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HEATING CAUSED BY A NON-PERIODIC ULTRASOUND. THEORY AND CALCULATIONS ON PULSE AND STATIONARY SOURCES

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General formulae on the heat release caused by any sound in the thermoviscous flow are derived, the well-known limit of the periodic source being traced. Some illustrations based on the calculations on pulse sound and stationary shock wave as acoustic sources are presented.

1. Introduction

Acoustic heating is a secondary process occurring at the background of the intense acoustic wave passing. The phenomenon is well-known and exhibits a slow increase of the temperature under the constant pressure behind the passing ultrasound. The reason of this increase are losses of energy during the wave propagation in the thermoviscous fluid. Recently, the role of changes in the background density, not only in temperature, was pointed out [1,2].

Traditionally, quasi-periodic acoustic waves generated by transducer are thought as an acoustic source. Though quite realistic there is a wide variety of non-periodic (including impulses) sources that are of great importance, for example, in medicine when internal parts of the human body are investigated by ultrasound. There is also a theoretical achievement in studying secondary processes caused by non-periodic ultrasound since the standard approach is to average the overall field over temporal interval including the integer number of sound periods which is much less then the characteristic time of heating. The averaging of the total energy conservation law $\partial E/\partial t + \nabla \mathbf{J} = 0$ gives for the rate of energy change per unit volume: $\langle q \rangle = -\nabla \langle \mathbf{J}_a \rangle$ [3]. Here, $E = \rho e + \rho(\mathbf{v} \cdot \mathbf{v})/2$ is the total energy volume density, $\mathbf{J} = p\mathbf{v} + E\mathbf{v}$ is the energy flux density vector, e, ρ, $\mathbf{v}$, p are internal energy per mass unit, mass density, velocity, and pressure, correspondingly (bold symbols denote vectors). Averaging over the integer number of sound periods is marked by square brackets, $\mathbf{J}_a$ is an acoustic quantity.

2. Theory

Let us start with the basic system of conservation equations in the differential form:

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_k}(T_{ik} + \rho v_i v_k) = 0,$$

$$\frac{\partial}{\partial t}\left(\rho\frac{v^2}{2} + \rho e\right) + \frac{\partial}{\partial x_k}\left((\rho\frac{v^2}{2} + \rho e)\, v_k + S_k\right) + \frac{\partial}{\partial x_k}(v_i T_{ik}) = 0, \tag{1}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_k}(\rho v_k) = 0,$$

where among the already mentioned variables, x_i – Cartesion co-ordinates, $T_{ik} = p\delta_{ik} - s_{ik}$ means the surface stress tensor which consists of scalar pressure p and the viscous stress tensor s_{ik}, δ_{ik} being the Kronecker symbol: $\delta_{ik} = 1$ if $i = k$ and $\delta_{ik} = 0$ otherwise. The value S is the heat flow depending on the temperature gradient $S_k = -\chi\frac{\partial T}{\partial x_k}$ with χ being a coefficient of thermal conductivity. For the uniform homogeneous liquid, the tensor s_{ik} relates to the deformations v_{ik} in the following manner:

$$s_{ik} = 2\mu v_{ik}\ (i \neq k), \quad s_{ii} = 2\mu v_{ii} + \Gamma \mathrm{div}\,\mathbf{v},$$

where μ is viscosity of fluid, and v_{ik} is the tensor of deformations defined as follows:

$$v_{ik} = 0.5\,(\partial v_i / \partial x_k + \partial v_k / \partial x_i).$$

$\Gamma = \mu' - \frac{2}{3}\mu$, and μ' is the so-called second viscous coefficient responsible for the transformation of the microscopic energy of the fluid particles to the energy of the internal degrees of freedom of molecules which should be accounted for the ultrasound of extremely high frequency. In the most cases, $\mu' = 0$, $\Gamma = -\frac{2}{3}\mu$, the values accepted in the theory by Stockes [4]. Equations (1) are known to be rewritten in the vector form. For the plane dynamics of a fluid equations (1) look quite simple and need two algebraic equations of state to be complete, the caloric one $e = e(p, \rho)$, and the thermal one $T = T(p,\rho)$:

