\ (\mathbf{I}+\mathbf{D})^{-1}\mathbf{K}_s \end{bmatrix}, \qquad \mathbf{V} = \begin{bmatrix} \mathbf{0} \\ (\mathbf{I}+\mathbf{D})^{-1}\mathbf{K}_w \end{bmatrix}. \end{aligned} \tag{16}$$

In the above expression $\mathbf{I}$ denotes the identity matrix, $\mathbf{K}_s$ and $\mathbf{K}_w$ are the coefficient vectors, $\mathbf{D}$ represents the fluid-plate interaction matrix, $\mathbf{\Omega} = \operatorname{diag}[\omega_1, \omega_2, .., \omega_N]$ [4]. It is assumed, that the response of the considered plate to the applied force distribution is measured by a set of linearly independent point sensors situated at locations $\mathbf{r}$ on the plate. The output equation in the matrix form is

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t), \tag{17}$$

where

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_a & 0 \\ 0 & \mathbf{C}_v \end{bmatrix} = \begin{bmatrix} w_1(r_1) & \cdots & w_N(r_1) & 0 & \cdots & 0 \\ \vdots & \vdots & & & \vdots & \\ w_1(r_{Nc}) & \cdots & w_N(r_{Nc}) & 0 & \cdots & 0 \\ 0 & & & w_1(r_1) & \cdots & w_N(r_1) \\ \vdots & & & \vdots & & \vdots \\ 0 & & & w_1(r_{Nv}) & \cdots & w_N(r_{Nv}) \end{bmatrix}, \tag{18}$$

N_c and N_v denote the number of displacements and velocity sensors, respectively, $w_i(r_j)$ is a value of the i-th eigenfunction at the j-th measurement point.

The above state-space model of the considered system will be used in the process of designing optimal feedback control so as to suppress the plate vibrations.

5. Computer simulation of the feedback control

The goal of the control problem is to determine a voltage $u(t)$ which, when applied to the actuators, leads to a significantly reduced level of vibration. For the system described above, one possible approach is to obtain a solution by applying the well-known linear-quadratic regulator (LQR). The LQR method consists in using a control law [1, 3]

$$\mathbf{u}(t) = -\mathbf{K}\mathbf{x}(t), \tag{19}$$

which minimize the cost function given by

$$J = \frac{1}{2}\int_0^{\infty} \left(\mathbf{x}^{\mathrm{T}}\mathbf{Q}\mathbf{x} + \mathbf{u}^{\mathrm{T}}\mathbf{R}\mathbf{u}\right) \mathrm{d}t, \tag{20}$$

where $\mathbf{Q}$, $\mathbf{R}$ denote weighting matrices chosen as follows:

$$\mathbf{R} = \left[\frac{\beta}{u_{\max}}\right], \qquad \mathbf{Q} = \begin{bmatrix} \mathbf{\Omega}^2 & \mathbf{0} \\ 0 & \alpha\mathbf{\Omega}^2 \end{bmatrix} \tag{21}$$

and α and β are the weight coefficients. The problem is to determine the gain matrix $\mathbf{K}$ that facilitates our requirements. The optimal solution is [1, 3]:

$$\mathbf{K} = \mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}, \tag{22}$$

where the matrix $\mathbf{P}$ is the unique, positive definite solution of the algebraic Riccatti equation

$$\mathbf{A}^{\mathrm{T}}\mathbf{P} + \mathbf{PA} - \mathbf{PBR}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P} + \mathbf{Q} = 0. \tag{23}$$

Figures 2–4 show the tests of computer simulations of the active control of plate vibration with the use of a point control force (shaker). In the simulations, the model including the first four modes of the aluminium plate with a 0.46 m diameter and 1 mm thickness was applied. In order to determine the dynamics of the fluid-plate system, the model obtained was first subject to a rectangular periodic signal with constant amplitude and frequency. The acoustic pressure and the displacement of the plate were calculated at the axial line in the Fraunhofer's zone and at the first point sensor on the plate, respectively.

a) b)

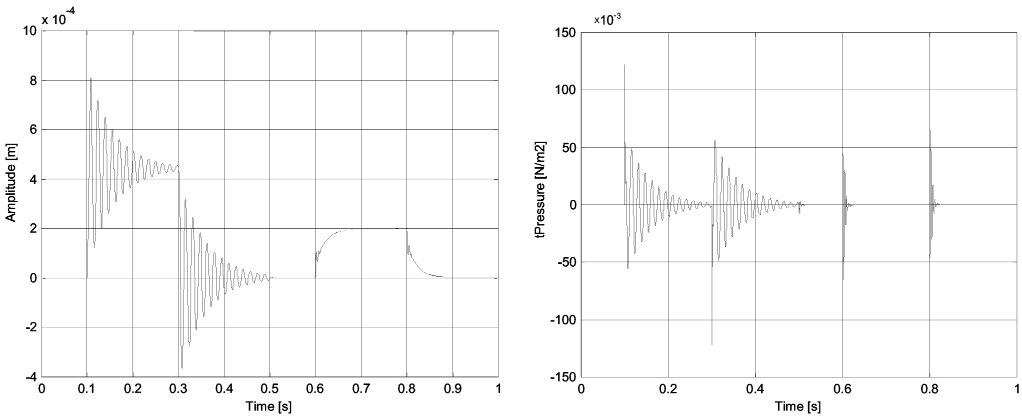


Fig. 2. Simulation results for LQR controller with sampling time 0.0001 sec. The time response of the open-loop system (0–0.5 sec) and the close-loop system (0.5–1 sec) to a rectangular periodic signal with constant amplitude; a) the plate displacement (sum of four modes); b) the acoustics pressure generated by the plate at Fraunhofer's zone.

