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Sound Power Radiated from Rectangular Plates

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An equation for calculating the sound power radiated from a rectangular plate with arbitrary boundary conditions is derived, in which the sound power radiated from the plate is represented in terms of the normal velocity distribution on the plate and a coupling matrix. The velocity distribution on the plate is expressed in terms of the modal amplitudes and normal modes. The coupling matrix for arbitrary boundary conditions is developed mathematically using the Rayleigh integral. Finally, an approach to compute the radiation efficiency for modes of vibration is presented and the radiation efficiency of the first four most efficient vibration modes for six different boundary conditions is presented.

Keywords: plate vibration, normal modes, boundary conditions, coupling matrix, sound power radiated by plates, Rayleigh integral, radiation efficiency, shape functions, noise control, vibration control.

1. Introduction

The control of noise and vibration is of concern in industrial machines, appliances and different forms of transportation vehicles [1–8]. In many cases the machines or vehicles have machine enclosures, vehicle cabin enclosures or other structural systems comprised of beams, panels or plates which have geometries approximating those of circular or rectangular shapes [9–14]. Thus the radiation of sound from circular and rectangular plates and its suppression is of interest in several practical problems. The sound radiation is sometimes suppressed by damping the plate vibration through passive or active damping measures, once the plate system parameters have been properly identified [15–18]. Alternatively attempts can also be made to suppress the vibration and sound radiation through a thorough understanding of the vibration and its optimized control through approaches such as statistical energy analysis [19].

The radiation of the sound from a plate can be determined from knowledge of the plate vibration and its radiation properties, usually known as radiation resistance, impedance or efficiency. Many papers have been published in the last 50 years concerning these problems. Vogel and Skinner and Leissa published well known early papers on plate vibration [20–22]. Pritchard, Greenspon and Sherman, Thompson, Stephanishen and others published early work on the radiation of sound from pistons [23–26]. More recently, papers have been published by Rdzanek and others on these topics [27–44]. In the present paper we study the sound power radiated from rectangular vibrating plates based on the work of Wallace, Cunefare, Curry and Cunefare, Nikiforov, Gomperts, Magrab and Clark and Fuller [45–51]. The motivation behind this study is the reduction of the noise radiated by plates.

If the sound power radiated by a plate can be represented in terms of the normal velocity distribution on the plate, a way can be found to drive the plate to a state (specific normal velocity distribution) to minimize the sound power radiated. In this paper, the sound power radiated is represented in terms of the normal velocity distribution of the plate and the radiation efficiency is determined for arbitrary boundary conditions.

The calculation of the sound power and radiation efficiency for simply supported beams and plates was discussed by Wallace [45], Cunefare [46] and Currey and Cunefare [47]. Currey and Cunefare have shown that the sound power radiated from a plate can be represented in terms of the modal amplitudes and a coupling matrix. The eigenvalue (of the coupling matrix) associated with a given radiation mode is directly proportional to the radiation efficiency of that radiation mode.

The calculation of the sound power and the radiation efficiency for a plate with arbitrary boundary conditions becomes very complicated. Nikiforov has shown that the radiation resistance of a clamped plate at low frequency is twice that of a simply-supported plate [48]. Gomperts has presented an approach for the determination of the radiation efficiencies of baffled, thin, rectangular plates carrying two-dimensional resonant vibration patterns for five different kinds of boundary conditions [49]. In the present paper, Cunefare's equation for a plate with simply supported boundary conditions is extended to six different boundary conditions. The sound power radiated from a plate with arbitrary boundary conditions can be represented in terms of the normal velocity distribution on the plate and a coupling matrix. The velocity distribution on the plate is expressed in terms of the modal amplitudes and normal modes. The key issue is to find the coupling matrix for the boundary conditions of interest. In this paper, a method to calculate the coupling matrix for six boundary conditions is described. The method used to calculate the sound power radiated from a plate and the radiation efficiency of the plate is based on the coupling matrix.

