ARCHIVES OF ACOUSTICS
31, 1, 47–56 (2006)

COMPARISON OF THE CONE-TRACING SIMULATION AND THE ROOM IMPULSE RESPONSE MEASUREMENTS

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(*received March 2, 2005; accepted December 9, 2005*)

The experimental verification of the acoustic field model based on the cone-tracing method [1, 2] is described. With simulated *echograms* and the impulse responses obtained experimentally, a set of parameters: *Early Decay Time* (*EDT*), C_{50} and C_{80} were obtained. The parameters measured at many receiving sites and those calculated were compared. Mean differences of less than 2 dB were observed for the octave band mid-frequencies above 125 Hz.

Key words: cone-tracing modelling, sound field optimisation, measurement verification.

1. Introduction

The cone-tracing algorithm used for computer simulation of the acoustic field in a room has been designed for automatic optimisation of the sound reinforcement system. Discussion of the optimisation model and the results obtained with it will be a subject of a separate study.

Both the measurements and the simulations were performed in a lecture hall (Fig. 1). Its volume was about 2450 m^3, the total walls area was 1400 m^2 and the number of seats 500. The number of test sites in the hall was 13.

The co-ordinates of vertices of the room were set in such a way that none of the room wall edges was shorter than 0.5 m. All the co-ordinates of the source and the test sites were measured and entered into the calculations with up to 1 cm accuracy.

The values of the reverberation absorption coefficients of the materials of the enclosure walls were adopted from the published data [3, 4] as well as from our own measurements. During the measurements there was no audience in the room.

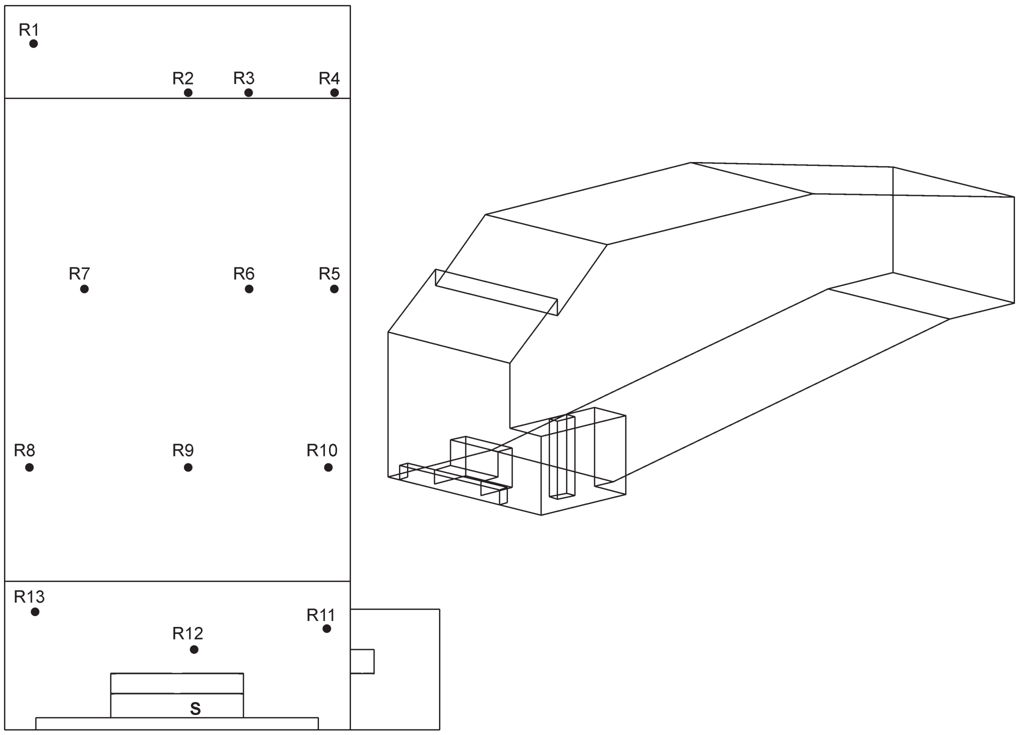

Fig. 1. Schematic view of the enclosure (lecture hall), the sound source (S) and location of the measuring sites (R).

