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ACOUSTICAL DIAGNOSTICS OF CRACKS IN BEAM LIKE STRUCTURES

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In this paper, the acoustical symptoms of a constructional element with an edge non-propagating crack on the example of a cantilever beam are searched. In this work the influence of a crack on flexural natural frequency was analysed. The crack is substituted by a rotational spring, which flexibility is calculated using the Castigliano theorem and the laws of the fracture mechanics.

In this work the changes in the first and second natural frequency of the flexural vibrations are showed as a function of location and depth of the crack. The acoustic signal measured with a microphone placed above the beam is used for diagnostic systems.

Changes in the natural frequency cannot be used for the identification of a small crack (the depth is less than 10% of beam height). For the detecting of a smaller crack, the effect of coupled different modes of vibration is presented. The paper presents a modelling and analysis algorithm for cracked Euler–Bernoulli beams by considering the coupling between the bending and axial vibration modes. The analysis of the coupled vibrations showed that additional resonance frequencies appeared in the acoustic spectrum.

Key words: acoustical diagnostic, crack, crack detection, vibration, coupled vibration.

1. Introduction

Structural elements and systems are very frequently subject to loads changing in time. Changing mechanical or kinematic loads induce stresses varying in time, producing complex coincidence of effects and fatigue changes in the material depending on the volume of stresses and the number of their cycles. Fatigue changes are visualized by a crack of the structural material and its continued use lead to element damaging. Therefore, the question about the crack depth becomes essential with reference to the design safety. This quantity enables to evaluate the suitability of the construction for further operations.

Technical devices generate (emit) acoustic and vibration signals containing essential information on the condition of the device as a source [2]. The bibliography includes many references for crack identification methods based on vibration measurements. Diagnostic symptoms can be found in the changes of: natural frequency [9–11], forced vibration amplitudes [10, 12, 17] and mechanical impedance [1], but also in the wavelet analysis [18] and vibration under white noise [3].

In many structural elements (e.g. turbogenerator shafts) the sole information on crack occurrence is essential (even if it is not possible to identify the crack depth or location), but simple measurement procedures are used.

This paper describes such "quick" crack diagnostics for a structural element on the example of a cantilever beam. In the diagnostic test, an acoustic signal of the impulse response of the beam is recorded (without the need to measure the force) using a microphone placed above the beam. We will look for such diagnostic symptoms in the spectrum (distribution of resonance frequencies) of the recorded signal.

The crack has been modelled as a flexible joint, the flexibility of which is determined basing on fracture mechanics relations between the strain energy release rate and the stress intensity factor (SIF) and the Castigliano's theorem.

Recently authors consider the effect of coupling of various vibration modes on the parameters of dynamic characteristics of cracked beam [4, 5]. Such a coupling has a slight effect on the change of the natural frequency, but in the case of a structural element with crack transverse vibrations induced by transverse loads generate also longitudinal vibrations, which in turn generate transverse vibrations. A diagnostic procedure based on the analysis of the coupled vibration model allows detecting cracks (with small depth) that cannot be detected basing on changes of the natural frequency.

2. Description of the problem

The problem discussed in this paper has been described by the Bernoulli–Euler's beam model without taking into consideration the crack closing effect during vibrations as shown schematically in Fig. 1.

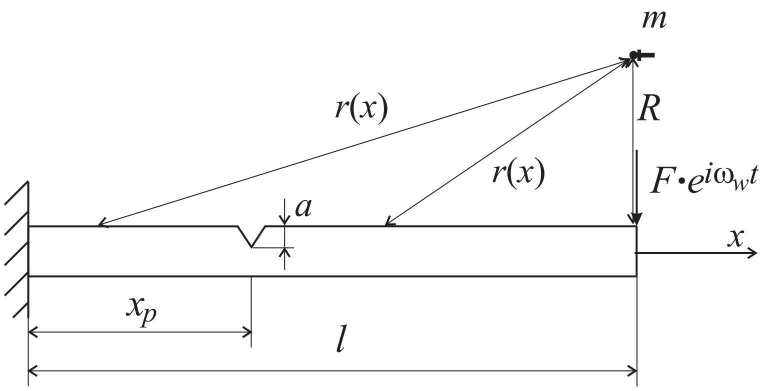

Fig. 1. The model of a cantilever beam with a transverse open and non-propagating crack.