2. Velocity distribution

Currey and Cunefare assumed that any arbitrary normal velocity distribution on the surface of a baffled and finite rectangular plate of length L_x and width L_y, as shown in Fig. 1, vibrating at a single frequency, may be represented by a series expansion [47]. Omitting the simple harmonic time dependence and higher-order terms, the normal velocity distribution can be written as

$$u(x,y) = u^{\mathrm{T}}\phi, \tag{1}$$

where

$$u^{\mathrm{T}} = [A_{11} \quad A_{12} \ \dots \ A_{1N} \quad A_{21} \quad A_{22} \ \dots \ A_{2N} \ \dots \ A_{M1} \quad A_{M2} \ \dots \ A_{MN}], \tag{2}$$

where u^{T} represents a vector of modal amplitude coefficients, and ϕ is the vector of corresponding basis functions. The only requirement on the selection of the basis functions is that they be complete. One set of basis functions used by Currey and Cunefare is [47],

$$\begin{aligned} \phi^{\mathrm{T}} = [&\sin(k_{x_1}x)\sin(k_{y_1}y) \ \dots \ \sin(k_{x_1}x)\sin(k_{y_N}y) \\ &\sin(k_{x_2}x)\sin(k_{y_1}y) \ \dots \ \sin(k_{x_2}x)\sin(k_{y_N}y) \\ &\dots\dots \\ &\sin(k_{x_M}x)\sin(k_{y_1}y) \ \dots \ \sin(k_{x_M}x)\sin(k_{y_N}y)], \end{aligned} \tag{3}$$

where $k_{x_m} = m\pi/L_x$ may be interpreted as the structural wavenumber of the *mth* basis function in the x direction and $k_{y_n} = n\pi/L_y$ as the structural wavenumber of the *nth* basis function in the y direction. The vectors u and ϕ are of length *MN*, where M and N denote the indices for the highest-order basis function in each dimension included in the summation. Although Currey and Cunefare emphasize that this analysis is completely independent of the physical characteristics and boundary conditions of a physical plate, and only depends on the

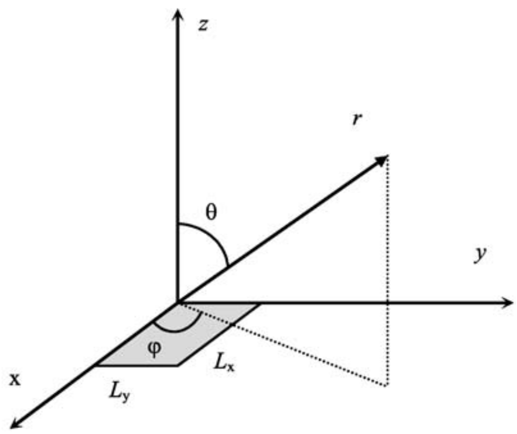

Fig. 1. Rectangular plate in an infinite baffle.

geometry of its "wetted" surface, they did not show how the analysis can be used for plates with other boundary conditions, and they did not discuss how to choose the basis functions for other boundary conditions [47].

It is well known that the normal velocity distribution on a plate with some specific boundary conditions can be expressed as the summation of the products of modal amplitudes and normal modes [50]. If the vector u represents the modal amplitude coefficients as defined in Eq. (2) and the vector ϕ is defined as a vector of normal modes, not basis functions, then Eq. (1) can be used to represent the normal velocity distribution of a plate with arbitrary boundary conditions. The vector ϕ is defined as:

$$\phi^{\mathrm{T}} = [W_{11} \;\; \ldots \;\; W_{1N} \;\; W_{21} \;\ldots\; W_{2N} \quad \ldots\ldots \quad W_{M1} \;\;\ldots\;\; W_{MN}], \tag{4}$$

where W_{mn} are the normal modes.

Equation (4) reduces to Eq. (3) for the case of a simply supported plate.

3. Sound pressure distribution

Using the Rayleigh integral, the sound pressure at a point in the far field is given in terms of the surface velocity,

$$p = -ik\rho\, c\frac{e^{ikr}}{2\pi\, r}\int_0^{L_x}\int_0^{L_y} u(x,y)\exp\left(-i\left(\alpha\left(\frac{x}{L_x}\right)+\beta\,\left(\frac{y}{L_y}\right)\right)\right)\mathrm{d}x\,\mathrm{d}y, \tag{5}$$