2. The experimental method

Measurements of the impulse response of the room were performed using a computer set-up including an A2D–160 DSP board. The specialised DSP was controlled by MLSSA software. Schematic layout of the experiment is shown in Fig. 2.

The sound source for the impulse response measurements was a speaker complex in the shape of a regular dodecahedron; 15 cm diameter loudspeakers are mounted on each face of the column [5]. The dodecahedron source of a diameter of 65 cm was fixed to a stand which allowed the adjustment of its height above the floor in the range from 1.4 to 2.2 m. Directional characteristics of the sound source corresponding to octave band mid-frequencies from 125 Hz to 8 kHz are shown in Fig. 3.

All the measurements were performed with a B&K microphone model 4155. The Maximum Length Sequence was used as the exciting signal. There were 13 test sites in the room (Fig. 1). For each site an impulse response was measured and within all the octave bands the parameters *EDT*, C_{50}, C_{80} were determined. The impulse response recording time was approx. 1.8 sec.

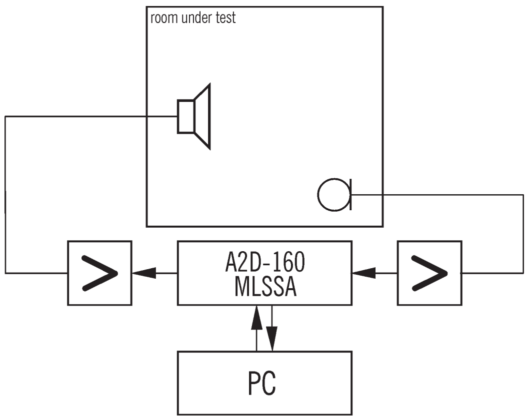

Fig. 2. Schematic drawing of the setup used to measure the impulse response.

