Response of a Plate to PZT Actuators

Dingguo ZOU, Malcolm J. CROCKER

Auburn University
Mechanical Engineering Department
Auburn, AL 36849, USA
e-mail: jameszou@live.com, mcrocker@eng.auburn.edu

In this paper, the response of a plate with arbitrary boundary conditions to PZT actuators is derived. It is assumed that the plate and the actuators are rectangular and the edges of the PZT actuators are parallel to the respective edges of the plate. The response of the plate is decomposed into normal modes. The modal amplitude of the normal mode is represented in terms of the shape function of the actuator and the normal mode. The shape function of the actuator is given as a singularity function. The normal modes for the boundary conditions with which we are concerned are calculated based on theoretical analyses of Magrab. The results of this paper are useful in designing an active noise control system in which the PZT actuators are used as the control sources.

Keywords: piezoelectric actuators, plate vibration, normal modes, singularity functions, noise control.

1. Introduction

Noise and vibration are of concern with many mechanical systems including industrial machines, home appliances, transportation vehicles, and building structures [1–3]. Many such structures are comprised of beam and plate like elements. The vibration of beam and plate systems can be reduced by the use of passive damping, once the system parameters have been identified [3–7]. In some cases of forced vibration, the passive damping that can be provided is insufficient and the use of active damping has become attractive. The rapid development of micro-processors and control algorithms has made the use of active control feasible in many practical situations [8]. The field of active control is now of considerable interest to researchers, for example in ships, rotating systems, industrial machinery and flexible structures [9–13].

In vibration problems, the vibration must be sensed and control forces must be applied with actuators using suitable control algorithms [14–18]. One possibility is to use piezoelectric materials for the sensors and actuators needed. PZT actuators and PVDF sensors have been shown to have potential to reduce the sound power radiated from vibrating structures. It has been shown by Jones and Fuller that if the primary source is structural vibration, structural control sources offer better control than acoustic control sources [19]. This is because the secondary structural sources are more likely to produce a sound field that matches the primary sound field. The distributed actuators and sensors can reduce spillover into higher order modes. Also the PZT actuators and PVDF sensors can be embedded into the vibrating structure and the control system can be made very compact. Therefore, PZT actuators and PVDF sensors are of great interest in active noise control.

Most of the previous research studies have concentrated on one-dimensional vibration excitation and control problems. CRAWLEY and LUIS analyzed the stresses, strains and loads generated on a cantilevered beam when piezoelectric segments were bonded symmetrically to both sides [20]. They reported that the effective moments resulting from the piezo-actuators can be seen as concentrated on the two ends of the actuator. BAILEY and HUBBARD developed an equation for the response of a cantilevered beam with a layer of PVDF bonded to one complete side of the beam [21]. CLARK *et al.* analytically and experimentally studied the response of a simply supported beam driven by multiple piezoelectric actuators [22].

Two-dimensional excitation and control problems were investigated by DIMITRIADIS and FULLER [23] and LEE and MOON [24]. Dimitriadis and Fuller derived a dynamic analysis model for an undamped thin rectangular plate with simply supported boundary conditions excited by a rectangular piezoelectric patch. Lee and Moon presented an important equation to model the effect of actuators.

Because a PZT ceramic actuator can generate a stronger force or moment than a PVDF actuator, usually actuators are made from PZT ceramic material. However, PZT ceramic material is very fragile, and it is hard to cut it into complicated shapes. In practice, rectangular PZT actuators are used extensively in active noise control applications. The dynamic analysis model for a vibrating plate excited by rectangular actuators is of considerable importance.

In this paper, DIMITRIADIS and FULLER'S dynamic analysis model [23] is extended to other boundary conditions.

2. Mathematical preliminaries

To help readers understand the following derivation, the properties of singularity functions (generalized functions) are discussed first.

The family of singularity functions can be written as [25]

$$f_n = \langle x - a \rangle^n = \left\{ \begin{array}{ll} 0 & \text{if } x < a \\ (x-a)^n & \text{if } x \geq a \end{array} \right\}. \tag{1}$$

The function f_n is defined to have a value only when the argument is positive. Singularity functions obey the laws of integration.

$$\int_{-\infty}^{x} \langle x - a \rangle^n \, \mathrm{d}x = \frac{\langle x - a \rangle^{n+1}}{n+1}, \quad n \geq 0. \tag{2}$$

The functions $\langle x - a \rangle^{-1}$ and $\langle x - a \rangle^{-2}$ are exceptions. They are equal to zero everywhere except when x equals a, where they are infinite, so that Eqs. (1) and (2) are valid.

