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COMPUTATIONAL COMPLEXITY OF THE ALGORITHM CREATING HYPERMETRIC RHYTHMIC HYPOTHESES

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This study presents the algorithm creating rhythmic hypotheses worked out by the authors, and then addresses the problem of determining its computational complexity. A short review of rhythm extraction methods is presented, first. Then, three phases of the algorithm engineered by the authors, namely creating periods, creating simplified hypotheses and creating full hypotheses are examined. The analyses of computational complexity of the method proposed assume that the engineered method is expected to rank rhythmic hypotheses formed of three rhythmic levels above meter. This proved to be sufficient for providing automatic drum accompaniment for a given melody without delay.

Keywords: (MIR) music information retrieval, rhythm retrieval, metric rhythm.

1. Introduction

A constantly growing number of musical documents stored in electronic libraries and in the Internet induced intensified scientific interest in the retrieval of music information. Methods of retrieval based on the content of multimedia files are most often computationally complex. However, the advancement in the computer processing power has now significantly grown, and combined with the scientific efforts to simplify the algorithms computational complexity let us expect that the systems employing existing methods will produce results in acceptable time. The subject of this paper is the analysis of the computational complexity of a method creating rhythmic hypotheses within the aspect of using the set of obtained hypotheses to rank them, the feature which is frequently neglected in the research of musical information retrieval.

The scope of the studies concerning rhythm is very wide and, among other issues, involves the quantization process of the beginnings and lengths of notes, the extraction of rhythm events from audio recordings, and the search for meter of compositions. Rhythm is an element of a piece determining musical style, which may be valuable in retrieval. The rhythmic structure together with patterns retrieved carry information about the genre of a piece. The time signature 3/4 for example determines that this piece may be waltz but cannot be tango, because time signatures of both styles are different.

One of the first methods of creating and ranking hypermetric hypotheses was proposed by ROSENTHAL in his works [6, 7]. The method was implemented in the "*Machine Rhythm*" system. It takes unquantized musical data presented in a symbolic format as inputs. The model is able to cope with slight changes in tempo. TEMPERLEY and SLEATOR [8], the authors of the preference-rule method that searches for metric rhythm, based their system on three rules. The adjustment of ascents to the beginnings of musical events is called *event rule*. The rule that favors the accents covering long sounds is called *length rule*. The last of the three rules (*regularity rule*) promotes accents that are evenly remote. Rhythmic levels of periods between 400 to 1600 ms are recognized by the authors as candidates for a rhythmic level related to the tactus level. The hypotheses obtained through the work of Temperley and Sleator contain two levels below meter, which evaluates to five rhythmic levels [8].

Existing metric rhythm research usually focuses on retrieving low rhythmic levels – usually to the level of a measure, those methods are enough to emulate human perception of a local rhythm. According to MCAULEY and SEMPLE [5] trained musicians perceive more levels, though. High-level perception is required from drum players, thus computational approach needs to retrieve a so-called hypermetric structure of a piece. If it reaches high rhythmic levels such as phrases, sentences and periods, then automatic drum accompaniment applications can be developed. In the authors' approach the hypermetric structure is searched.

2. Outline of the method of creating rhythmic hypothesis

The objective of this section is to present shortly a method for retrieval of hypermetric rhythm on the basis of melody. A stream of sounds in MIDI format is introduced at the system input, on the basis of a musical content the method retrieves a hypermetric structure of rhythm of a musical piece consisting rhythmic motives, phrases, and sentences. A method does not use any information about rhythm (time signature), which is present often in MIDI files. Neither rhythmic tracks nor harmonic information are used to support the method. The only information analyzed is a melody, which might be monophonic as well as polyphonic. The method starts to analyze a piece after the entire piece is played. Two elements are combined, namely recurrence of melodic and rhythmic patterns and the rhythmic salience of sounds to create a machine able to find the metric structure of rhythm to a given melody.

The method proposed by the authors of this paper generates rhythmic levels first, and then uses these levels to compose rhythmic hypotheses. The lowest rhythmic level has a phase of the first sound from the piece and its period is atomic. The following levels have periods of values achieved by recursive multiplication of periods that have already been calculated (starting from the atomic value) by the most common prime numbers in Western music, i.e. 2 and 3. The process of period generation may be illustrated as a process of a tree structure formation (Fig. 1) with a root representing the atomic period equal to 1. Each node is represented by a number which is the node ancestor number multiplied by either 2 or 3.