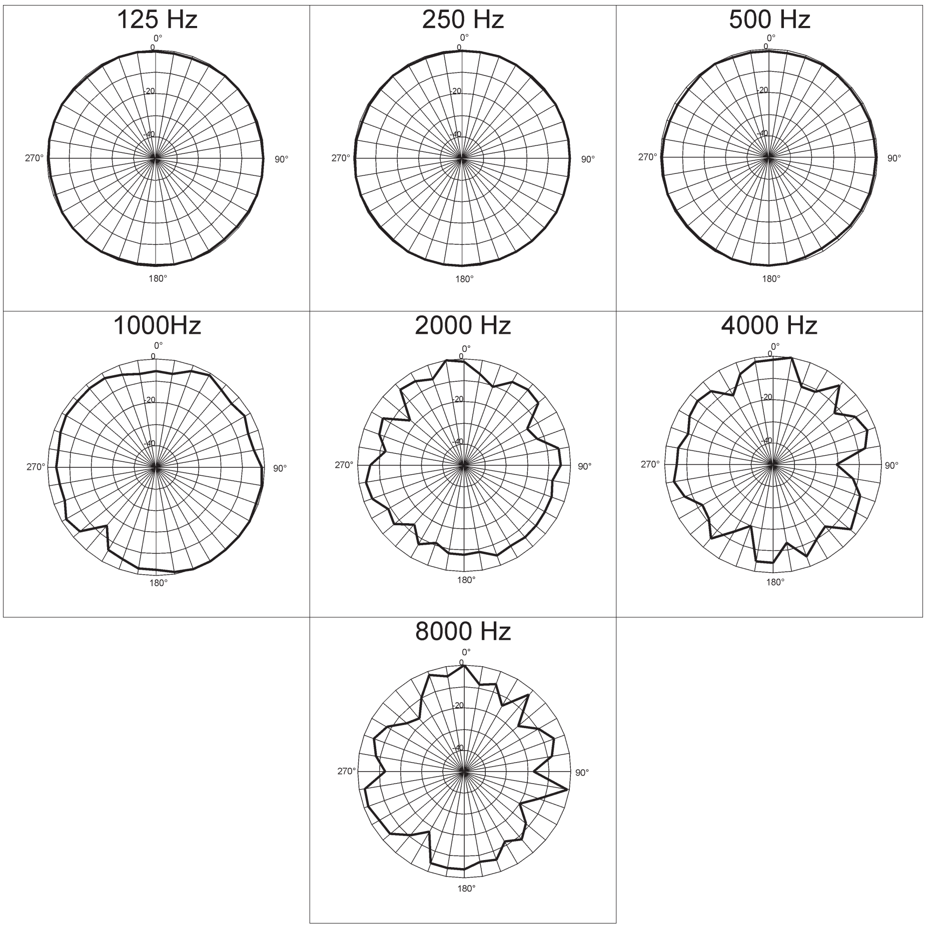

Fig. 3. Horizontal directional characteristics of the sound source at 125 Hz to 8 kHz octave bands.

3. Computer simulation

The simulation program is based on the *cone-tracing* method where the cone tracing is performed to determine a set of image sources apparent from a given observation site. Then, the calculations were made according to the image source method. The simulation software has been developed by the author using a sound field generation engine for automatic optimisation of the sound reinforcement system in the room.

3.1. The cone-tracing method

It has been assumed that the sound source can be regarded as a point source and that all the measuring sites are located in the far field:

$$\overline{p}_i^2 = \frac{P\rho_0 c}{4\pi\, d_i^2}\,,$$

where $\overline{p}_i^2$ – mean square of the acoustic pressure at the i-th receiving site, d_i – the distance of this site from the sound source measured along the acoustic ray, P – acoustic power of the source.

Taking into account the specular reflection, air absorption, and the directional characteristics of the source leads to the formula below:

$$\overline{p}_i^2 = \sum_k \left[\frac{PQ(\vartheta_k,\varphi_k)\rho_0 c}{4\pi\, d_{ik}^2} \cdot e^{-md_{ik}} \prod_{l=1}^{R_k}(1-\alpha_{ikl})\right],$$

where k is the number of an image source at the distance d_{ik}, l – reflection index, R_k – the number of reflections at each image source, $Q(\vartheta_k, \varphi_k)$ – directional characteristics of the sound source, m – sound absorption coefficient in air.

The summation symbol refers to the superposition of all contributions from the image sources in the echogram. Each k – index corresponds to a separate time coordinate of a vertical line in the echogram. The distance d_{ik} was calculated using the data from the *cone-tracing* algorithm.

3D directional characteristics of the sound source were calculated basing on two-dimensional characteristics (Fig. 3). It was assumed that the vertical and horizontal directional characteristics are similar enough within the accuracy of the model.

3.2. Acoustic field parameters

Values of the parameters measured previously in the experiment were then calculated from the echograms. The Early decay time (*EDT*) was calculated by a linear regression fit to the reverberation curve [6]. The parameters C_{50} and C_{80} were directly determined from the echogram according to the well known formulas:

$$C_{50} = 10 \log_{10} \frac{\sum\limits_{k=1}^{k_{50\,\mathrm{ms}}} \overline{p}_k^2}{\sum\limits_{k_{50\,\mathrm{ms}}}^{k_{mx}} \overline{p}_k^2} \,,$$

and

$$C_{80} = 10 \log_{10} \frac{\sum\limits_{k=1}^{k_{80\,\mathrm{ms}}} \overline{p}_k^2}{\sum\limits_{k_{80\,\mathrm{ms}}}^{k_{mx}} \overline{p}_k^2} \,,$$

where $k_{50\,\mathrm{ms}}$ and $k_{80\,\mathrm{ms}}$ are the indexes of these lines in the echogram, which occur after at least 50 ms and 80 ms, on the time scale, respectively. k_{mx} denotes the maximum line index in the echogram.