where

$$\alpha = kl\sin(\theta)\cos(\varphi), \tag{6}$$

$$\beta = kw\sin(\theta)\sin(\varphi), \tag{7}$$

and

$$\text{acoustic wavenumber} \qquad k = \frac{\omega}{c}. \tag{8}$$

Substituting Eqs. (1), (2) and (4) into Eq. (5) yields

$$p = -i\frac{\rho c}{\pi}u^{\mathrm{T}}b, \tag{9}$$

where

$$b = \frac{ke^{ikr}}{2r}\int_0^{L_x}\int_0^{L_y}\phi\exp\left(-i\left(\alpha\left(\frac{x}{L_x}\right)+\beta\,\left(\frac{y}{L_y}\right)\right)\right)\mathrm{d}x\,\mathrm{d}y, \tag{10}$$

and

$$b_{mn} = \frac{ke^{ikr}}{2r}\int_0^{L_x}\int_0^{L_y}W_{mn}\exp\left(-i\left(\alpha\left(\frac{x}{L_x}\right)+\beta\,\left(\frac{y}{L_y}\right)\right)\right)\mathrm{d}x\,\mathrm{d}y. \tag{11}$$

For a simply supported plate, Eq. (11) reduces to Cunefare's equation

$$b_{mn} = -\frac{k}{k_{xm}k_{yn}}\frac{e^{ikr}}{2r}\left(\frac{1-(-1)^m e^{-i\alpha}}{1-(\alpha/m\pi)^2}\right)\left(\frac{1-(-1)^n e^{-i\beta}}{1-(\beta/n\pi)^2}\right). \tag{12}$$

4. Radiated sound power

For a baffled finite flat plate, the total sound power radiated from the plate can be obtained by integrating the sound intensity over a hemisphere in the far field,

$$W = \int_0^{2\pi}\int_0^{\pi/2} \frac{|p|^2}{2\rho c} r^2 \sin(\theta)\,\mathrm{d}\theta\,\mathrm{d}\varphi, \tag{13}$$

where p is the sound pressure, r is the distance, ρ is the density of air, and c is the speed of sound.

Substituting Eq. (9) into Eq. (13) yields

$$W = \rho c u^{\mathrm{T}} C' u^*, \tag{14}$$

where

$$C' = \frac{1}{2\pi^2}\int_0^{2\pi}\int_0^{\pi/2} bb^H r^2 \sin(\theta)\,\mathrm{d}\theta\,\mathrm{d}\varphi, \tag{15}$$

and the θ, φ and r are shown in Fig. 1.

The coupling matrix C may be defined to be

$$C = \rho c C'. \tag{16}$$

Then the sound power radiated from the vibrating plate may be represented as

$$W = u^{\mathrm{T}} C u^*. \tag{17}$$

The coupling matrix is dependent on the frequency of excitation and the characteristics of the vibrating plate. In an active control system, with PZT actuators applied on a plate, the velocity vector u is dependent on the vibrating noise source and the placement of the actuators, assuming that the other features of the actuators, such as their width, length, and thickness and the piezoelectric strain constants are fixed. If the exciting frequency and the characteristics and boundary conditions of the vibrating plate are known, the optimal placement of the actuators to minimize the sound power radiated from the vibrating plate can be found.

It can be seen from Eq. (13) that W is always real and greater than zero so that the coupling matrix is real and positive definite. It is very easy to prove that the coupling matrix is symmetric (it is obvious from Eq. (15)). Therefore, the coupling matrix C can be diagonalized.

5. Radiation efficiencies

If the radiation efficiencies for the vibration modes of concern are known, more effort can be spent to control the efficient radiating modes than the inefficient ones in active noise control studies. Also, knowledge of the radiation efficiencies for a range of the vibration modes can help with the analysis of noise control problems in which structural radiation of noise is important. Normally, it is difficult to reduce the amplitudes of all of the structural modes simultaneously. In many cases, a spillover problem will be encountered. When the amplitudes of some of the modes are reduced, the amplitudes of some of the other modes will be increased at the same time. The amplitudes of the vibration modes can be plotted for the control system on and off. If the amplitudes of the modes with high radiation efficiencies decrease and the amplitudes of the modes with low radiation efficiencies increase, the control system can be said to be really working.