The subject of this paper is the finding in the acoustic signal, recorded with the microphone m, the symptoms of crack in the beam as shown in Fig. 1. In the studies the beam has a constant cross-section $A = b \times h$ and area moment of inertia I. The Young modulus E and material density ρ are also constant.

The diagnostic procedure described in the paper involves the analysis of vibrations induced by force-impulse and determining the acoustic pressure at point m. The analysis was carried out in the frequency domain by setting a beam vibration velocity function (for different crack depths) for each excitation frequency. The Green's function set for open space was used for the description of the acoustic wave propagation.

3. Transverse vibration

In order to estimate the effect of crack on beam vibration, the latter has been simulated as a rotational spring. The spring flexibility c_g binds the bending moment in the cross-section with the coordinate $x = x_p$ and the angles of rotation (angular displacement) from the right and left side of the cross-section in which the crack exists, i.e.:

$$y'(x_p^+) - y'(x_p^-) = c_g \, EI \, y''(x_p^-). \tag{1}$$

The equation binding the spring flexibility and crack depth based on fracture mechanics will be shown in the next section of the article.

4. Flexibility at the cracked place

The fracture mechanics studies allow finding relations between the global quantity G – Energy Release Rate determining the increase in the elastic strain energy for an infinitesimal crack surface increase:

$$G = \frac{\partial U}{\partial A_p}$$

and the local quantity K – Stress Intensity Factor (SIF), which is a function of the crack depth a:

$$G = \frac{1-\nu^2}{E} K_I^2, \tag{2}$$

where G – energy release rate represents the elastic energy per unit crack surface area, A_p – area of the crack, ν – Poisson ratio, E – Young modulus, K_I – Stress Intensity Factor (SIF) of mode I (opening the crack) for pure bending.

After simple transformation, the elastic strain energy due to the crack has the form (b, h, a, α are explained in Fig. 2):

$$U = \int\limits_{A_p} G \, \mathrm{d}A_p = \frac{1-\nu^2}{E} b \int\limits_0^a K_I^2 \, \mathrm{d}\alpha. \tag{3}$$

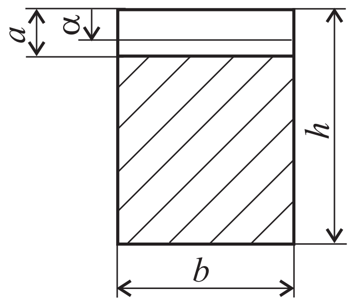

Fig. 2. Geometry of the cracked section.

SIF can be expressed as follows:

$$K_I = \sigma\sqrt{\pi\alpha}\, F_I\left(\frac{\alpha}{h}\right),$$

where $\mathrm{d}A_p = b\,\mathrm{d}\alpha$ – elementary crack area, b, h – cross-section dimensions, a – depth of the crack, σ – normal stress, F_I – correction function also called "shape functions".

The relation between the function F_I and the crack depth for different geometries of the elements with crack and different ways of loading can be found in catalogs, e.g. [16]. It takes the following form for pure bending:

$$F_I\left(\frac{\alpha}{h}\right) = 1.122 - 1.40\left(\frac{\alpha}{h}\right) + 7.33\left(\frac{\alpha}{h}\right)^2 - 13.08\left(\frac{\alpha}{h}\right)^3 + 14.0\left(\frac{\alpha}{h}\right)^4.$$

The error made when using the above mentioned formula is less than 0.2% for a crack depth not larger than 60% of the beam height.