4. Results of the measurement and simulation of the impulse response

In order to verify the acoustic field model, the impulse response has been measured and the echograms were numerically simulated. Both, the simulation parameters and the experimental conditions were deliberately chosen so as to imitate best the sound source as well as the propagation conditions of the real audience hall. A large number (of the order of 10^6) of the cones was used in the simulation to reduce the undesirable effects of the *cone-tracing* method inaccuracy. The origin of this inaccuracy is the difficulty to reproduce exactly some elementary phenomena and the unavoidable mapping of the sound wave into a finite number of cones. Their bases were allowed to overlap (up to the 2-nd order [7]). If the overlap of those bases of the cones caused an observation point belonging to two or more cones, the cone nearest (with respect to the symmetry axis) to the observation point was selected.

The parameters representing the wall size of the room, the number of cones, their overlap and the order of the image sources in the numerical model were selected in such a way that the cone base diameter was never larger than the wall dimensions. The least possible number of acoustic rays that satisfy these conditions was adopted in the simulation.

To compare the simulated and measured quantities, the logarithmic amplitudes of the echograms were displayed using the following definition:

$$LR_k = 10 \log_{10} \frac{p_k^2}{p_{mx}^2} \,,$$

where p_k^2 – square of the room impulse response of the k-th sample or the height of the k-th line in the echogram, p_{mx}^2 – maximum p_k^2 value in the given range.

Figure 4 shows the initial 100 ms time span of the squared impulse response, measured at the R12 point of the room at the octave band mid-frequency of 1 kHz. Simulation results for the same point are plotted in Fig. 5.

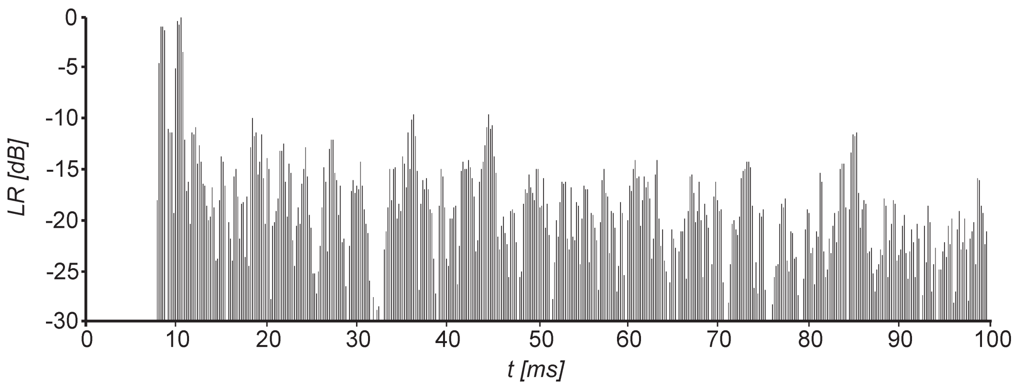

Fig. 4. Initial 100 ms of the squared impulse response measured at R12 and 1 kHz mid band octave frequency.

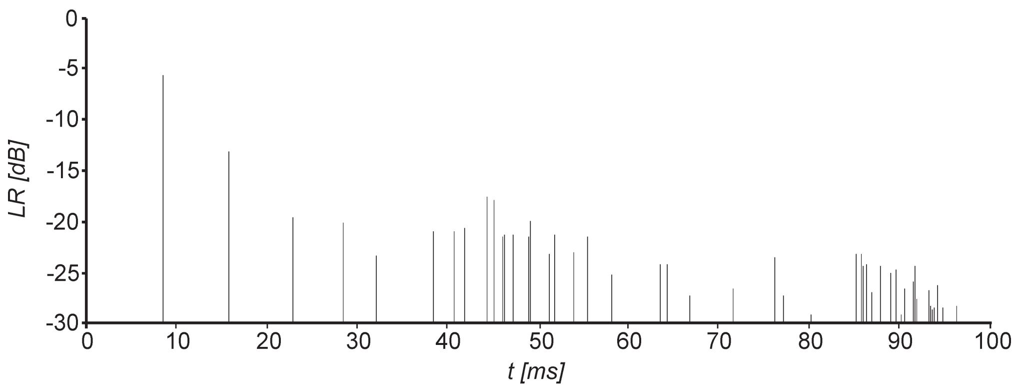

Fig. 5. Initial 100 ms of the simulated echogram at R12 site and 1 kHz mean octave band frequency.