$$\rho_0 e' = E_1 p' + \frac{E_2 p_0}{\rho_0}\rho' + \frac{E_3}{p_0}p'^2 + \frac{E_4 p_0}{\rho_0^2}\rho'^2 + \frac{E_5}{\rho_0}p'\rho' + \dots,$$

$$T' = \frac{\theta_1}{\rho_0 C_v} p' + \frac{\theta_2 p_0}{\rho_0^2 C_v} \rho' + \ldots \tag{2}$$

In the series of the internal energy and temperature in the vicinity of the equilibrium point (p_0, ρ_0) (2), $\theta_1, \ldots E_1$ are dimensionless constants depending on the pair (p_0, ρ_0), C_V means heat capacity under constant volume, perturbed values are primed. Quadratic and higher order terms in the temperature expansion are ignored. Therefore losses due to the thermal conductivity are supposed to be small. For an ideal gas obeying the relation $e = \frac{p}{\rho(\gamma - 1)}$, the coefficients are (see [5]):

$$E_1 = E_4 = \theta_1 = \frac{1}{\gamma - 1}, \quad E_2 = E_5 = \theta_2 = -\frac{1}{\gamma - 1}, \quad E_3 = E_6 = 0. \tag{3}$$

In many sources [2, 6], the series of entropy are used when slowly distorting from isentropic processes. That is considered reasonable for acoustic waves propagation. It seems however more physical to start with series of the internal energy which are not related to the possible processes in the fluid but are essential feature of any medium. Recent advances in numerical methods of physical chemistry allow to calculate constants of intermolecular interactions with great accuracy in order to get the free energy and therefore high order coefficients in both series (2) theoretically. See [7] concerning liquid water and the papers referred there. It should be stressed that the constants of Eq. (2) are not independent. This follows from the relation

$$\left[-\rho^2 \left(\frac{\partial e}{\partial \rho}\right)_T + p\right] = T\left(\frac{\partial p}{\partial T}\right)_\rho \tag{4}$$

which appears as a condition for compatibility of the caloric and thermal equations of state and follows from the fact that a change in entropy is fully differential. The relation (4) is obeyed automatically if both the equations of state are expressed through the derivatives of free energy. Considering (4) in the vicinity of the equilibrium point (p_0, ρ_0) yields also:

$$\theta_2 = \frac{C_V \rho_0 T_0}{E_1 p_0} - \frac{(1 - E_2)\,\theta_1}{E_1}. \tag{5}$$

Indeed, the formulae (2) allow to consider the wide variety of fluids that do not obey the equation of state for an ideal gas. For the plane flow depending on one spatial co-ordinate x over uniform background (p_0, ρ_0) the system (1) goes to the equivalent system in non-dimensional variables $v_*, p_*, \rho_*, x_*, t_*$ ($v_* = v/c$, $p_* = (p - p_0)/c^2\rho_0$, $\rho_* = (\rho - \rho_0)/\rho_0$,

$x_* = x/\lambda$, $t_* = tc/\lambda$, asterisks for dimensionless variables will be later omitted) [8]:

$$\frac{\partial}{\partial t}\psi + L\psi = \tilde{\psi}. \tag{6}$$

Here, λ means a characteristic scale of disturbance, c is adiabatic sound velocity, $c = \sqrt{\frac{p_0(1 - E_2)}{\rho_0 E_1}}$, $\psi = (v\, p\, \rho)^T$ is a column of dimensionless perturbations,

$$L = \begin{pmatrix} -\delta_1 \partial^2/\partial x^2 & \partial/\partial x & 0 \\ \partial/\partial x & -\delta_2^1 \partial^2/\partial x^2 & -\delta_2^2 \partial^2/\partial x^2 \\ \partial/\partial x & 0 & 0 \end{pmatrix} \tag{7}$$

is a linear matrix operator with

$$\delta_1 = \frac{4\mu}{3\rho_0 c\lambda}, \quad \text{and} \quad \delta_2^1 = \frac{\theta_1}{E_1\rho_0 c\lambda C_v}\chi, \quad \delta_2^2 = \frac{\theta_2}{(1 - E_2)\rho_0 c\lambda C_v}\chi. \tag{8}$$