The time response (Fig. 2a) shows that the plate displacement can be considerably suppressed but the negative feature of the control system is a non-zero error signal for 0.6–0.8 sec. The reduction of the acoustic pressure observed in Fig. 2b is also significant.

a) b)

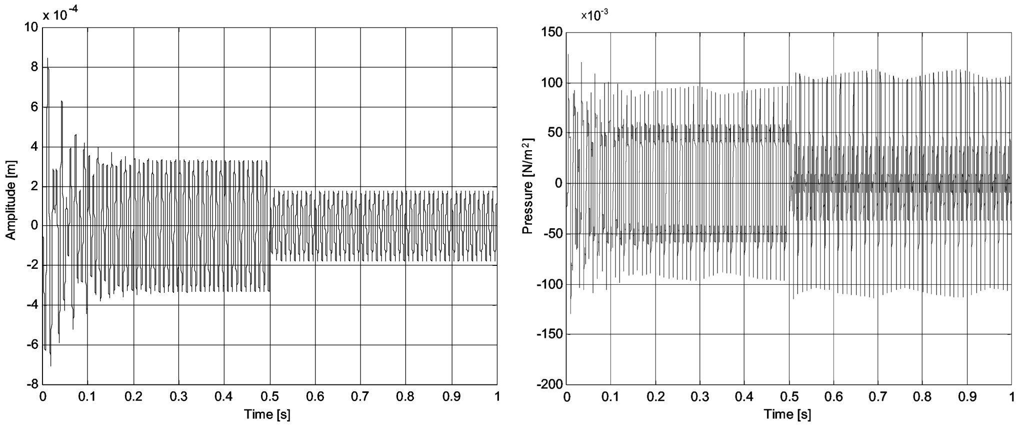


Fig. 3. The time response of the open-loop system (0–0.5 sec) and the close-loop system (0.5–1 sec) to rectangular disturbance of 100 Hz; a) the plate displacement (sum of four modes); b) the acoustics pressure generated by the plate at Fraunhofer's zone.

However, if the disturbance frequency increases (Fig. 3), the efficiency of the vibration cancellation decreases, while the accompanying sound radiation is not attenuated (Fig. 3b).

The system response to sinusoidal excitation over the plate surface is presented in the next figure.

In the case of sinusoidal disturbance, it can be seen (Fig. 4a, b) that the vibration suppression for low frequency is also better, whereas, the acoustic pressure generated by the plate was not minimized satisfactorily. In fact, for low frequencies, the level of the

a) b)

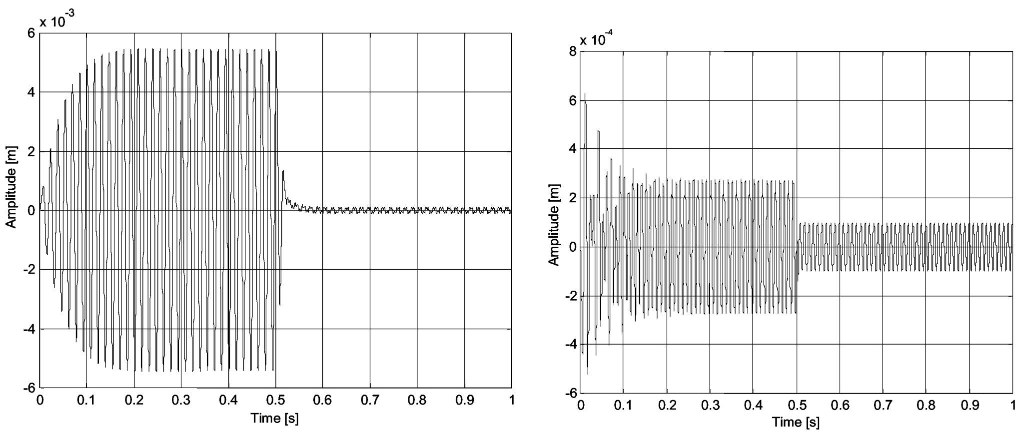


Fig. 4. Simulation results for LQR controller with sampling time 0.0001 sec. The time response of the open-loop system (0–0.5 sec) and the close-loop system (0.5–1 sec) to sinusoidal disturbance: a) 65 Hz; b) 100 Hz.

acoustic pressure decreases when the controller is started, but for higher frequencies, it exceeds the value generated in the open-loop system, similarly as in the case of a rectangular disturbance.

a) b)