Table 1 presents a list of singularity functions of different degrees and their common properties as given by Burke *et al.* after some corrections [25].

Table 1. Singularity functions and their properties.

Names	Definition and integration property	Graphical representation
Doublet Dipole Concentrated Moment function	$\langle x - a \rangle^{-2} = 0 \quad \text{if} \quad x \neq a$ $\int_{-\infty}^{x} \langle x - a \rangle^{-2} \, \mathrm{d}x = \langle x - a \rangle^{-1}$	
Dirac delta Delta function Impulse Concentrated force function	$\langle x - a \rangle^{-1} = 0 \quad \text{if} \quad x \neq a$ $\int_{-\infty}^{x} \langle x - a \rangle^{-1} \, \mathrm{d}x = \langle x - a \rangle^{0}$	
Unit step function	$\langle x - a \rangle^{0} = \begin{cases} 0 & \text{if} \quad x < a \\ 1 & \text{if} \quad x \geq a \end{cases}$ $\int_{-\infty}^{x} \langle x - a \rangle^{0} \, \mathrm{d}x = \langle x - a \rangle^{1}$	
Ramp function	$\langle x - a \rangle^{1} = \begin{cases} 0 & \text{if} \quad x < a \\ x - a & \text{if} \quad x \geq a \end{cases}$ $\int_{-\infty}^{x} \langle x - a \rangle^{1} \, \mathrm{d}x = \frac{\langle x - a \rangle^{2}}{2}$	
General Macauley notation	$\langle x - a \rangle^{n} = \begin{cases} 0 & \text{if} \quad x < a \\ (x - a)^{n} & \text{if} \quad x \geq a \end{cases}$ $\int_{-\infty}^{x} \langle x - a \rangle^{n} \, \mathrm{d}x = \frac{\langle x - a \rangle^{n+1}}{n+1}$	

Assume that a shape function is defined as

$$F_1 = \left(\langle x - x_1 - \Delta \rangle^0 - \langle x - x_2 + \Delta \rangle^0 \right). \tag{3}$$

The parameter Δ is an infinitesimally small quantity that vanishes in the limit. This means that $F_1(x)$ is identically equal to zero at the system boundaries. A continuous function is given as $R_1(x)$. Then the following equation is valid.

$$\int_0^L R_1(x) F_1''(x)\, \mathrm{d}x = \int_{x_1}^{x_2} \frac{\partial^2 R_1(x)}{\partial x^2}\, \mathrm{d}x, \tag{4}$$

where $F_1''(x)$ is the second derivative of $F_1(x)$ with respect to x.

3. Differential equation of motion of a plate with piezo-actuator patches

Using classical thin plate theory [25, 26], the equation of motion of the plate with piezoelectric patches can be written as

$$D\nabla^4 \eta + \rho\, \ddot{\eta} = \frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_y}{\partial y^2}, \tag{5}$$

where M_x and M_y are the effective bending moments applied to the plate by the piezo-actuators, D is the plate flexural rigidity and η is the plate transverse displacement.

Equation (5) can be developed further as [24, 25]

$$D\left(\frac{\partial^4 \eta}{\partial x^4} + \frac{\partial^4 \eta}{\partial^2 x \partial^2 y} + \frac{\partial^4 \eta}{\partial y^4} \right) + \rho\frac{\partial^2 \eta}{\partial t^2}$$

$$= G(t) h_p \bar{Z}_p \left(e_{31}\frac{\partial}{\partial x^2}(FP_0) + e_{32}\frac{\partial}{\partial y^2}(FP_0) \right), \tag{6}$$

where the terms e_{31} and e_{32} can be considered as the electric constants with respect to the x and y axes, $G(t)$ is the time signal of the applied electric field, h_p is the thickness of the PZT lamina and $\bar{Z}_p$ is the moment arm of PZT. The polarization profile $P_0 = P_0(x, y)$ is introduced to model the effect that PVDF or PZT are ferroelectric, which means that the piezoelectric strength can be changed or reversed by poling. In our case, P_0 is set equal to one. The $F(x, y)$ is equal to one if (x, y) is covered by an electrode on both sides of the lamina. Otherwise $F(x, y)$ is zero.