Also, a complete term caused by thermal conductivity is introduced: $\delta_2 = \delta_2^1 + \delta_2^2$ as well as an overall thermoviscous constant $\beta = \delta_1 + \delta_2$. Taking into account (5), it is clear that the value δ_2 depends only on the series of internal energy, but not on the temperature: $\delta_2 = \frac{T_0\chi}{E_1(1 - E_2)p_0 c\lambda}$ though it is caused by the thermal conductivity. The right-hand nonlinear vector

$$\tilde{\psi} = \begin{pmatrix} -v\frac{\partial}{\partial x}v + \rho\frac{\partial}{\partial x}p \\ -v\frac{\partial}{\partial x}p + (N_1 p + N_2\rho)\frac{\partial}{\partial x}v + \frac{\partial}{\partial x}v + \frac{\delta_1}{E_1}\left(\frac{\partial v}{\partial x}\right)^2 \\ -v\frac{\partial}{\partial x}\rho - \rho\frac{\partial}{\partial x}v \end{pmatrix}$$

includes only quadratic nonlinear terms that are of major importance in the nonlinear acoustics. Constants N_1, N_2 are evaluated in [8]:

$$N_1 = \frac{1}{E_1}\left(-1 + 2\frac{1 - E_2}{E_1}E_3 + E_5\right),$$

$$N_2 = \frac{1}{1 - E_2}\left(1 + E_2 + 2E_4 + \frac{1 - E_2}{E_1}E_5\right).$$

It is of the great importance to consider also a cross viscous-nonlinear term $\frac{\delta_1}{E_1}\left(\frac{\partial v}{\partial x}\right)^2$. The possible linear motions of the fluid follow from the linearized version of the system (6):

$$\frac{\partial}{\partial t}\psi + L\psi = 0. \tag{9}$$

There are two acoustic modes relating to the rightwards and leftwards progressive (acoustic) waves as well as the heat (called also entropy) mode:

$$\psi_1 = \begin{pmatrix} v_1(x,t) \\ p_1(x,t) \\ \rho_1(x,t) \end{pmatrix} = \begin{pmatrix} 1 - (\beta/2)\,\partial/\partial x \\ 1 - \delta_2\,\partial/\partial x \\ 1 \end{pmatrix}\rho_1(x,t), \quad \psi_2 = \begin{pmatrix} -1 - (\beta/2)\,\partial/\partial x \\ 1 + \delta_2\,\partial/\partial x \\ 1 \end{pmatrix}\rho_2(x,t),$$

$$\psi_3 = \begin{pmatrix} \delta_2^2\,\partial/\partial x \\ 0 \\ 1 \end{pmatrix}\rho_3(x,t). \tag{10}$$

The specific modes are separated from the overall flow by orthogonal matrix projectors

$$P_n\,\psi(x,t) = \psi_n(x,t) \tag{11}$$

(n = 1, 2, 3), see [8] for more details.

$$P_{1,2} = \begin{pmatrix} \frac{1}{2} \pm \left(\frac{\delta_2}{2} - \frac{\beta}{4}\right)\partial/\partial x & \pm\frac{1}{2} + \frac{\delta_2^2}{2}\partial/\partial x & -\frac{\delta_2^2}{2}\partial/\partial x \\ \pm\frac{1}{2} & \frac{1}{2} \pm \left(\frac{\beta}{4} - \frac{\delta_2^1}{2}\right)\partial/\partial x & \mp\frac{\delta_2^2}{2}\partial/\partial x \\ \pm\frac{1}{2} + \frac{\delta_2}{2}\partial/\partial x & \frac{1}{2} \pm \left(\frac{\beta}{4} + \frac{\delta_2^2}{2}\right)\partial/\partial x & \mp\frac{\delta_2^2}{2}\partial/\partial x \end{pmatrix},$$

$$P_3 = \begin{pmatrix} 0 & -\delta_2^2 \partial/\partial x & \delta_2^2 \partial/\partial x \\ 0 & 0 & 0 \\ -\delta_2 \partial/\partial x & -1 & 1 \end{pmatrix}. \tag{12}$$