The normal stress in the section $x = x_p$ (Fig. 1) of the beam with no crack (the change in stress distribution caused by crack is given by the K_I coefficient) is:

$$\sigma = \frac{M_g(x_p)}{W_g} = \frac{M_g(x_p)}{2I}h\,.$$

After simple transformations the increase in the elastic strain energy in the beam, connected with the appearance of a crack of depth a in the beam of rectangular section $b \times h$, is:

$$\begin{aligned} U \;=\; 3h\pi\frac{1-\nu^2}{EI}M_g^2(x_p)\left(\frac{a}{h}\right)^2 \Bigg[0.6294 - 1.0472\frac{a}{h} + 4.6021\left(\frac{a}{h}\right)^2 \\ - 9.9751\left(\frac{a}{h}\right)^3 + 20.2948\left(\frac{a}{h}\right)^4 - 32.9933\left(\frac{a}{h}\right)^5 + 47.0408\left(\frac{a}{h}\right)^6 \\ - 40.6933\left(\frac{a}{h}\right)^7 + 19.6\left(\frac{a}{h}\right)^8\Bigg]. \qquad (4) \end{aligned}$$

The section with the crack should be replaced with a flexible joint (rotational spring) having the same potential energy [13].

In order to determine the value of flexibility, c_g, of such a joint, the Castigliano's theorem is used, from which the additional generalized displacement can be determined (angle of rotation $\theta = y'(x_p^+) - y'(x_p^-)$) resulting from the increase of the potential energy of deformation of the beam (4):

$$\theta = \frac{\partial U}{\partial M_g}\,.$$

Taking into account:

$$c_g = \frac{\partial \theta}{\partial M_g}\,,$$

we obtain after simple transformations an equation, which binds the spring flexibility with the increase in the potential energy of deformation U:

$$c_g = \frac{\partial^2 U}{\partial M_g^2(x_p)}\,,$$

hence the flexibility is:

$$\begin{aligned} c_g \;=\; 6h\pi\frac{1-\nu^2}{EI}\left(\frac{a}{h}\right)^2 \Bigg[& 0.6294 - 1.0472\frac{a}{h} + 4.6021\left(\frac{a}{h}\right)^2 \\ & - 9.9751\left(\frac{a}{h}\right)^3 + 20.2948\left(\frac{a}{h}\right)^4 - 32.9933\left(\frac{a}{h}\right)^5 \\ & + 47.0408\left(\frac{a}{h}\right)^6 - 40.6933\left(\frac{a}{h}\right)^7 + 19.6\left(\frac{a}{h}\right)^8 \Bigg]. \end{aligned} \tag{5}$$

The flexibility of the elastic element modelling of the cracked cross-section will be used to find diagnostic symptoms of the crack in the beam.

5. Flexural vibration equation of the cracked beam

The equation of the forced vibration amplitudes including the exciting force applied in the point with the coordinate $x = x_f$, amplitude F and frequency ω_w has the following form:

$$X^{(4)} - \lambda^4 X = c_g\, X''(x_p)\, \delta''(x, x_p) - F\, \delta(x, x_f). \tag{6}$$

The solution of Eq. (6) can be found in the class of a generalized function, which gives the solution in the finite form in contrast to the standard method which leads to a solution in the form of an infinite sum of eigenfunctions.

The following function is a solution of Eq. (6):

$$\begin{aligned} X(x) \;=\; & X_0(x) + \frac{c_g}{2\lambda} X''(x_p)\left[\mathrm{sh}\lambda(x-x_p) + \sin\lambda(x-x_p)\right] H(x,x_p) \\ & - \frac{F}{2EI\lambda^3}\left[\mathrm{sh}\lambda(x-x_f) + \sin\lambda(x-x_f)\right] H(x,x_f), \end{aligned} \tag{7}$$

where $\delta(x, x_p)$ – Dirac delta function at $x = x_p$, $H(x, x_p)$ – Heaviside step function at $x = x_p$, $\lambda = \omega_w^2 \rho A/EI$, ρ – beam material density, A – cross-sectional area,

$$X_0 = P\,\mathrm{ch}\lambda x + Q\mathrm{sh}\lambda x + R\cos\lambda x + S\sin\lambda x,$$

P, Q, R, S – integration constants.