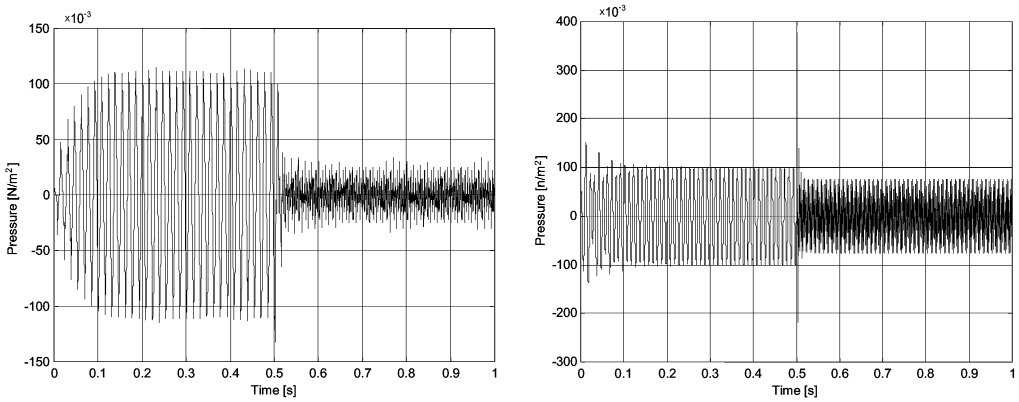


Fig. 5. The acoustic pressure in the Fraunhofer’s zone in the case of the open-loop system (0–0.5 sec) and the close-loop one (0.5–1 sec). Simulation results for LQR controller to sinusoidal disturbance: a) 65 Hz; b) 100 Hz.

The reason for the poor far field suppression, despite the plate vibration suppression, is a new control force calculated in the close-loop system. As a consequence of this force, the vibration frequency is much higher because the controller acts a lot of times per period of the disturbance. This process results in an immediate radiation of corresponding values of the acoustic pressure, but the radiation efficiency of the new, higher modes is greater, hence the value of the acoustic pressure are also greater despite of the significantly lower vibration amplitude.

a) b)

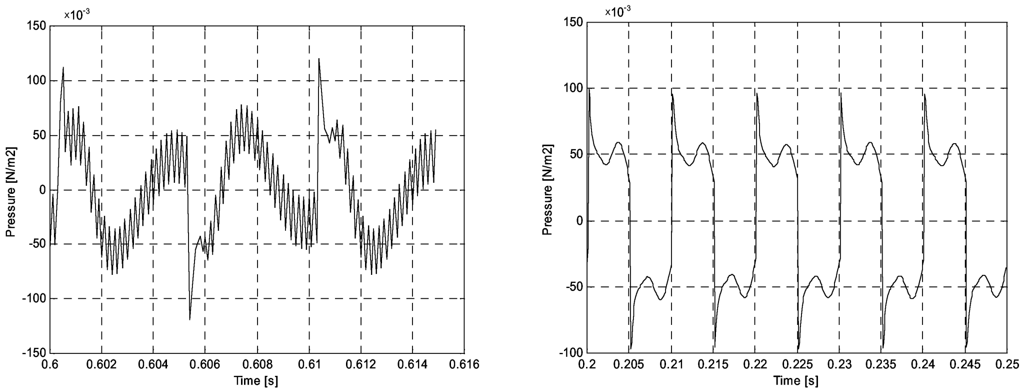


Fig. 6. Part of the far field response with and without control; a) close-loop system; b) open-loop system.

Comparing the acoustic pressure attenuation for both kinds of disturbances of 100 Hz (sinusoidal and rectangular), it can be concluded, that the rectangular periodic signal introduces much higher modes into plate vibrations. For this reason, the related radiation is more complicated and its reduction is worse. For comparison, a part of the far field response, caused by the rectangular disturbance, for the system with and without control force, is showed in the Fig. 6.

It can be seen from the plots in Fig. 6a that several system modes of higher frequencies are excited in the close-loop system.

6. Conclusions

In this paper, the active control of vibration suppression of a circular, fluid-loaded planar structure was analytically studied. The purpose of this work was to examine the effect of the active vibration control strategy on the sound generated by a structure in question.

The structure under consideration was a thin circular plate located in a finite baffle and interacting with a fluid. The plate, excited by a harmonic force acting on its whole surface, has been controlled by a point force located at the centre. The application of the optimal linear quadratic theory to the problem of plate vibrations inducing acoustic noise shows, that this control technique led to a substantial reduction of plate vibration, however, if the disturbance frequency increases, the efficiency of the vibration cancellation decreases. In the case of the accompanying sound radiation, the attenuation of radiated noise was satisfactorily for low frequencies, while it was poor or even completely bad for higher ones. This is so because a lot of high modes were induced, as well as the chosen cost function for the LQR controller did not minimize the far-field radiated pressure but it was derived from the plate displacement and velocity only. Finally, the numerical results demonstrate that while a control law provided a significant reduction of the plate vibrations, it is rather ineffective for the noise attenuation. It means, that the problem of constructing an acceptable control law for vibroacoustic systems requires of a compensator design which would minimize both the components: the plate vibrations and its acoustic radiation.

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