The octave band characteristics shown in (Fig. 4) were obtained through digital filtering of the broadband impulse response. In the range of low order reflections, the time domain patterns of those measured and computer simulated echograms exhibit remarkable similarity. Minor differences that occur between certain lines are due to several simplifications made in the geometric representation of the room and result probably also from the point source approximation. Physically, the real source diameter was about 60 cm. Therefore, the spectral lines in the experimental echogram become broadened as well as slight shifts (order of 2 ms) between some of these lines may occur. Inaccuracies could also originate from the model of the audience seats: they were only represented by their appropriate *reverberation absorption coefficients*, the shapes of the particular chairs or tables were not taken into account. Such an approach to the presentation of these elements of the interior follows from the restriction of the rectilinear sound wave propagation and from neglecting the diffraction in the reflection process. Some lines in the echogram may have been missed since the number of walls assumed in the model is smaller. There is yet another mechanism leading to the omission of some of the image sources, namely the lines in the echograms simulated with the *cone-tracing* method. The effect, described in literature as *missing-images* [7], has been attributed to the finite angular aperture of the cones, which do not "illuminate" the room wall surfaces in 100%. In the extremely disadvantageous cases, hardly less than 40% of the wall surface is analysed as the viable sound wave reflective site, which may lead to missing of some image sources essential for the process. To lessen this error (which is similarly present in other geometrical models of the acoustic field analysis), one applies an additional overlap of the cones, i.e. they are attributed to a larger angular aperture than necessary to achieve a total coverage of the sphere around the source. Application of this method causes an enlargement of the wall area "illuminated" by a cone. Unfortunately, excessive enlargement of the cone aperture can generate additional image sources in the numerical model, which are unrelated to the actual sound propagation path in a room. Therefore, the overlap of cones adopted in these calculations to validate the model was limited to a double value of the minimum aperture.

Arithmetic mean deviations of the C_{50}, C_{80} and *EDT* parameters, the simulated *vs*. the measured ones in this room, are shown in Fig. 6 and Fig. 7.

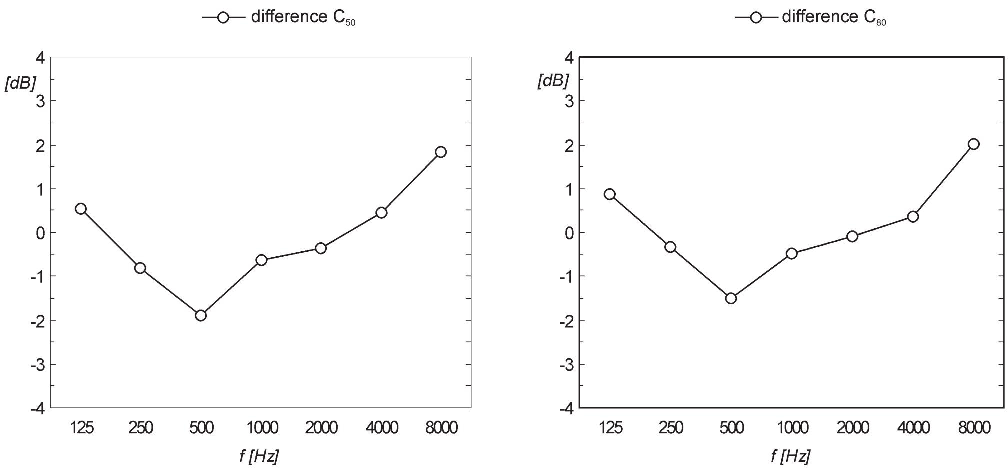

Fig. 6. Site averaged differences between the measured and simulated values of C_{50} and C_{80}.

In Table 1 there is a list of the differences between the parameters C_{50}, C_{80} and *EDT*, all determined at octave band mid-frequencies from 125 Hz to 8 kHz. Average values of these differences were calculated at the successive measuring sites. The arithmetic mean was used for all the averages and mean standard deviations are included in the table.