The panel and the PZT ceramic patch mounted on it are shown in Fig. 1. The shape function F of the PZT ceramic patch for this configuration is given as

$$F = \left(\langle x - x_1 \rangle^0 - \langle x - x_2 \rangle^0 \right)\left(\langle y - y_1 \rangle^0 - \langle y - y_2 \rangle^0 \right). \tag{7}$$
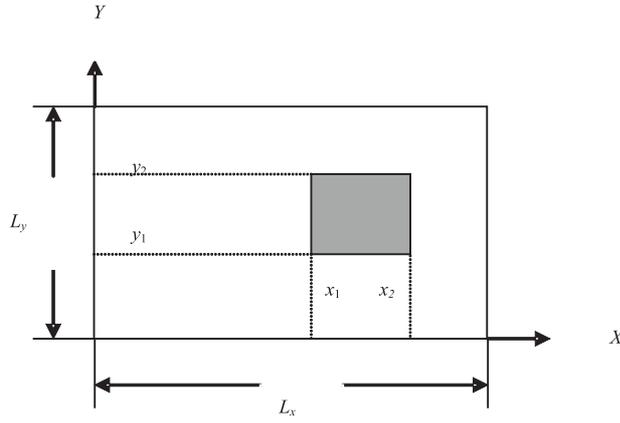
Fig. 1. Plate and piezoelectric actuator.

Here the MACAULEY notation has been used to represent the generalized functions [28]. From the properties of the singularity functions above, the second derivatives of $F(x, y)$ with respect to x and y can be developed as

$$\frac{\partial F^2(x, y)}{\partial x^2} = \left(\langle x - x_1 \rangle^{-2} - \langle x - x_2 \rangle^{-2}\right)\left(\langle y - y_1 \rangle^0 - \langle y - y_2 \rangle^0\right), \qquad (8)$$

and

$$\frac{\partial F^2(x, y)}{\partial y^2} = \left(\langle x - x_1 \rangle^0 - \langle x - x_2 \rangle^0\right)\left(\langle y - y_1 \rangle^{-2} - \langle y - y_2 \rangle^{-2}\right). \qquad (9)$$

To make the derivation clear, the right hand side of Eq. (6) is defined as

$$PZTR(x, y, t) = G(t)h_p\overline{Z}_p\left(e_{31}\frac{\partial}{\partial x^2}(FP_0) + e_{32}\frac{\partial}{\partial y^2}(FP_0)\right). \qquad (10)$$

Assuming $G(t) = G_0 e^{j\omega t}$, and substituting Eqs. (8) and (9) into Eq. (10) gives

$$PZTR(x, y, t) = G_0 e^{j\omega t} h_p\overline{Z}_p\left(e_{31}\left(\langle x - x_1 \rangle^{-2} - \langle x - x_2 \rangle^{-2}\right)\left(\langle y - y_1 \rangle^0 - \langle y - y_2 \rangle^0\right)\right.$$
$$\left. + e_{32}\left(\langle x - x_1 \rangle^0 - \langle x - x_2 \rangle^0\right)\left(\langle y - y_1 \rangle^{-2} - \langle y - y_2 \rangle^{-2}\right)\right)$$
$$= PZTR(x, y)e^{j\omega t}, \qquad (11)$$

where

$$PZTR(x, y) = G_0 h_p\overline{Z}_p\left(e_{31}\left(\langle x - x_1 \rangle^{-2} - \langle x - x_2 \rangle^{-2}\right)\left(\langle y - y_1 \rangle^0 - \langle y - y_2 \rangle^0\right)\right.$$
$$\left. + e_{32}\left(\langle x - x_1 \rangle^0 - \langle x - x_2 \rangle^0\right)\left(\langle y - y_1 \rangle^{-2} - \langle y - y_2 \rangle^{-2}\right)\right). \qquad (12)$$

4. General solution of the differential equation

The solution of Eq. (6) can be written as

$$w(x,y,t) = \frac{e^{j\omega t}}{\rho} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} A_{mn} W_{mn}(x,y), \tag{13}$$

where

$$A_{mn} = \frac{\int_0^{L_x} \int_0^{L_y} PZTR(x,y) W_{mn}(x,y)\,\mathrm{d}x\,\mathrm{d}y}{\rho N_{mn}(\omega_{mn}^2 - \omega^2)}, \tag{14}$$

L_x is width of the plate, L_y is length of the plate, ω_{mn} is the natural frequency, N_{mn} is the norm, which is the integration of the square of the normal mode over the plate, W_{mn} is the normal mode, $PZTR(x,y)$ is defined in Eq. (12).