The boundary conditions for the beam in Fig. 1 are described by the equations: $X(0) = 0$, $X'(0) = 0$, $X''(l) = 0$ and $X'''(l) = 0$. These equations could be written in a matrix form:

$$\mathbf{M} \cdot \mathbf{C} = \mathbf{W}, \tag{8}$$

where

$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ \mathrm{ch}\lambda l & \mathrm{sh}\lambda l & -\cos\lambda l & -\sin\lambda l & \dfrac{c_g}{2\lambda}\big[\mathrm{sh}\lambda(l-x_p) - \sin\lambda(l-x_p)\big] \\ \mathrm{sh}\lambda l & \mathrm{ch}\lambda l & \sin\lambda l & -\cos\lambda l & \dfrac{c_g}{2\lambda}\big[\mathrm{ch}\lambda(l-x_p) - \cos\lambda(l-x_p)\big] \\ \mathrm{ch}\lambda x_p & \mathrm{sh}\lambda x_p & -\cos\lambda x_p & -\sin\lambda x_p & -\dfrac{1}{\lambda^2} \end{bmatrix},$$

constants vector $\mathbf{C}$:

$$\mathbf{C}^T = \big[P \;\; Q \;\; R \;\; S \;\; X''(x_p)\big]^T$$

and excitation vector $\mathbf{W}$:

$$\mathbf{W} = \begin{bmatrix} 0 \\ 0 \\ \dfrac{F}{2EI\lambda^3}\big[\mathrm{sh}\lambda(l-x_f) - \sin\lambda(l-x_f)\big] \\ \dfrac{F}{2EI\lambda^3}\big[\mathrm{ch}\lambda(l-x_f) - \cos\lambda(l-x_f)\big] \\ \dfrac{F}{2EI\lambda^3}\big[\mathrm{sh}\lambda(x_p-x_f) - \sin\lambda(x_p-x_f)\big]H(x_p,x_f) \end{bmatrix}.$$

After calculating the integration constants, the function of the forced vibration amplitudes is described by Eq. (7), and the vibration velocity amplitudes by the following function:

$$\begin{aligned} v(x) \;=\; & \omega\,(P\,\mathrm{ch}\lambda x + Q\,\mathrm{sh}\lambda x + R\cos\lambda x + S\sin\lambda x) \\ & + \omega\frac{c_g}{2\lambda}X''(x_p)\big[\mathrm{sh}\lambda(x-x_p) + \sin\lambda(x-x_p)\big]H(x,x_p) \\ & - \frac{\omega F}{2EI\lambda^3}\big[\mathrm{sh}\lambda(x-x_f) + \sin\lambda(x-x_f)\big]H(x,x_f). \end{aligned} \tag{9}$$

The vibrating beam is a sound source with a pressure described by relation (10) as a function of vibration excitation frequency resulting directly from the Green's function [7, 15]:

$$p(\omega) = \frac{j\omega\rho_0}{2\pi} b \int\limits_0^l \frac{v(x)\, e^{jkr(x)}}{r(x)}\, \mathrm{d}x, \tag{10}$$

where ρ_0 – density of air, b – width of beam, $v(x)$ – function of velocity amplitudes described by (9), k – wave number, l – length of the beam, $r(x) = \sqrt{R^2 + (l-x)^2}$ – distance between the measured point and the beam point of coordinate x.