Table 1. Mean differences between the basic acoustic parameters obtained in the measurement and the simulation. S_{C50}, S_{C80}, S_{EDT} and $S_{EDT\%}$ refer to the standard mean deviations of C_{50}, C_{80}, EDT and $EDT\%$, respectively.

Octave band mid-frequency [Hz]	ΔC_{50} [dB]	S_{C50} [dB]	ΔC_{80} [dB]	S_{C80} [dB]	ΔEDT [sec]	S_{EDT} [sec]	$\Delta EDT\%$ [sec]	$S_{EDT}\%$ [sec]
125	0.53	1.89	0.87	1.57	0.24	0.33	16.2	9.1
250	−0.82	1.08	−0.34	0.91	0.28	0.21	12.6	5.6
500	−1.88	1.29	−1.5	1.04	0.34	0.12	12.8	4.2
1000	−0.64	1.14	−0.47	0.94	0.46	0.15	15.8	3.9
2000	−0.36	1.28	−0.08	1.09	0.49	0.11	16.3	3.2
4000	0.45	0.95	0.36	0.9	−0.06	0.05	3.9	1.8
8000	1.82	1.18	2.00	1.13	−0.32	0.23	45.4	21.3

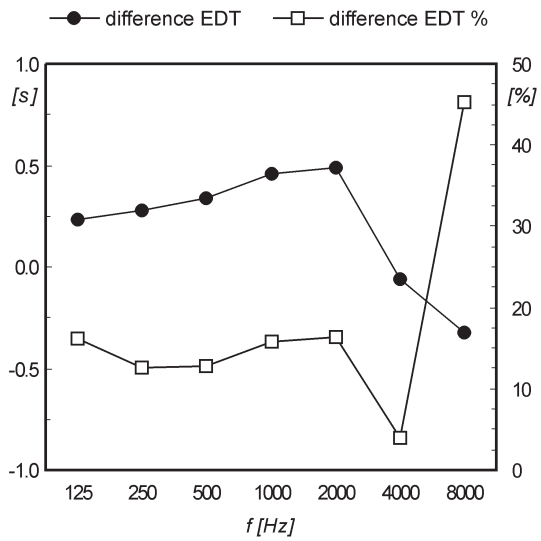

Fig. 7. Site averaged differences between the measured and simulated EDT value. “EDT %” denotes the difference relative to the measured value of EDT.

In Fig. 8, the parameters C_{50}, C_{80} and *EDT* are shown for the extreme cases of the best and the worst match between the simulation and measurement. Their analysis leads to the conclusion that the general form of the variation of these parameters in the frequency domain is preserved.

A satisfactory agreement was found between the real and the simulated values within a limited frequency range. The best match is observed for the 250 Hz, 1 kHz, 2 kHz and 4 kHz octave bands, where the discrepancies are of the order of 1.5 dB for C_{50} and C_{80}. For *EDT* (early decay time) the relative differences are less than $1/4$ of the

