VIBRATORY ENERGY FLUX STUDY IN A HERMETIC COMPRESSOR BY STATISTICAL ENERGY ANALYSIS

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This paper presents a study of the vibratory energy transmission from the block to the shell of a hermetic compressor by the statistical energy analysis. The loss factor and modal density have been obtained experimentally. The vibratory energy of the block and shell, and the sound power radiation were also measured. The coupling loss factors from the block to shell and from the shell to the reverberation chamber have been calculated from the parameters above.

1. Introduction

The traditional analysis of vibratory systems, machines or structures have been placed on the first modes (at low frequencies). Although vibration analysis and sound radiation at high frequency have already provoked interest at some time, it was only with the big aerospacial structures advent that the researches have begun because of the need of predicting structural fatigue, equipments fails, noise levels and vibrations of these structures. The uncertainness in the prediction of these parameters grow up with the frequency increase and the difficulty of prediction grows up with the mechanical and geometrical complexity degree of the system.

The statistical energy analysis (SEA) forms the basis for calculus and prediction of the high frequency vibration levels in systems which are subordinated to large band random excitation, providing valuations of the vibratory energy level of the system (or subsystem) as a whole. The denomination of the statistical energy analysis appeared in the beginning of the 60's from LYON researches [1, 2, 7]: *Analysis* — More than a particular technique, it is an analysis methode; *Statistical* — The system in study is described by the statistical knowledge of its dynamics parameters; *Energy* — The primary variable of interest is the energy.

The energy use as a primary variable to the analysis allows to remove, from the beginning, the existing differences between several vibratory systems and propitiates the analysis of mixed systems, as the response of a structure to an acoustic field.

2. Energy flux between vibratory systems

To analyse the energy flux between coupled vibratory systems, the parameters that describe their dynamic properties must be known and the two most important are: i) Modal density (D) or the number (n) of modes per band, and ii) Loss factor (η_i). The energy flux from the i-th to the j-th system is related to the coupling factor η_{ij}. As the loss factor η_{ij} if E_{it} is the total energy of the i-th system so (1) represents the power (W_{id}) dissipated in the i-th system and (2) represents the transfered energy from the i-th to the j-th system (ω is the circular frequency).

$$W_{id} = \eta_i \cdot \omega \cdot E_{it}, \qquad (1)$$

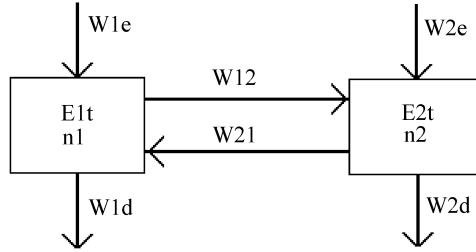$$W_{ij} = \eta_{ij} \cdot \omega \cdot E_{it}. \qquad (2)$$

Fig. 1. Energy flux between coupled vibratory systems.

Consider the coupled systems 1 and 2 shown in Fig. 1. We can affirm that (W_{ie} being the entrance power of the i-th system) in permanent state (or regime):

$$W_{ie} = W_{id} + W_{12} - W_{21}, \qquad (3)$$

$$W_{2e} = W_{2d} + W_{21} - W_{12}. \qquad (4)$$

Considering the case where only system 1 is externaly excited we come to:

$$\frac{E_{2t}}{E_{1t}} = \frac{\eta_{12}}{(\eta_2 + \eta_{21})}. \qquad (5)$$

We can prove that: [1]

$$\frac{n_i}{n_j} = \frac{\eta_{ji}}{\eta_{ij}} = \frac{D_i}{D_j}, \qquad (6)$$

where n_i — number of modes and D_i — modal density of the i-th system in the considered band.

From (5) and (6) we come to (7). Defining (8) the modal energy becomes (9):

$$\frac{E_{2t}}{E_{1t}} = \left(\frac{n_2}{n_1}\right)\frac{\eta_{21}}{(\eta_2 + \eta_{21})}, \tag{7}$$

$$E_i = \frac{E_{1t}}{n_i}, \tag{8}$$

$$\frac{E_2}{E_1} = \frac{\eta_{21}}{(\eta_2 + \eta_{21})}. \tag{9}$$

From (9) and precedents we can affirm that, if:

i) $\eta_2 \ll \eta_{21}$ then E_2/E_1 tends towards $1, i$ and, the energy of the system 2 tends to be the same as system 1. An important practical application is that the inclusion of additional damping to the system will only make sense if this value (of the loss factor) is of the same order of magnitude as coupling factor (CLF);

ii) $E_2 < E_1$ since $\eta_2 > 0$, always.