In this section, the radiation efficiencies of the first four most efficient vibration modes are plotted for six boundary conditions. The abbreviations which are used in Magrab's study [50] are followed here: SS = simply supported edge; F = free edge; C = clamped edge and ES = elastically supported edge. Then the abbreviation SS-C-SS-F, for example, identifies a plate that is simply supported, clamped, simply supported and free along each of the four edges, respectively. The following dimensions and properties are assumed for the plate: 1) the width (L_x) and length (L_y) of the plate are equal to 1 m, 2) the thickness of the plate is 0.002 m, 3) the plate is made of steel, 4) the Young's modulus is 19.5×10^{10} Pa, and 5) Poisson's ratio is 0.3.

The general expression for the radiation efficiency σ of an acoustic radiator is

$$\sigma = \frac{W}{\rho\, c L_x L_y \langle |u(x,y)|^2 \rangle}, \tag{18}$$

where $\langle |u(x,y)|^2 \rangle$ is the spatial mean-square velocity and W is the sound power radiated from the plate. To calculate the first four most efficient vibration modes, it is sufficient to use a 4 by 4 coupling matrix C. To obtain the radiation efficiencies of the odd-odd, odd-even, even-odd and even-even modes, set the vector $u^{\mathrm{T}} = [1,\ 0,\ 0,\ 0]$, $[0,\ 1,\ 0,\ 0]$, $[0,\ 0,\ 1,\ 0]$ and $[0,\ 0,\ 0,\ 1]$ respectively. The spatial mean square velocity for individual vibration modes is also required for the computation of the radiation efficiency of the individual modes.

5.1. SS-SS-SS-SS

For this special SS-SS-SS-SS plate case, which represents a plate simply supported on all four edges, the calculation of the radiation efficiencies of the first four most efficient radiating modes is discussed by Wallace [45], Currey and

Cunefare [47] and Clark and Fuller [51]. The radiation efficiencies for the SS-SS-SS-SS case for the first four most efficient radiating modes are shown in Fig. 2.

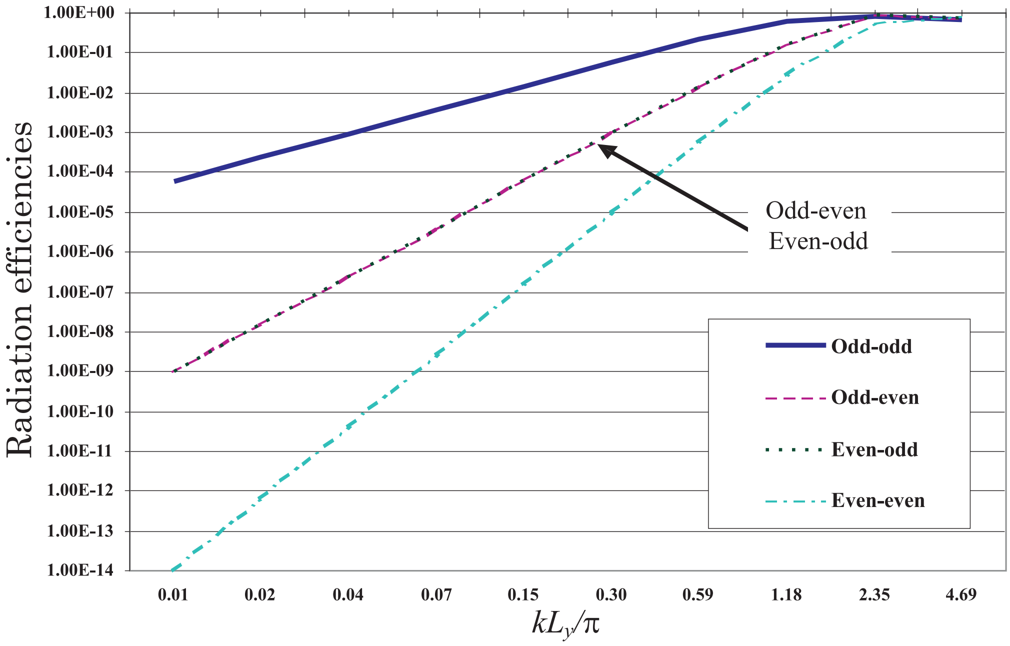

Fig. 2. Radiation efficiencies of the first four most efficient radiating modes of a simply-supported plate for increasing frequency with $L_x/L_y = 1$.