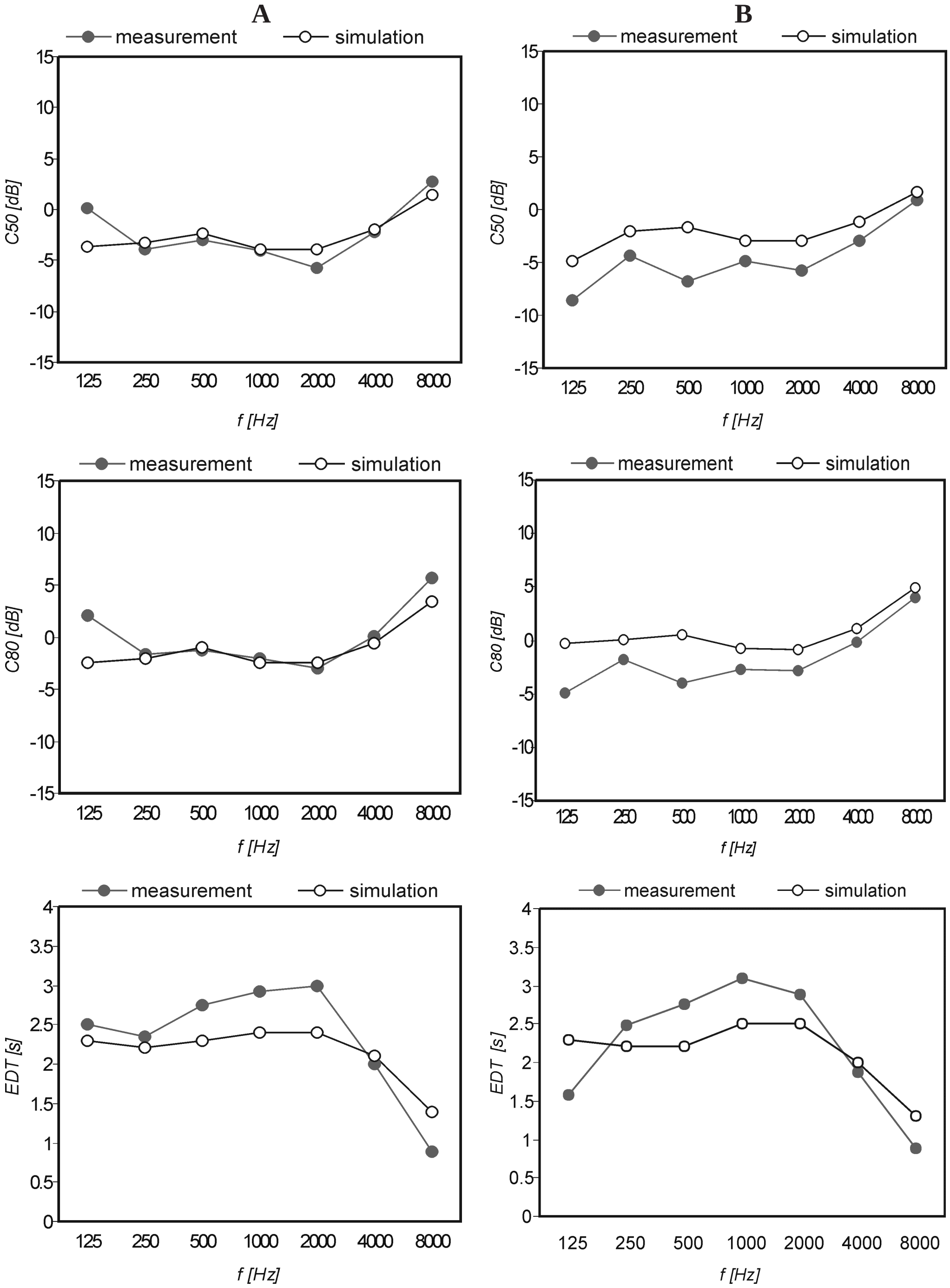

Fig. 8. Comparison of the parameters C_{50}, C_{80} and EDT for the receiving points of the best agreement – **A** and the worst agreement – **B** between simulation and measurement. Receiving points are R9 and R1, respectively.

measured values. The origin of the larger discrepancies at the remaining octave bands are caused probably by the failure of the adopted model to account for the diffraction and diffused reflection phenomena, which both play a considerable role in the sound propagation within the lowest and the highest frequency bands.

The amplitude differences of the lines occurring at the same time moments in the simulated and measured echograms appear larger than it could be anticipated when comparing the magnitudes of the C_{50}, C_{80} and *EDT* parameters. A possible origin of this inconsistency is the time sampling ($f_s = 36$ kHz is the sampling frequency) featured in the measured impulse response but which is absent in the simulated echogram.

5. Concluding remarks

The results of measurements of a selection of acoustic field parameters for the authentic lecture room have been reported, which verify the computer model of the same room. The results indicate a satisfactorily correspondence of this model with the actual measurements: the discrepancies of the measured parameters do not exceed 1.5 dB in the frequency range 250 Hz–4 kHz (except the 500 Hz octave band). In the frequency range from 125 Hz to 8 kHz, the differences did not exceed 2 dB. Thus, the verified computer model can be employed for simulations of an acoustic field. It seems also feasible to apply this model of the acoustic field to optimise a sound reinforcement system basing on the parameters available from echograms.

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