The numerator of A_{mn} can be developed further.

$$\int_0^{L_x} \int_0^{L_y} PZTR(x,y) W_{mn}(x,y)\,\mathrm{d}x\,\mathrm{d}y$$

$$= G_0 h_p \overline{Z}_p \Bigg(e_{31} \int_0^{L_x} \int_0^{L_y} \left(\langle x-x_1 \rangle^{-2} - \langle x-x_2 \rangle^{-2} \right) \left(\langle y-y_1 \rangle^0 - \langle y-y_2 \rangle^0 \right) W_{mn}(x,y)\,\mathrm{d}x\,\mathrm{d}y$$

$$+ e_{32} \int_0^{L_x} \int_0^{L_y} \left(\langle x-x_1 \rangle^0 - \langle x-x_2 \rangle^0 \right) \left(\langle y-y_1 \rangle^{-2} - \langle y-y_2 \rangle^{-2} \right) W_{mn}(x,y)\,\mathrm{d}x\,\mathrm{d}y \Bigg). \tag{15}$$

Apparently, to compute A_{mn}, it is necessary to find the normal mode $W_{mn}(x,y)$ for the boundary conditions with which we are concerned. Fortunately, we can calculate the normal mode $W_{mn}(x,y)$ based on Magrab's analysis for seven special cases [29]. Additional results for various other combinations of boundary conditions can be found in Ref. [30].

For the seven special cases given by Magrab, the edges $x = 0$ and $x = L_x$ are hinged (simply supported). The edges $y = 0$ and $y = L_y$ can be free edges, clamped edges, elastically supported edges, simply supported edges or some other combinations.

When $\Omega_{mn} > M$, where Ω_{mn} is the natural frequency coefficient, which is proportional to the square root of the natural frequency [29], and $M = m\pi L_y/L_x$, the corresponding normal modes are

$$W_{mn}(x,y) = \big(C_{1mn} \cos \mathrm{h}(\delta_{mn}y/L_y) + C_{2mn} \sin \mathrm{h}(\delta_{mn}y/L_y)$$

$$+ C_{3mn} \cos(\varepsilon_{mn}y/L_y) + \sin(\varepsilon_{mn}y/L_y) \big) \sin(m\pi\, x/L_x). \tag{16}$$

Substituting Eq. (16) into (15) and applying Eq. (4) yields

$$
\int_0^{L_x} \int_0^{L_y} PZTR(x,y) W_{mn}(x,y) \,\mathrm{d}x\,\mathrm{d}y
$$

$$
= G_0 h_p \overline{Z}_p \left(e_{31} \int_{y_1}^{y_2} \left(C_{1mn} \cosh(\delta_{mn} y/L_y) + C_{2mn} \sinh(\delta_{mn} y/L_y) \right.\right.
$$

$$
\left. + C_{3mn} \cos(\varepsilon_{mn} y/L_y) + \sin(\varepsilon_{mn} y/L_y) \right) \mathrm{d}y
$$

$$
\int_{x_1}^{x_2} \left(\langle x - x_1 \rangle^{-2} - \langle x - x_2 \rangle^{-2} \right) \sin(m\pi x/L_x) \,\mathrm{d}x
$$

$$
+ e_{32} \int_{x_1}^{x_2} \sin(m\pi x/L_x) \,\mathrm{d}x \int_{y_1}^{y_2} \left(\langle y - y_1 \rangle^{-2} - \langle y - y_2 \rangle^{-2} \right) \left(C_{1mn} \cosh(\delta_{mn} y/L_y) \right.
$$

$$
\left.\left. + C_{2mn} \sinh(\delta_{mn} y/L_y) + C_{3mn} \cos(\varepsilon_{mn} y/L_y) + \sin(\varepsilon_{mn} y/L_y) \right) \mathrm{d}y \right), \quad (17)
$$

$$
= G_0 h_p \overline{Z}_p \left(e_{31} \left(C_{1mn} \frac{L_y}{\delta_{mn}} \left(\sinh\left(\delta_{mn} \frac{y_2}{L_y} \right) - \sinh\left(\delta_{mn} \frac{y_1}{L_y} \right) \right) \right.\right.
$$

$$
+ C_{2mn} \frac{L_y}{\delta_{mn}} \left(\cosh\left(\delta_{mn} \frac{y_2}{L_y} \right) - \cosh\left(\delta_{mn} \frac{y_1}{L_y} \right) \right)
$$