Note that in Fig. 2 and the rest of the figures in this paper, "odd-odd" means an odd mode in the x axis and an odd mode in the y axis direction, "odd-even" means an odd mode in the x axis and an even mode in the y axis direction, "even-odd" means an even mode in the x axis and an odd mode in the y axis direction, and "even-even" means an even mode in x axis and an even mode in y axis direction.

5.2. SS-C-SS-C

The radiation efficiencies for the SS-C-SS-C plate case (a plate simply supported on two opposite edges and clamped on the other two opposite edges) for the first four most efficient radiation modes are shown in Fig. 3. It can be seen that when kL_y/π is less than two, the radiation efficiency of the mode (1, 1) is a little smaller than in the SS-SS-SS-SS case. However, when kL_y/π is greater than two, the radiation efficiency of the mode (1, 1) is a little larger than that in the SS-SS-SS-SS case. The radiation efficiency of the mode (1, 2) is a little larger than that in the SS-SS-SS-SS case when kL_y/π is greater than four. The radiation efficiency of the mode (2, 1) is a little smaller than in the SS-SS-SS-SS case. The radiation efficiency of the mode (2, 2) is larger than in the SS-SS-SS-SS case when kL_y/π is greater than four.

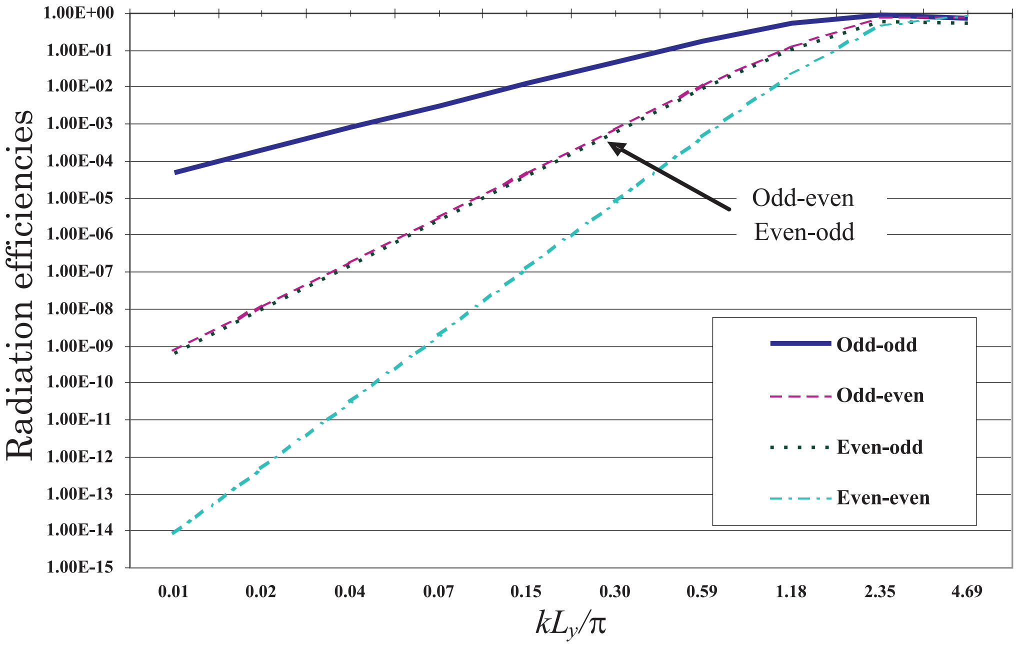

Fig. 3. Radiation efficiencies of the first four most efficient radiating modes for SS-C-SS-C boundary condition with $L_x/L_y = 1$, where L_x is the width of the plate and L_y is the length of the plate. The kL_y/π in the diagram represents the nondimensionalization of the wavenumber, $k = \omega/c$.