Among the conditions that favor the energy transfer we can emphasize:

i) high modal density;

ii) little damping in the system to which the energy flows;

iii) strong coupling between the oscillators.

In general, the coupling is weak when the rate η_{ij}/η_i is substantially smaller than unity $((\eta_{jj}/\eta_i) \ll 1)$. In the analysis considering weak damping, the energy of a mode transferred from the i-th system to another one, for example the j-th one, can be evaluated without considering the influence of other systems.

3. Loss factor determination (η)

The loss factor relates the dissipated energy, per oscillation radian, in the system with maximum vibratory energy. The experimental procedure for its determination is relatively simple, however, depending on the system, significant differences can be found in the loss factors determined by different methods or by the same method in different experiments. The most important methods to determine the loss factor are the following ones:

3.1. Decaying method

This method is much more used because it is the most simple one. The decaying time of the vibration level is measured after the excitation is suddenly removed. It is written as the reverberation time (T) and the loss factor is calculated using (10) where f is the band central frequency [1, 4, 6]

$$\eta = \frac{2,2}{(f \cdot T)}. \tag{10}$$

This method, despite of being simple and popular, has limitations that must be considered in its application:

i) the first one refers to the damping level of the structure in context; the method is not recomended to structures with loss factors greater than 0.1;

ii) in the same band there are generally several modes, however the decaying time is dominated by the smaller damping mode.

3.2. Half power band method

The loss factor calculated by the half power band method consists in, in measuring the largeness (Δf) of the band 3 dB below the natural frequency f_n peak from the structure response spectrum; the loss factor is calculated using [1, 4, 6]

$$\eta = \frac{\Delta f}{f_n} \, . \tag{11}$$

At high frequency there is, in general, an increase of the number of modes which turns the spectrum "dense". Many times a mode overlaps another one, and this limits the methode applications at low frequencies.

3.3. Entrance power method

The entrance power method, supposing that no energy is transferred out of the system, considers that the dissipated power (W_d) equals the supplied power (from the entrance) to the system and involves the inner power and the vibratory energy (E_{vib}) of the system measurement. (12) is used to calculate the loss factor: [1, 4, 6]

$$\eta = \frac{W_d}{(2 \cdot \pi \cdot f \cdot E_{\mathrm{vib}})} \, . \tag{12}$$

The main inconvenience are the problems due to the phase measurement between the force and response. Working with medium values, in space ($<>$) and time, if there is a minimum of five natural frequencies in the band, Clarckson [3] suggests the determination of the loss factor by:

$$\eta = \frac{\int\limits_{f_1}^{f_2} |F(if)|^2 \, df}{\int\limits_{f_1}^{f_2} |A(if)|^2 \, df} \frac{\pi f D}{2m^2} \, , \tag{13}$$

where $F(if)$ is the Fourier transform of the force; $A(if)$ is the Fourier transform of the acceleration and m is the mass.

4. Modal density determination

First the modal density can be determined exciting the system with a pure tone of variable frequency or by an impulsive force (if the spectrum is large) and counting the number of resonances which are excited in each band. In practice several problems appear due to the interaction of the structure with its supports, to their excitation and response points, to the high modal density and damping. This method can be used provided the modal density and the damping are not very high.

If there is a minimum of five natural frequencies in the band, CLARCKSON proposes [3], the modal density determination using (14), where Re[ ] = real part of [ ]

$$D = \frac{1}{(f_2 - f_1)} \int\limits_{f_1}^{f_2} 4m \cdot \text{Re}\left[\frac{-iA(if)}{2\pi f F(if)}\right] \cdot df. \tag{14}$$

The modal density can be determined also by the mathematic modeling of the system and using either the analytic solution (the references [1, 7, 4] show several formulas to simple systems) or a numeric method by the finite elements program.

5. The hermetic compressor

The hermetic compressor is, as at home refrigerator systems, the most important component, and also the main source of noise and vibration.