$$
+ C_{3mn} \frac{L_y}{\varepsilon_{mn}} \left(\sin\left(\varepsilon_{mn} \frac{y_2}{L_y} \right) - \sin\left(\varepsilon_{mn} \frac{y_1}{L_y} \right) \right)
$$

$$
\left. - \frac{L_y}{\varepsilon_{mn}} \left(\cos\left(\frac{\varepsilon_{mn} y_2}{L_y} \right) - \cos\left(\frac{\varepsilon_{mn} y_1}{L_y} \right) \right) \right)
$$

$$
\left(\frac{m\pi}{L_x} \right) \left(\cos\left(m\pi x_2/L_x \right) - \cos\left(m\pi x_1/L_x \right) \right)
$$

$$
- e_{32} \left(\frac{L_x}{m\pi} \right) \left(\cos\left(m\pi x_2/L_x \right) - \cos\left(m\pi x_1/L_x \right) \right)
$$

$$
\left(C_{1mn} \frac{\delta_{mn}}{L_y} \left(\sinh\left(\delta_{mn} \frac{y_2}{L_y} \right) - \sinh\left(\delta_{mn} \frac{y_1}{L_y} \right) \right) \right.
$$

$$
+ C_{2mn} \frac{\delta_{mn}}{L_y} \left(\cosh\left(\delta_{mn} \frac{y_2}{L_y} \right) - \cosh\left(\delta_{mn} \frac{y_1}{L_y} \right) \right)
$$

$$
- C_{3mn} \frac{\varepsilon_{mn}}{L_y} \left(\sin\left(\varepsilon_{mn} \frac{y_2}{L_y} \right) - \sin\left(\varepsilon_{mn} \frac{y_1}{L_y} \right) \right)
$$

$$
\left.\left.\left. + \frac{\varepsilon_{mn}}{L_y} \left(\cos\left(\frac{\varepsilon_{mn} y_2}{L_y} \right) - \cos\left(\frac{\varepsilon_{mn} y_1}{L_y} \right) \right) \right) \right) \right). \quad (18)
$$

The norm (N_{mn}), the natural frequency (ω_{mn}), C_{1mn}, C_{2mn}, C_{3mn}, ε_{mn} and δ_{mn} for seven boundary conditions have been discussed by MAGRAB [29]. Thus the modal amplitude of the mode (m, n) caused by the rectangular piezoelectric actuator can be obtained accordingly.

When $\Omega_{mn} < M$, the corresponding normal modes are

$$
\begin{aligned}
W_{mn}(x, y) = ((&C'_{1mn}\cosh(\delta_{mn}y/L_y) + C'_{2mn}\sinh(\delta_{mn}y/L_y) \\
&+ C'_{3mn}\cosh(\varepsilon'_{mn}y/L_y) + \sinh(\varepsilon'_{mn}y/L_y))\sin(m\pi x/L_x),
\end{aligned} \tag{19}
$$

Substituting Eq. (19) into (15) and applying Eq. (4) yields

$$
\int_0^{L_x}\int_0^{L_y} PZTR(x, y)W_{mn}(x, y)\,\mathrm{d}x\,\mathrm{d}y
$$

$$
\begin{aligned}
= G_0 h_p \overline{Z}_p \Bigg(& e_{31}\int_{y_1}^{y_2}\big(C'_{1mn}\cosh(\delta_{mn}y/L_y) + C'_{2mn}\sinh(\delta_{mn}y/L_y) \\
&+ C'_{3mn}\cosh(\varepsilon'_{mn}y/L_y) + \sinh(\varepsilon'_{mn}y/L_y)\big)\,\mathrm{d}y \\
&\int_{x_1}^{x_2}(\langle x - x_1\rangle^{-2} - \langle x - x_2\rangle^{-2})\sin(m\pi x/L_x)\,\mathrm{d}x
\end{aligned}
$$

$$
\begin{aligned}
+ e_{32}\int_{x_1}^{x_2}\sin(m\pi x/L_x)\,\mathrm{d}x\int_{y_1}^{y_2}&(\langle y - y_1\rangle^{-2} - \langle y - y_2\rangle^{-2})\big(C'_{1mn}\cosh(\delta_{mn}y/L_y) \\
+ C'_{2mn}\sinh(\delta_{mn}y/L_y) + C'_{3mn}&\cosh(\varepsilon'_{mn}y/L_y) + \sinh(\varepsilon'_{mn}y/L_y)\big)\,\mathrm{d}y\Bigg),
\end{aligned}
$$