5.3. SS-C-SS-SS

The radiation efficiencies of the first four most efficient radiating modes of a SS-C-SS-SS plate (a plate simply supported on three edges and clamped on the other edge) are shown in Fig. 4. Comparing this figure with Figs. 2 and 3, it is seen that when kL_y/π is small, the radiation efficiency of the mode (1, 1) is a little smaller than in the SS-SS-SS-SS case and it is larger than in the SS-C-SS-C case. However, when kL_y/π is large, the radiation efficiency of the mode (1, 1) is a little larger than in the SS-SS-SS-SS case and it is smaller than in the SS-C-SS-C case. The radiation efficiency of mode (1, 2) is larger than in the SS-SS-SS-SS case or the SS-C-SS-C case when kL_y/π is small. It is seen that plates with greater boundary constraints do not always have larger radiation efficiencies than plates that have smaller boundary constraints.

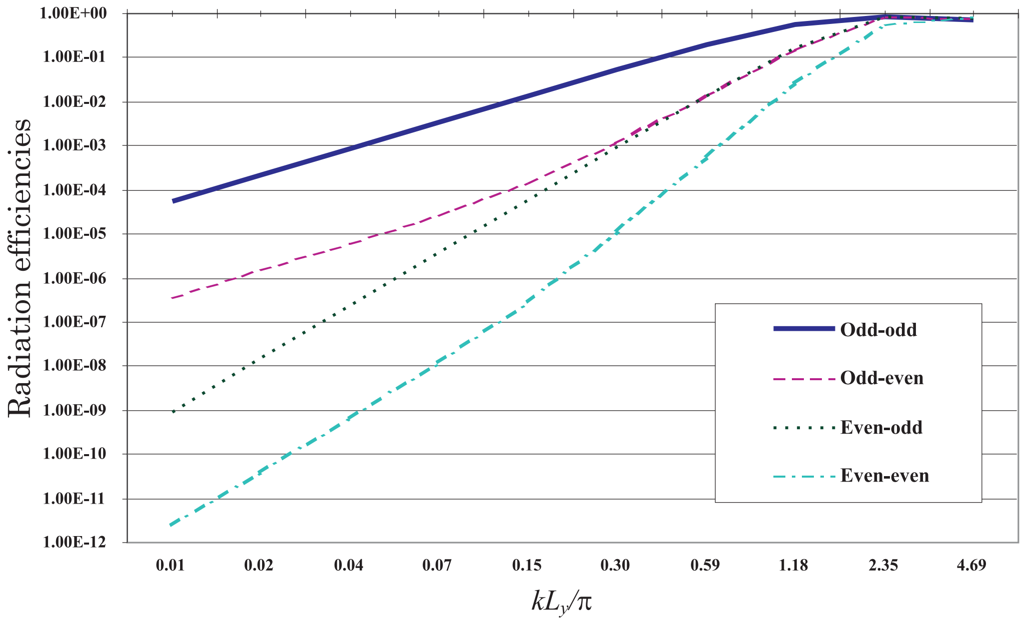

Fig. 4. Radiation efficiencies of the first four most efficient radiating modes for SS-C-SS-SS boundary condition with $L_x/L_y = 1$.

5.4. SS-C-SS-F

The radiation efficiencies of the first four most efficient radiating modes of the SS-C-SS-F plate (a plate simply supported on two opposite edges and clamped on one of the other edges and free on remaining edge) are shown in Fig. 5. The radiation efficiency of the mode (1, 1) is higher than in SS-C-SS-SS case. The radiation efficiencies of the mode (1, 2) is lower than in the SS-C-SS-SS case when kL_y/π is less than 0.03. The radiation efficiencies of the mode (2, 1) is greater than in the SS-C-SS-SS case when kL_y/π is less than one.

5.5. SS-SS-SS-F

The radiation efficiencies of the first four most efficient radiating modes of the SS-SS-SS-F plate (a plate simply supported on three edges and free on the other edge) are shown in Fig. 6. Comparing this figure with Fig. 5, it can be seen how the radiation efficiencies change when a clamped edge is replaced with a simply supported edge.

In this case, the radiation efficiency of the mode (1, 1) is smaller than in the SS-C-SS-F case. The radiation efficiency of the mode (1, 2) increases dramatically and reaches that of the most efficient radiating mode (1, 1). Also the radiation efficiency of the mode (2, 2) reaches that of the more efficient radiating mode (2, 1).