The nonlinear evolution equations for interacting modes follow from Eq.(6) when the corresponding projector acts at the both sides of this equation:

$$\frac{\partial}{\partial t}\psi_n + L\psi_n = P_n\tilde{\psi}. \tag{13}$$

The right-hand nonlinear vector $\tilde{\psi}$ represents an input of all modes. The well-known nonlinear evolution equations like the Earnshow one follows from (13) [9].

3. Heating generated by any acoustic source

An equation for the slow change in the background density corresponding to the entropy mode may be obtained if one acts by the projector P_3 at the system (6) assuming that the nonlinear right-hand vector $\tilde{\psi}$ consists of the rightwards progressive acoustic mode inputs only. So the rightwards acoustic mode imposed to be dominant and the evolution equation is expected to be proper when the heating is small in comparison to the dominant mode and therefore the induced effects may be ignored. In terms of the pressure of the dominant mode, the evolution equation looks

$$\frac{\partial \rho_3}{\partial t} + \delta_2^2 \frac{\partial^2 \rho_3}{\partial x^2} = -(N_1 + N_2 + 1)p_1\frac{\partial p_1}{\partial x}$$
$$+ \left(\frac{\beta(N_1 + N_2 + 1)}{2} - \delta_2(N_1 + N_2 + 2)\right)p_1\frac{\partial^2 p_1}{\partial x^2} + (-\delta_2(N_2 + 1) - \delta_1/E_1)\left(\frac{\partial p_1}{\partial x}\right)^2. \tag{14}$$

Note that though the overall coefficient responsible for the thermal attenuation δ_2 depends only on the internal energy series, first from (2), the equation of thermal conductivity like this (14) possesses a viscous term with the negative multiplier δ_2^2 which actually needs also a least value of θ_1 or θ_2. Unfortunately, the experimental data even for the most known liquid, such as water, concern only linear fluctuations, so there may be extracted data on E_1, E_2 and completely unavailable on higher order coefficients (see [7] and referred papers). It was already mentioned that all coefficients in series (2) in any equilibrium point may be provided analytically by the theory of modern physical chemistry.

In the case of the perfect gas without thermal conductivity, $N_1 = -\gamma$, $N_2 = 0$, $\delta_2 = 0$ (14) goes to the following equation:

$$\frac{\partial \rho_3}{\partial t} = (\gamma - 1)\left[p_1 \frac{\partial p_1}{\partial x} - \frac{\beta}{2} p_1 \frac{\partial^2 p_1}{\partial x^2} - \beta \left(\frac{\partial p_1}{\partial x}\right)^2\right]. \tag{15}$$

All calculations below will be done for the perfect gas for simplicity.

3.1. Periodic acoustic source

To trace a quasi-periodic acoustic source given by the general theory for small Reynolds numbers, one may take the acoustic pressure $p_1(x,t)$ as follows (see [6]):

$$p_1(x,t) = p_{10} \exp(-\beta x/2) \sin(x - t). \tag{16}$$

The formula (16) is correct beyond some vicinity of the transducer where the nonlinear distortions are strong. Temporal averaging of both sides of (15) gives:

$$\left\langle \frac{\partial \rho_3}{\partial t} \right\rangle = (\gamma - 1)\left\langle \frac{\partial p_1^2}{\partial x} \right\rangle = -\frac{\beta}{2}(\gamma - 1) p_{10}^2 \exp(-\beta x). \tag{17}$$