The noise and vibration, produced inside of the hermetic compressor, comes, mainly, from [5, 8, 9]:

i) abrupt pressure variation occured in the cylinder;

ii) vibration due to slacknesses;

iii) the suction chamber systems.

And the main transmition ways to the shell are:

i) the mechanical one, by the block suspention springs and the discharge tube;

ii) acoustic excitation of the shell through the gas medium.

5.1. Hermetic compressor modeled by SEA

The hermetic compressor analysed (FFE 8A – 115 V – 60 Hz) was divided into two subsystems: the block and shell. A third subsystem is added to the analysis: the reverberating chamber.
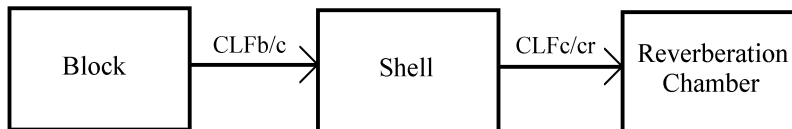

Fig. 2. Hermetic compressor modeled by SEA.

6. Experimental data

6.1. Loss factor

The loss factors were determined by the decaying method applying an impulsive force, filtering the 1/3 octave band response and registering it in a HP 5451C Fourier analyser. Figure 3 shows the loss factors determined for block and shell.
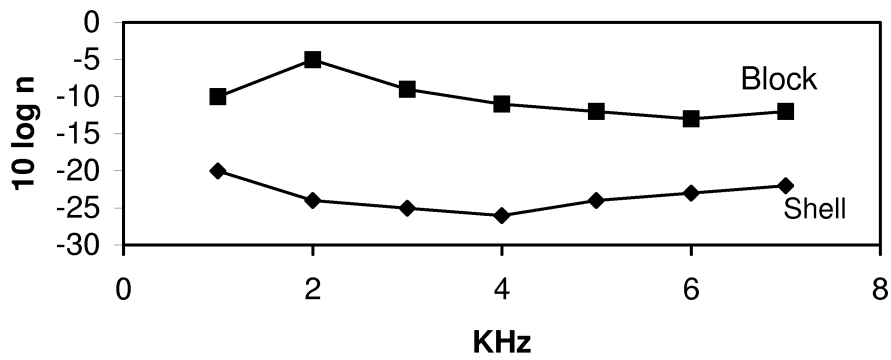
Fig. 3. Loss factor.

6.2. Number of modes

The number of modes per band was determined counting the peaks of the frequency response function obtained experimentaly by applying an impulsive force to the system and registering, simultaneously, the force and the acceleration response in a HP 5451C Fourier analyser by two entrance channels. The result of the measurements is shown in Fig. 4.
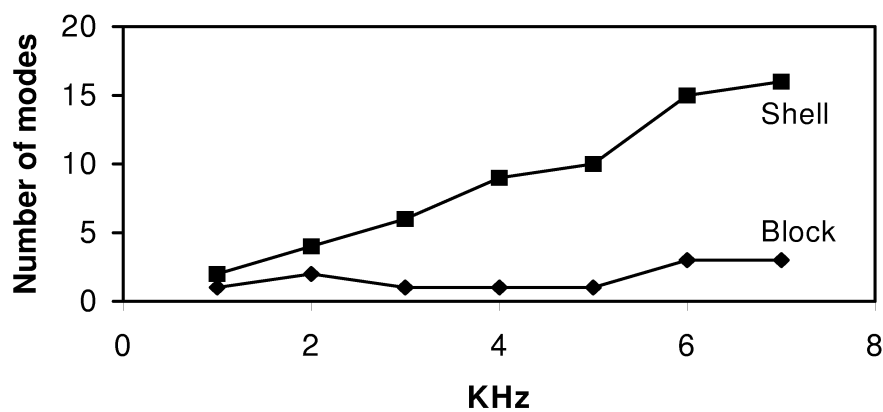

Fig. 4. Number of modes per 1/3 octave band.