$$
\begin{aligned}
= G_0 h_p \overline{Z}_p \Bigg(e_{31}\Bigg(&C'_{1mn}\frac{L_y}{\delta_{mn}}\left(\sinh\left(\delta_{mn}\frac{y_2}{L_y}\right) - \sinh\left(\delta_{mn}\frac{y_1}{L_y}\right)\right) \\
+ &C'_{2mn}\frac{L_y}{\delta_{mn}}\left(\cosh\left(\delta_{mn}\frac{y_2}{L_y}\right) - \cosh\left(\delta_{mn}\frac{y_1}{L_y}\right)\right) \\
+ &C'_{3mn}\frac{L_y}{\varepsilon_{mn}}\left(\sinh\left(\varepsilon'_{mn}\frac{y_2}{L_y}\right) - \sinh\left(\varepsilon'_{mn}\frac{y_1}{L_y}\right)\right) \\
+ &\frac{L_y}{\varepsilon'_{mn}}\left(\cosh\left(\frac{\varepsilon'_{mn}y_2}{L_y}\right) - \cosh\left(\frac{\varepsilon'_{mn}y_1}{L_y}\right)\right)\Bigg)\Bigg) \\
&\left(\frac{m\pi}{L_x}\right)\left(\cos\left(m\pi x_2/L_x\right) - \cos\left(m\pi x_1/L_x\right)\right) \\
&- e_{32}\left(\frac{L_x}{m\pi}\right)\left(\cos\left(m\pi x_2/L_x\right) - \cos\left(m\pi x_1/L_x\right)\right)
\end{aligned}
$$

$$\cdot \left(C'_{1mn} \frac{\delta_{mn}}{L_y} \left(\sinh\left(\delta_{mn}\frac{y_2}{L_y}\right) - \sinh\left(\delta_{mn}\frac{y_1}{L_y}\right) \right) \right.$$

$$+ C'_{2mn} \frac{\delta_{mn}}{L_y} \left(\cosh\left(\delta_{mn}\frac{y_2}{L_y}\right) - \cosh\left(\delta_{mn}\frac{y_1}{L_y}\right) \right)$$

$$+ C'_{3mn} \frac{\varepsilon'_{mn}}{L_y} \left(\sinh\left(\varepsilon'_{mn}\frac{y_2}{L_y}\right) - \sinh\left(\varepsilon'_{mn}\frac{y_1}{L_y}\right) \right)$$

$$\left. + \frac{\varepsilon'_{mn}}{L_y} \left(\cosh\left(\frac{\varepsilon'_{mn}y_2}{L_y}\right) - \cosh\left(\frac{\varepsilon'_{mn}y_1}{L_y}\right) \right) \right) \right). \tag{20}$$

5. Special case for simply supported boundary condition

For the special case of simply supported boundary conditions,

$$\varepsilon_{mn} = n\pi, \qquad n = 1, 2 \dots \tag{21}$$

Substituting Eqs. (18), (21)–(23) into (14), the following are obtained:

$$C_{1mn} = C_{2mn} = C_{3mn} = 0, \tag{22}$$

$$N_{mn} = \frac{1}{4} L_x L_y, \tag{23}$$

$$A_{mn} = \frac{4G(t)h_p\overline{Z}_p}{\rho\, L_x L_y(\omega_{mn}^2 - \omega^2)} \left(e_{31}\frac{\gamma_m}{\gamma_n} + e_{32}\frac{\gamma_n}{\gamma_m} \right) \big(\cos(\gamma_m x_1)$$
$$- \cos(\gamma_m x_2)\big)\big(\cos(\gamma_n y_1) - \cos(\gamma_n y_2)\big), \tag{24}$$

where

$$\gamma_m = (m\pi/L_x) \qquad \text{and} \qquad \gamma_n = (n\pi/L_y). \tag{25}$$

Equation (24) is identical to the equation derived by DIMITRIADIS and FULLER [23].

6. Conclusions

An equation for the response of a plate excited by a rectangular PZT actuator has been derived. The theory can be applied to a plate supported with any of seven boundary conditions. The theory can be extended to other boundary conditions in a similar manner.

If the size and position of the actuators are known in addition to the input voltages applied to the actuators, the response of the plate can be computed. This equation provides a mathematical foundation for calculating the optimal placement of actuators to minimize the sound power radiated from the plate.

It can be concluded from Eq. (13), that the modal amplitudes excited by the PZT actuator depend very much on the position of the actuator and the

input voltage applied to the actuator. In principle, some modes can be selectively excited or suppressed.

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