In all calculations the terms of order β^2 as well as the cubic nonlinear ones were left out of account. To calculate (17), note that the periodic perturbation (16) in the leading order (up to quadratic nonlinear terms) satisfies the relations below:

$$\left\langle p_1 \frac{\partial p_1}{\partial x} \right\rangle = \frac{\beta}{2}\left\langle p_1 \frac{\partial^2 p_1}{\partial x^2} \right\rangle = -\frac{\beta}{2}\left\langle \left(\frac{\partial p_1}{\partial x}\right)^2 \right\rangle. \tag{18}$$

The formula (17) goes to the known result for the periodic ultrasound given in introduction. It is known that heating is an isobaric process as proved also by the eigenvector $\psi_3(x,t)$ defined by (10). The rate of heat and temperature distortions (both dimensional) per unit volume in dimensionless variables are (isobaric heating):

$$q = \frac{c}{\lambda} \rho_0 C_p \left\langle \frac{\partial T}{\partial t} \right\rangle = -\frac{c^3 \rho_0}{\lambda(\gamma - 1)} \left\langle \frac{\partial \rho_3}{\partial t} \right\rangle. \tag{19}$$

(in dimensional ρ, t the normalizing value λ disappears: $q = -\dfrac{c^2}{(\gamma - 1)}\left\langle \dfrac{\partial \rho_3}{\partial t} \right\rangle$). On the other hand, a dimensionless gradient (in dimensional co-ordinate x, λ disappears as well) of the acoustic energy flux relating to the rightwards plane wave is

$$\frac{1}{\lambda}\frac{\partial}{\partial x}\langle J_1\rangle = \frac{\rho_0 c^3}{\lambda}\frac{\partial}{\partial x}\langle p_1 v_1\rangle = \frac{\rho_0 c^3}{\lambda}\frac{\partial}{\partial x}\langle p_1^2 - (\beta/2 + \delta_2)p_1 \partial p_1/\partial x\rangle = \frac{\rho_0 c^3}{\lambda}\frac{\partial}{\partial x}\langle p_1^2\rangle, \quad (20)$$

where the last calculation ignores terms of order β^2. Formulae (17), (19), (20) result in relation $q = -\frac{1}{\lambda}\frac{\partial}{\partial x}\langle J_1\rangle$ presented at the introduction in the dimensional form.

3.2. Heating generated by pulses

As it was proved in 3.1., the evolution equation (15) goes to the known case of a periodic acoustic source but is suitable for any ultrasound source including a non-periodic one. The next step is to calculate the equation (15) with an acoustic source being a solution of the Burgers equation

$$\frac{\partial}{\partial t}p_1 + \frac{\partial}{\partial x}p_1 + \frac{\gamma+1}{2}p_1\frac{\partial}{\partial x}p_1 - \frac{\beta}{2}\frac{\partial^2}{\partial x^2}p_1 = 0. \quad (21)$$

Equation (15) gives a formula for the rate of the heat release per unit volume:

$$q(x,t) = -\frac{\partial \rho_3}{\partial t} = -(\gamma-1)\left[p_1\frac{\partial p_1}{\partial x} - \frac{\beta}{2}p_1\frac{\partial^2 p_1}{\partial x^2} - \beta\left(\frac{\partial p_1}{\partial x}\right)^2\right]. \quad (22)$$

An example of a mono-polar source is given by a self-similar solution of the Burgers equation, see [6]:

$$p_1(\xi,\tau) = -\frac{2\beta}{\sqrt{(\xi+\xi_0)}}\cdot\frac{\mathrm{Exp}\left(\frac{-(\tau+\tau_0)^2}{2(\xi+\xi_0)}\right)}{2\sqrt{\beta}C - \frac{\gamma+1}{2}\cdot\sqrt{2\pi}\cdot Erf\left(\frac{\tau+\tau_0}{\sqrt{2(\xi+\xi_0)}}\right)}, \quad (23)$$