Figure 3 shows the acoustic pressure curve $p(\omega)$ set for a beam with no crack (Fig. 3a) and for that with a crack at point $x_p = 0.9$ m and the depths $a = 0.1h$

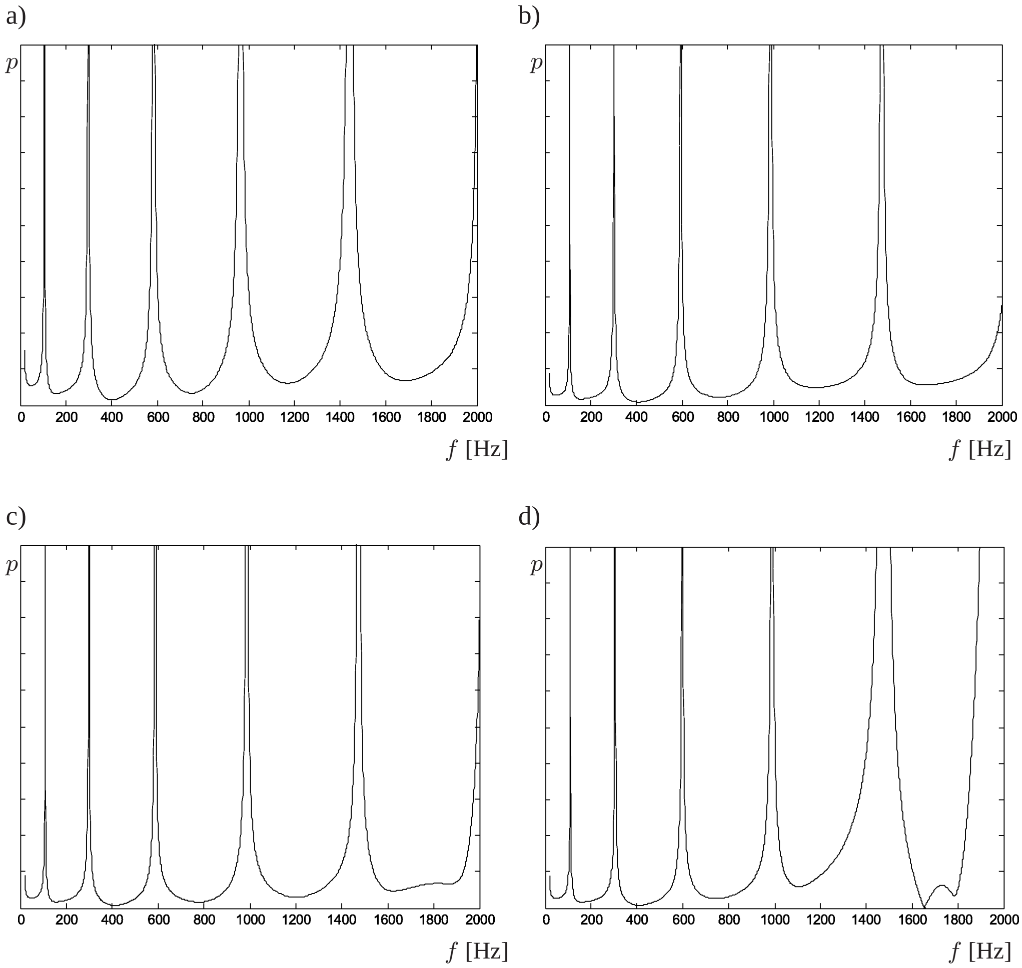

Fig. 3. Acoustic pressure as function of frequency the source of which is an impulse forced beam with no crack (a) and with different crack depths (b, c and d).

(h – beam height) (Fig. 3b), $a = 0.3h$ (Fig. 3c) and $a = 0.5h$ (Fig. 3d). The calculations were performed for a beam with the following material data: Young modulus $E = 2.1 \cdot 10^{11}$ Pa; material density $\rho = 7860$ kg/m^3, and the geometric data: cross-section $b \times h = 0.03 \times 0.03$ m; beam length $l = 1.2$ m. The calculations were performed for the force with constant amplitude for each excitation frequency, which was changed from $\omega = 2\pi \cdot 20$ rad/s (frequency $f = 20$ Hz) to $\omega = 2\pi \cdot 2000$ rad/s (frequency $f = 2$ kHz) with a 1 Hz increment.