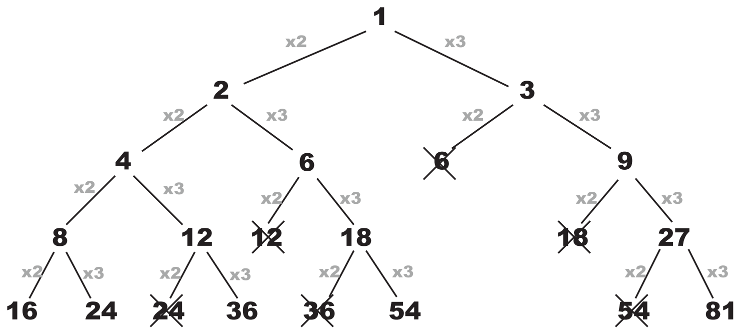

Fig. 1. The tree of periods.

The tree holds some duplicates. The node holding a duplicated value would generate a sub-tree whose all nodes would also be duplicates of already existing values. So duplicate subtrees are eliminated and we obtain a graphical interpretation in the form of the periods triangle (see Fig. 2).

When the phase of periods creation is completed, each period must have all its phases (starting from phase 0) generated. The last phase of a given rhythmic level has the value equal to the size of the period decreased by one atomic period. In order to achieve hypotheses from the generated rhythmic levels, it is necessary to find all families of related rhythmic levels. A level may belong to many levels. The generated hypotheses are instantly ranked to extract the one which designates the appropriate rhythm of the piece. The hypotheses that cover notes of significant rhythmic weights are ranked higher. The weights are calculated based on the knowledge gathered by learning systems that know how to asses the importance of physical characteristics of sounds that comprise the piece. Systems proposed by the authors employ rules obtained in the process of *data mining* [3, 9], as well as from the operation of neural networks [1], and through employing rough sets [4]. Taking a set of representative musical objects as grounds, these systems learn how to asses the influence of a sound relative frequency, amplitude and length on its rhythmic weight. The second group of methods used to rank hypotheses is based on one of the elementary rules known in music composition, i.e. recurrence of melodic and rhythmic patterns – the group is described in the authors' works [2] and [10].

3. Analysis of method complexity

The method creates all possible rhythmic structures. However, their number is limited and depends on the following factors:

- The level designated as the lowest among all the created hypotheses (this defines the parameter of sound length quantization). The authors observed that the quantization with the resolution of a quarter-note is sufficient.
- The intricacy of the hypotheses, i.e. how many levels they contain. The method was examined for at most three rhythmic levels above meter, similarly as in the research conducted by ROSENTHAL [7], and TEMPERLEY and SLEATOR [8].

Taking the above assumptions into consideration, i.e. the quantization parameter being a quarter-note and the analysis of a hypothesis concerning three levels above meter, we obtain the number of periods from the first 6 layers of the triangle shown in Fig. 2 (from the top rows refer to: quarter note, half note, whole note (motive), phrase, sentence, period).

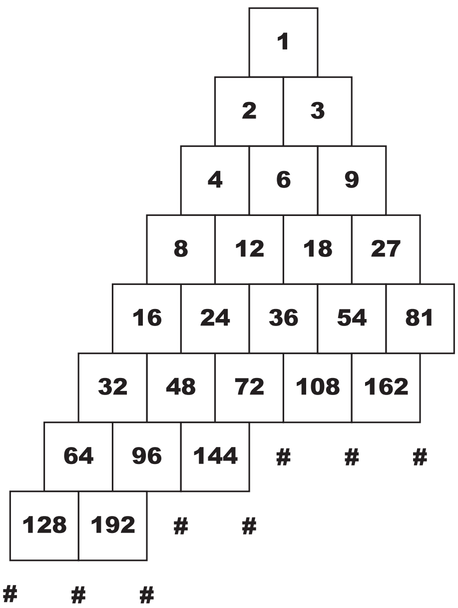

Fig. 2. Triangle of periods.