written in the new variables: slowly varying co-ordinate $\xi = \beta x$ and the retarded time $\tau = t - x$. Note that in contrast to the quasi-periodic sound (16), the solution (23) does not relate to large or small Reynolds numbers. The absolute value of C is responsible for the symmetry of the impulse: $|C| \gg 1$ gives a curve close to the Gauss one. In calculations of Eq.(22) with an ultrasound source given by (23), the next values of the parameters are taken: $\beta = 0.1$, $\gamma = 1.4$, $C = (\gamma+1)\cdot\sqrt{\frac{\pi}{2\beta}}$, $\xi_0 = \tau_0 = 0$. Figures 1, 2 show the pressure of the acoustic source (bold line) and the rate of heat production calculated due to (22), (23) as functions on x at $t = 1$, $t = 2$ respectively.

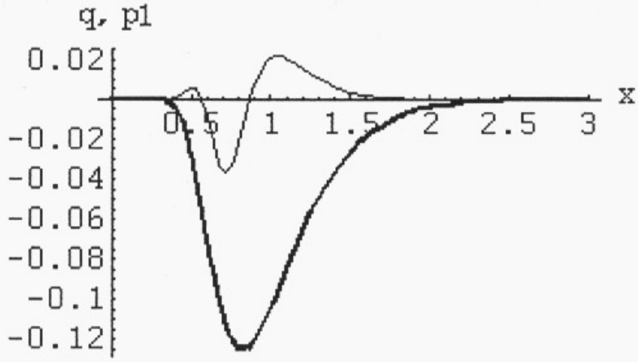

Fig. 1. The rate of heat production per unit volume q and pressure of the acoustic source p_1 (bold line) calculated due to (22), (23) as functions of x at $t = 1$.

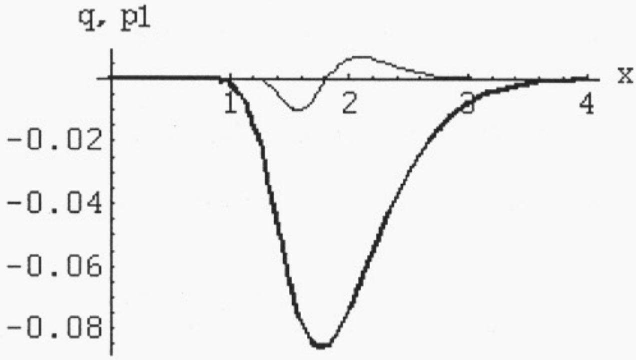

Fig. 2. The rate of heat production per unit volume q and pressure of the acoustic source p_1 (bold line) calculated due to (22), (23) as functions of x at $t = 2$.

Some comments on Figs. 1, 2 are useful. There are areas of negative and positive q at both figures representing space non-uniformity of heat release caused by dependence of q on $\frac{\partial p_1}{\partial x}, \frac{\partial^2 p_1}{\partial x^2}$ in accordance to Eq.(22). The overall heat which is released by the impulse in unit time is positive and may be found as an integral over volume V ($dV = Sdx$, S is cross section of flow): $\int q dV = S \int_0^\infty q dx$. Calculations of $\int_0^\infty q dx$ give values of $3 \cdot 10^{-3}$ at $t = 1$ and $8 \cdot 10^{-4}$ at $t = 2$, both positive as expected. Attenuation of a source during its propagation results in a decrease of heat production. Also, a stationary temperature increase as a trace after the source passing ($t \to \infty$) may be evaluated numerically accordingly to Eqs. (19), (22). Figure 3 shows the relative temperature $dT = T' / T_0$ vs. x. For a single impulse, a large growth of temperature is hardly expected. Nevertheless even a single pulse results in a new temperature and density of the background.