The sought-after diagnostic symptom is the change of the natural frequency value. The analysis of the characteristics shown in Fig. 3 leads to the conclusion that, despite the analysis of the beam vibration with a large crack (50% of beam height), the change in frequency is imperceptible. The visible widening of the spectrum, particularly for higher frequencies, is a diagnostic symptom hard to identify. The model considered does not include the always existing (internal and external) vibration damping, which increases with the increase of the excitation frequency (internal damping is proportional to deformation velocity). Therefore, on one hand the vibration velocity increases due to the crack side, on the other hand and it reduces due to the increase of damping. This causes problems in the evaluation of the value of the spectrum widening as a diagnostic symptom.

Due to the fact that the change of the free vibration frequency is a diagnostic symptom, this change has been shown in Figs. 4a and 4b in relation to the frequency of the beam with no crack as a function of the crack depth a for different crack locations x_p. The figures show, $r_i = \omega_{i\ pek}/\omega_i$, $\omega_{i\ pek}$ – i-th natural frequency of cracked beam, ω_i – i-th natural frequency of beam with no crack, $i = 1, 2$.

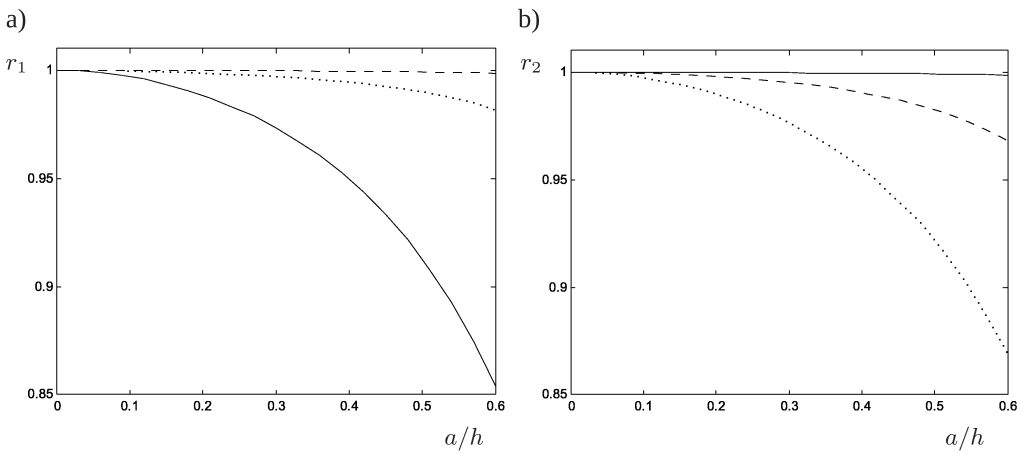

Fig. 4. Variations of the first two natural frequencies as functions of the crack depth: a) first natural frequency and, b) second natural frequency.

The results are shown for $x_p = 0.3$ m as solid lines, for $x_p = 0.6$ m as dotted lines and for $x_p = 0.9$ m as broken lines.

According to the analysis of the results presented in Fig. 4, the crack with a depth up to approx. 10% of beam height does not cause practically any detectable changes of the free vibration frequency, which makes the diagnostics of such fatigue cracks impossible by using the method of analysis of natural frequency change. Basing on the detected change of the natural frequency value, it is possible to identify the depth and location of the crack [10, 14].