The atomic period is a quarter-note (layer 1), the layer containing periods 4, 6, 9 is the level of meter, and the sixth layer holding the values of 32, 48, 72 . . . is the last examined rhythmic level.

3.1. *Algorithm of periods calculating*

The number of periods is $n{\cdot}(n{+}1)/2$, where n is the number of layers, so the algorithm is polynomial, the function of the computational complexity is of class $\mathbf{O(n^2)}$. The basic operation that calculates periods is multiplication. The number of periods calculated for 6 layers is 21, and these are the elements of a periods list.

3.2. Algorithm crating hypotheses (with periods only)

Hypotheses are lists of related rhythmic levels that include pairs of <period, phase> values. If we take only periods into consideration, the number of hypotheses is the number of paths starting from the highest rhythmic level (layer 6) and ending in the level of atomic period (layer 1). For assumed parameters, this gives 32 hypotheses if only periods defined. The number is a result of the following computations:

- from period 32 there is one path (32, 16, 8, 4, 2, 1),
- from period 48 there are 5 paths,
- from period 72 there are 10 paths.

Table 1. Rhythmic hypotheses (without phases) for a 6 layer triangle of periods.

Layer					
1	**2**	**3**	**4**	**5**	**6**
1	2	4	8	16	32
1	2	4	8	16	48
1	2	4	8	24	48
1	2	4	8	24	72
1	2	4	12	24	48
1	2	4	12	24	72
1	2	4	12	36	72
1	2	4	12	36	108
1	2	6	12	24	48
1	2	6	12	24	72
1	2	6	12	36	72
1	2	6	12	36	108
1	2	6	18	36	72
1	2	6	18	36	108
1	2	6	18	54	108
1	2	6	18	54	162
1	3	6	12	24	48
1	3	6	12	24	72
1	3	6	12	36	72
1	3	6	12	36	108
1	3	6	18	36	72
1	3	6	18	36	108
1	3	6	18	54	108
1	3	6	18	54	162
1	3	9	18	36	72
1	3	9	18	36	108
1	3	9	18	54	108
1	3	9	18	54	162
1	3	9	27	54	108
1	3	9	27	54	162
1	3	9	27	81	162
1	3	9	27	81	243

For the left half of the triangle we may specify 16 paths. The computations for the right half, i.e. the paths including periods 108, 162, and 243, are analogous. This gives 32 paths altogether in a 6 layer triangle.

The function of computational complexity is of class $\mathbf{O}(\mathbf{n^2})$, where n is the number of layers. Thus, the complexity is exponential which with n limited to 6 layers confines the number of hypotheses to 32. The rows of Table 1 show subsequent simplified hypotheses, i.e. the ones that contain only periods (phases are ignored) for the example from Figs. 1 and 2.

The algorithm that creates hypotheses with periods only ranks rhythmic hypotheses based on the recurrence of melorhythmic patterns (16 methods proposed in the thesis of WOJCIK [10]). The basic operation of patterns recurrence evaluation is in this case addition. The only hypotheses ranking method examined by the authors that requires the phases to be defined is the method based on rhythmic weights.

3.3. Algorithm creating hypotheses with phases

Each hypotheses may have as many versions, with regard to phases, as its longest period is, e.g. the first hypothesis from Table 1 (the first row: 1, 2, 4, 8, 16, 32) will have 32 different phases. On condition that $n = 6$, the number of all hypotheses for the discussed example will amount to 3125, which is the sum of all periods from layer 6. Thus, the number of all hypotheses is the sum of the values from the last column of Table 1.

The algorithm that forms hypotheses with phases is used in a method ranking rhythmic hypotheses based on rhythmic weight. The elementary operation of this method is addition.

4. Conclusions

Understanding the basic rules of human rhythm perception may result in designing various applications, for example an artificial drummer accompanying a given melody may be constructed [10].

To analyze a piece of music with regard to its motives, phases, phrases and periods when its atomic period is defined as a quarter-note, the number of 6 layers ($n = 6$) is sufficient. Despite the exponential complexity of the method, the number of elementary operations is not more than 10^4 on a 1.6 GHz computer. The total time of all operations for a single piece of music is imperceptible for a system user, which was proved by the experimental system, engineered by the authors. This means that the method provides high quality automatic drum accompaniment without delay.

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