6.3. Sound power

The sound power (NWS) was determined (1/3 octave band) inside a reverberating chamber using a BK 7507 calculator and a BK 3923 microphone rotating support. It is shown in Fig. 5.
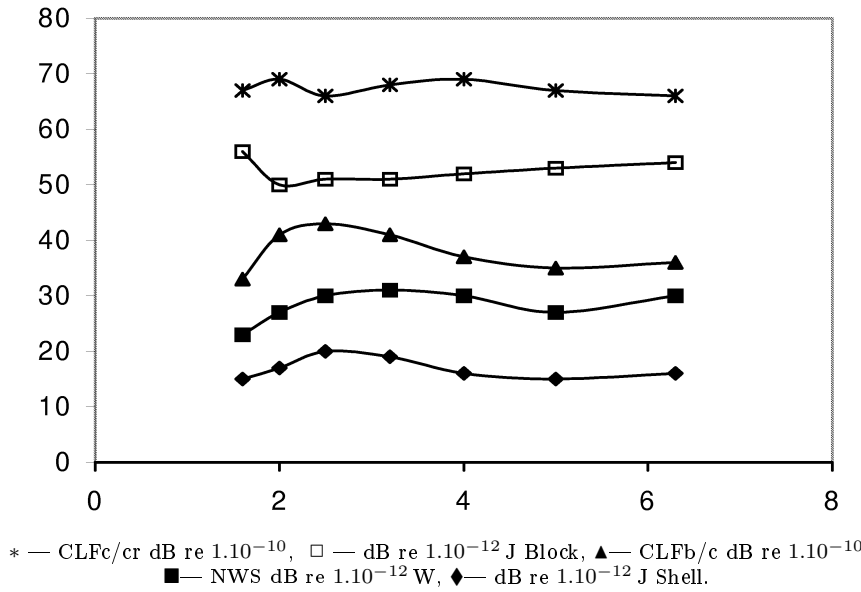
$* — \text{CLFc/cr dB re } 1.10^{-10}, \quad \square — \text{dB re } 1.10^{-12} \text{ J Block}, \quad \blacktriangle — \text{CLFb/c dB re } 1.10^{-10}$
$\blacksquare — \text{NWS dB re } 1.10^{-12} \text{ W}, \quad \blacklozenge — \text{dB re } 1.10^{-12} \text{ J Shell}.$

Fig. 5. Vibratory energy, sound power, and coupling factor.

6.4. Vibratory energy

The block and shell vibratory energies were determined at normal operation conditions of the compressor (suction $-23.3°$C and discharge $+54.4°$C) using a BK 2032 Fourier analyser. Twenty five points of the measurement were used in the shell and 5 points in the block.

6.5. Coupling factors

The block/shell coupling factor (CLFb/c) and the shell/reverberating chamber coupling factor (CLFc/cr) calculated [1] using (15) (where $R_{\rm rad}$ is the radiation resistance) are shown in Fig. 5.

$$\frac{\eta_c}{CR} = \frac{R_{\rm rad}}{(\omega \cdot m)} \, .$$ (15)

7. Conclusions

The block and shell loss factors determined by the decaying method are inside the expected values for this kind of structures. For the block at 2.0 kHz, the value is greater than the maximum recommended by the same method. The number of ressonance frequencies of the block and the shell are in accordance to the expected values.

In Figure 5 we can notice that:

i. The excitation block energy (dB re 1.10^{-12} J block) is flat except the point 1.6 kHz;

ii. The shell/reverberating chamber coupling factor (CLFc/crdBre 1.10^{-10}) is also completely flat;

iii. The curves of the shell energy (dBre 1.10^{-12} shell), the block/shell coupling factor (CLFb/c dB re 1.10^{-10}) and the sound power (NWS dB re 1.10^{-12} W) present similar forms. From Fig. 3 we can see that the flat block energy is transferred to the shell by the block/shell coupling factor, and the shell energy takes over this coupling factor form. From the shell the energy is transferred by the reverberating chamber and by the shell/reverberating chamber coupling factor, which is flat, so that the reverberating chamber energy takes over the same form as the shell energy. Therefore, in the bands considered, the final form of the sound power spectrum of this hermetic compressor is determined by the block/shell coupling factor.

Between the block and the shell there is a weak coupling. Most of the vibratory energy created inside the block is dissipated by its own. A great level difference also exists between the block/shell and shell/reverberating chamber coupling factors. This difference was expected since the high modal density of the reverberating chamber sustains the energy transfer. Again, we can emphasize that the coupling factor is characteristic of the system, being independent of the excitation and the response. Therefore, it is possible to reach the sound power of this hermetic compressor in the analysed bands changing the excitation by the coupling factors.

Acknowledgements

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