6. Coupled vibrations

The previous considerations regarding vibrations of a structural element such as a cracked cantilever beam relate to the simulation of the element as a bending element. The sought-after and described crack symptoms are the changes of the eigenfrequency value. In the latest literature on structural element vibration analysis with a crack, the authors consider the effect of coupling of various vibration modes on the parameters of dynamic characteristics. The coupling results from the fact that lateral vibrations induced by lateral external forces generate longitudinal vibrations, which in turn generate lateral vibrations. For lateral and longitudinal coupled vibrations of the beam with crack, the energy release rate could be expressed as follows:

$$G = \frac{1-\nu^2}{E}\left(K_{Ig} + K_{Iw}\right)^2, \tag{11}$$

where

$K_{Ig} = \sigma_g \sqrt{\pi\alpha} F_{Ig}\left(\frac{\alpha}{h}\right)$ – Stress Intensity Factor of mode I for bending moment M_g,

$\sigma_g = \frac{M_g(x_p)}{W_g}$ – normal stress,

$F_{Ig}\left(\frac{\alpha}{h}\right) = 1.122 - 1.40\left(\frac{\alpha}{h}\right) + 7.33\left(\frac{\alpha}{h}\right)^2 - 13.08\left(\frac{\alpha}{h}\right)^3 + 14.0\left(\frac{\alpha}{h}\right)^4$ – correction function,

$K_{Iw} = \sigma_w \sqrt{\pi\alpha} F_{Iw}\left(\frac{\alpha}{h}\right)$ – Stress Intensity Factor of mode I for axial force P_w,

$\sigma_w = \frac{P_w(x_p)}{A}$ – normal stress,

$F_{Iw}\left(\frac{\alpha}{h}\right) = 1.12 - 0.231\left(\frac{\alpha}{h}\right) + 10.55\left(\frac{\alpha}{h}\right)^2 - 21.72\left(\frac{\alpha}{h}\right)^3 + 30.39\left(\frac{\alpha}{h}\right)^4$ – correction function.

The above designations demonstrate that the elastic strain energy depend on: the square of the bending moment $M_g(x_p)$, the square of the longitudinal force $P_w(x_p)$ and the product of both $M_g(x_p)$ and $P_w(x_p)$.

Hence, the crack has been modelled as a $[2 \times 2]$ flexibility matrix containing c_g and c_w coefficients on the main diagonal, and flexibility coefficients c_{gw} and c_{wg} outside the diagonal. The relation between displacements (longitudinal $u(x)$ and lateral $y(x)$) from the right and left hand sides of the cross-section with crack, the longitudinal force $P_w(x_p)$ and the bending moment $M_g(x_p)$ in this cross-section is given by matrix relation [14]:

$$\begin{bmatrix} c_g & c_{gw} \\ c_{wg} & c_w \end{bmatrix} \begin{bmatrix} M_g(x_p) \\ P_w(x_p) \end{bmatrix} = \begin{bmatrix} y'(x_p^+) - y'(x_p^-) \\ u(x_p^+) - u(x_p^-) \end{bmatrix}. \tag{12}$$

Individual flexibilities included in the flexibility matrix can be calculated using the Castigliano theorem:

$$\begin{aligned} c_g &= \frac{\partial^2 U}{\partial M_g^2(x_p)}, \\ c_w &= \frac{\partial^2 U}{\partial P_w^2(x_p)}, \\ c_{gw} &= \frac{\partial^2 U}{\partial M_g(x_p)\,\partial P_w(x_p)}, \\ c_{wg} &= \frac{\partial^2 U}{\partial P_w(x_p)\,\partial M_g(x_p)}. \end{aligned}$$

According to the Schwarz's theorem, the sequence of differentiation has no effect on the final result, which means that $c_{gw} = c_{wg}$.

According to the analysis carried out by the author, the including of the coupling of lateral and longitudinal vibrations in the discussed model has an insignificant effect on the change of natural frequency value. This leads to the conclusion that for the identification of the parameters, i.e. the location and depth of the crack, basing on measurements of the free vibration frequency, there is no need to consider the coupled vibrations.

Figure 5 shows the acoustic pressure as function of frequency, the source of which is the beam, described with the model including the vibration coupling, vibration-excited by the force impulse in the lateral direction (in Fig. 5a for a beam with no crack, in Fig. 5b for a beam with a crack).