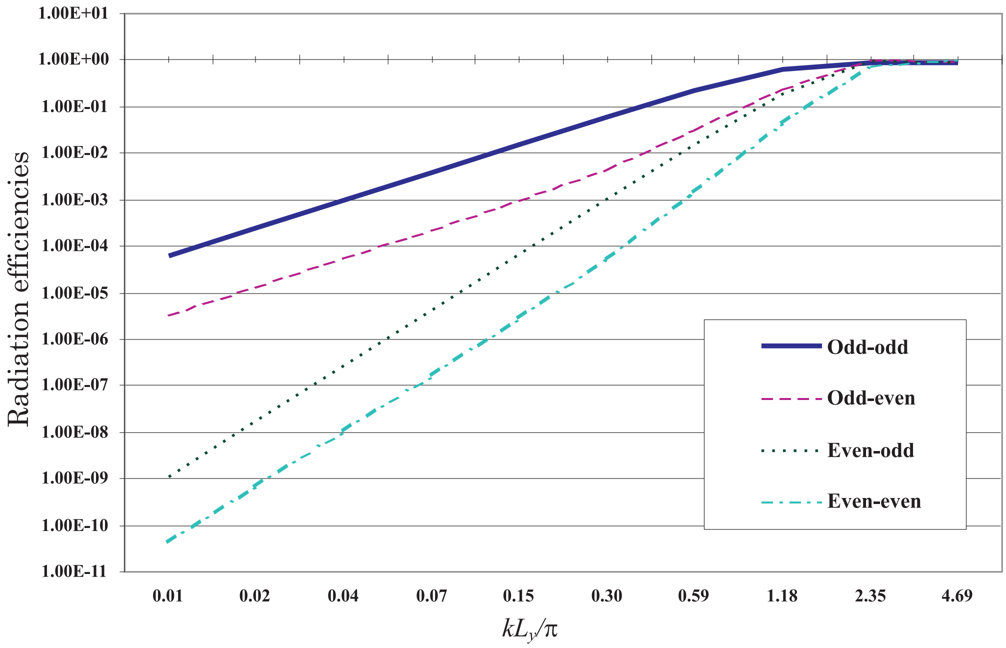

Fig. 5. Radiation efficiencies of the first four most efficient radiating modes for SS-C-SS-F boundary condition with $L_x/L_y = 1$.

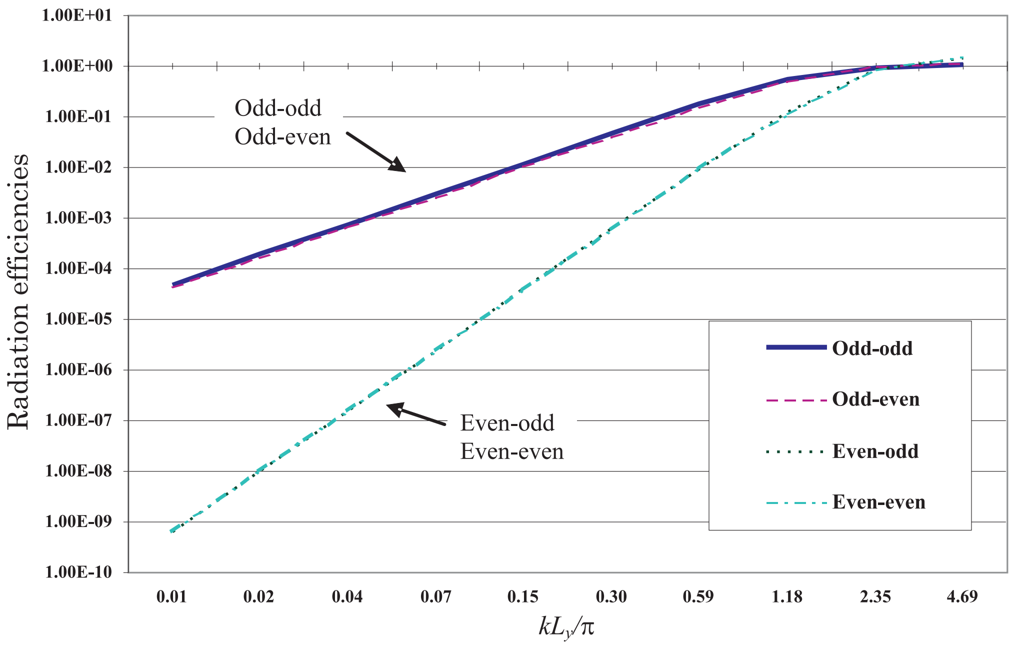

Fig. 6. Radiation efficiencies of the first four most efficient radiating modes for the SS-SS-SS-F boundary condition with $L_x/L_y = 1$.