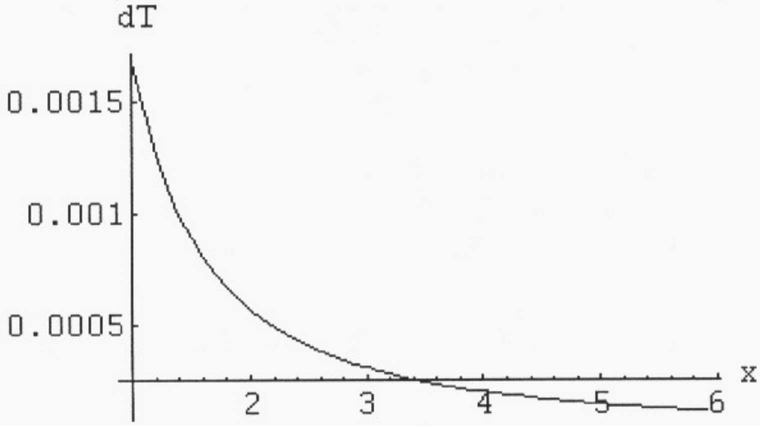

Fig. 3. Stationary relative temperature $dT = T'/T_0$ after the source passing vs. x.

3.3. Heating caused by a stationary wave

It is known that there is a class of a stationary solution of (21) in the form of a shock wave:

$$p_1(\xi) = \frac{T\exp\left(\frac{(c_1 - \tilde{c})(\xi - \xi_0)}{\beta}\right)}{1 - \frac{\gamma + 1}{4(c_1 - \tilde{c})} T\exp\left(\frac{(c_1 - \tilde{c})(\xi - \xi_0)}{\beta}\right)}, \tag{24}$$

with $\xi = x - \tilde{c}t$, $T = \frac{p_0}{1 - \frac{\gamma + 1}{4(\tilde{c} - c_1)} p_0}$, $p(\xi_0) = p_0$. The shock wave possesses velocity $\tilde{c}$ different from the velocity of an acoustic mode c_1 (equal to unit in the dimensionless variables). It is also known that equilibrium between dissipation and nonlinearity is possible if the energy losses are compensated by the background. To provide $\lim_{\xi \to \infty} p_1(\xi) = 0$, a value $\tilde{c} > c_1$ should be chosen. The background pressure in the front of the shock wave has a limit: $\lim_{\xi \to \infty} p_1(\xi) = -4(c_1 - \tilde{c})/(\gamma + 1)$. Constants β, γ, ξ_0 are the same as in calculations of 3.2., $\tilde{c} = 1.1$, $p_0 = 0.1$. Figure 4 presents the pressure of the source (bold line) and the rate of heat release per unit volume (multiplied by 10) as functions of ξ.

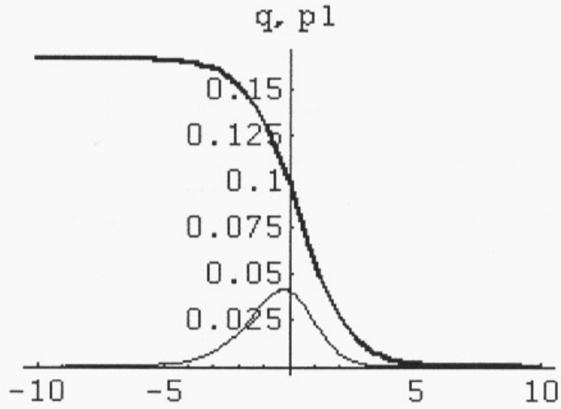

Fig. 4. The rate of heat production per unit volume q (multiplied by 10) and pressure of acoustic source p_1 (bold line) calculated due to (22), (24) as functions of ξ.

4. Conclusions

The paper continues previous investigations by the author developing the idea of applying projectors to the theory of nonlinear flow. The reader may find some ideas on interaction of modes leading to the coupled nonlinear evolution equations, as well as an approximate solution of these equations in the papers [10–12].

In the present paper, the heat generation as a secondary process caused by ultrasound is considered from the point of view of a nonlinear interaction of the specific modes. An equation for the heat generation with acoustic quadratic nonlinear source is derived. Since the very approach does not need temporal averaging, the results are suitable for any type of sound sources including non-periodic ones. The general formulae for any fluid are derived. Results of numerical calculations of heating following a single pulse and shock wave as sources are discussed.

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