As easily seen in the pressure spectrum, there is an additional resonance frequency of a value of approx. 1050 Hz. This frequency corresponds to the first frequency of the longitudinal free vibration. This means that for a structural element with crack, the lateral vibration induced by lateral external forces generate also longitudinal vibration, despite of that fact that excitation on these directions does not exist, whereas the latter generate lateral vibration of a frequency equal to the frequency of the longitudinal natural vibration.

The characteristics shown in Fig. 5b was obtained for the crack depth $a = 3\%$ of the beam height, from which results that the coupled vibration analysis allows to diagnose such cracks (of little depth) in beams, which are undetectable when measuring changes of the eigenfrequency, or amplitudes of harmonic vibration.

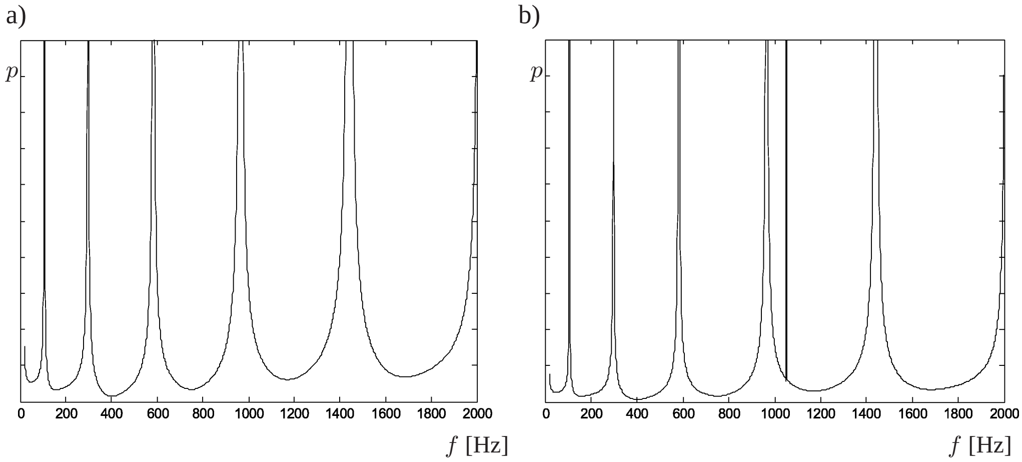

Fig. 5. Acoustic pressure as function of frequency, the source of which is the beam: a) for a beam with no crack, b) for a beam with a crack.

7. Conclusions

The paper describes acoustic diagnostics for crack detection in a structural element on the example of a cantilever beam. Acoustic diagnostics is one of non-invasive methods, which are based on the passive measurement, in which the interference of the measuring sensors is negligible. The advantage of non-invasive methods is the capability of diagnosing and monitoring (continuous control) without interrupting the normal operation of the device. An additional advantage of the acoustic diagnostics is the relatively easy recording of the diagnostic signals, which include acoustic emission signals.

The paper analyses the most commonly described in the reference material models, a beam model including only bending vibrations and a model including coupled bending and longitudinal vibrations.

It has been shown that for bending eigenvalues their change may be regarded as an identification diagnostic symptom. At the same time, the analysis of the results obtained shows that a crack with a depth up to approx. 10% of the beam height practically does not cause any detectable change of the natural frequency. This leads to the conclusion that the diagnostics of fatigue cracks with depths less than 10% of beam height, it is impossible to use this diagnostic symptom.

For the detection of cracks of smaller depth, the effect of coupling of different vibration modes was used. It has been proved that the crack is the source of coupling, i.e. in the case of a structural element with crack, the lateral vibrations induced by lateral external forces generate also longitudinal vibrations, contrary to the fact that excitation in these directions does not exist. It has been demonstrated that the presence of the additional resonance frequency, close to the longitudinal eigenfrequencies of the element in the spectrum of recorded acoustic signal, is a diagnostic symptom of crack, however it is not impossible to identify crack parameters, i.e. depth and location of crack, basing on this experiment.

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