5.6. *SS-F-SS-F*

The radiation efficiencies of the first four efficient radiating modes of the SS-F-SS-F plate (a plate simply supported on two opposite edges and free on the other two opposite edges) are shown in Fig. 7. Comparing this figure with Fig. 6, it can be seen that when kL_y/π is greater than 0.15, the radiation efficiency of the mode (1, 1) is smaller than in the SS-SS-SS-F case. The radiation efficiencies of mode (1, 2) and mode (2, 2) are smaller than those in the SS-SS-SS-F case.

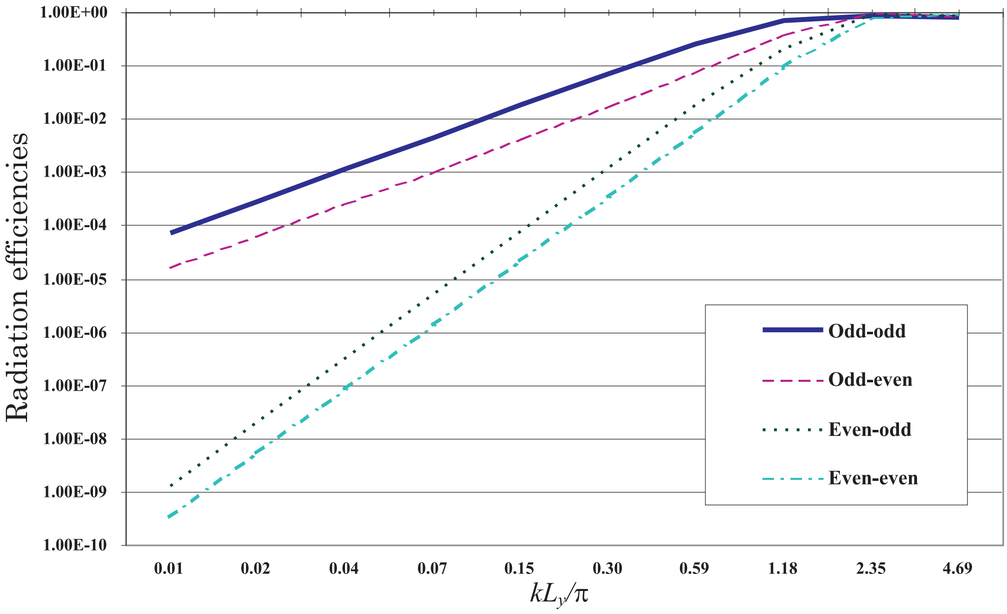

Fig. 7. Radiation efficiencies of the first four most efficient radiating modes for SS-F-SS-F boundary condition with $L_x/L_y = 1$.

6. Conclusions

Cunefare's equation has been extended to several new boundary conditions. The sound power radiated from a vibrating plate can be expressed in terms of the modal amplitudes and the coupling matrix. The computation of the coupling matrix is presented in this paper.

To help evaluate the usefulness of noise control measures, the radiation efficiencies have been plotted for the first four most efficient modes for the six boundary conditions of a vibrating plate. From these diagrams, it is evident why the active noise control system works well only in the low frequency range. At low frequency, only a few modes will radiate sound power very efficiently and these efficient modes can be controlled easily. At high frequency, all of the plate modes, especially the higher-order modes, radiate sound power very efficiently and it is very hard to control all of the modes at once.

Knowledge of the expression for the sound power radiated (Eq. (17)) in terms of modal amplitudes and the coupling matrix is useful in active noise control studies. It should be noted that the expression is approximate because the higher order terms are truncated. If it is desired to determine the sound power radiated more precisely, higher-order modes must be included, and then the dimension of the coupling matrix will increase. As the dimension of the coupling matrix increases, the computation effort to obtain the coupling matrix will increase dramatically.

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