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THERMAL EFFECTS IN SOFT TISSUES DEVELOPED UNDER THE INFLUENCE OF FOCUSED ULTRASONIC FIELDS OF SHORT DURATION

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Temperature increases in soft tissues developing under the influence of a concentrated ultrasonic beam, as used in ultrasonography, have been analytically determined. By solving the heat conductivity equation and subsequent use of the Laplace transformation, formulae have been obtained (30), (38) which permit the calculation of the approximate value and distribution of the temperature in the focus of the beam, perpendicular to the direction of propagation, as a function of time.

In the case of an ultrasonic impulse with a duration of 1 μs, the intensity of 20 W/cm², in soft tissue with an attenuation of 3 dB/cm and simplified shape of ultrasonic beam (Fig. 2), a maximum temperature increase equal barely to $3.3\cdot 10^{-6}$°C has been obtained. The temperature increase calculated for an ultrasonic impulse of 1 s duration gives a value of the same order as that obtained by other authors. In the case of an ultrasonic exposure of an intensity of 200 W/cm² and a duration of 1 s, which corresponds to the threshold from curves for irreversible changes in the brain published in the literature, a value of temperature increase in the tissue of 33°C has been obtained.

1. Introduction

In pulse-echo ultrasonography, used for the visualisation of internal organs of the human body, focused ultrasonic beams are used, the intensity of which, in the focus, may attain a value of 20 W/cm² for the short duration of the impulse about 1 μs.

In investigations concerning the influence of ultrasounds on biological structure this value may reach 1000 W/cm² [5]. No temperature increases are observed in tissues ultrasonically irradiated in this manner, because of the impulse character of radiating fields; the interruption between impulses is about 1 ms and is thus 1000 times as long as the duration of the pulse.

In such a situation the question arises as to whether, in the course of the impulse duration, the tissue region confined by the focus is not temporarily overheated. This question is important because it is known that even at a temperature of 50°C a permanent damage to the tissues can occur. For instance, we can consider the situation that, in the course of the impulse duration, the temperature has increased by 20°C, but after the longer pause between impulses it has decreased to the original temperature. The high inertia thermometer will, however, in this case indicate only a mean temperature increase three orders of magnitude smaller, and the overheating of the tissues may be not noticed.

We have, therefore, set ourselves the task of estimating the magnitude of the temperature effect arising in the soft tissue under the influence of a focused ultrasonic field of short duration. To simplify the problem we have made some assumptions which will be dealt with in the sequel.

2. Assumptions

Two measurements made of the spatial distribution of the focused ultrasonic field used in ultrasonography of the abdominal cavity at a frequency of 2.5 MHz gave the results shown in Fig. 1. The region of the focus defined by a 3 dB decrease of acoustic pressure relative to the maximum value can, with a good approximation, be described by a cylinder 0.25 cm in diameter and 6.2 cm in length (Fig. 2).

We assume that in the focal region thus defined sources of heat are located with a rate of heat generation per unit volume equal to $\dot{Q}_v$ cal/s·cm³. The size of these sources depends on the intensity of the incident ultrasonic wave as well as on the absorption properties of the tissue. Another simplifying assumption is that of an even distribution of these sources throughout the whole focal region.

In this situation it is convenient to consider the problem in a cylindrical coordinate system. By assuming that the focal region under consideration has a cylindrical shape of infinite lenght the problem is reduced to a two-dimensional one.

In view of the axial symmetry the phenomena considered are solely a function of one coordinate, the radius r. Fig. 3 shows the case considered for the focal region formed in an unlimited biological medium in the shape of an infinite cylinder with the radius R. The temperature of the medium outside the focus is T_o, whereas inside the focus it is T_i. We assume that at the initial moment $t = 0$ the temperatures are equal, i.e. $T_o = T_i = 0$.

3. Initial equations

The absorption of ultrasonic waves will release heat in the focus, thus causing a rise of temperature followed by a flow of heat from the focus into the surrounding biological medium by thermal conduction.

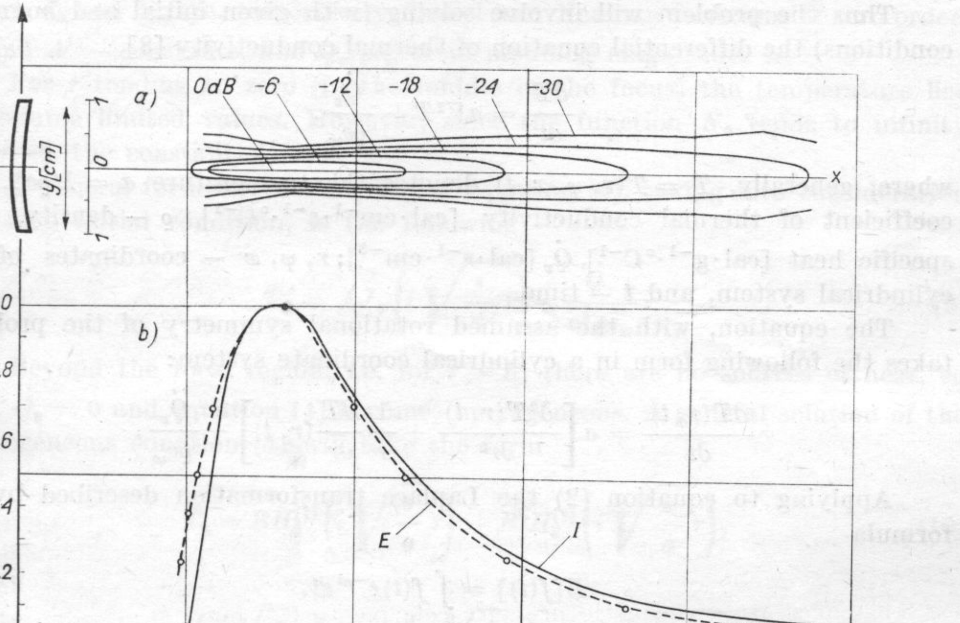

Fig. 1. Measured and calculated distributions of the focused ultrasonic field produced in water by the ultrasonograph UG-4 used for diagnostic examination of the abdominal cavity [2]

a – measured isoecho curves, b – computed (I) and measured (E) intensity distribution along the x-axis of ultrasonic beam

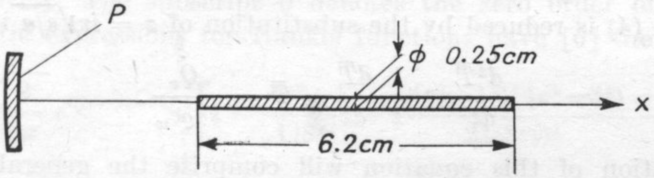

Fig. 2. Approximated shape of the focus region of ultrasonic beam in water described by the 3 dB curve relative to the maximum pressure

x – direction of ultrasonic wave propagation, P – transducer radiating the ultrasonic wave

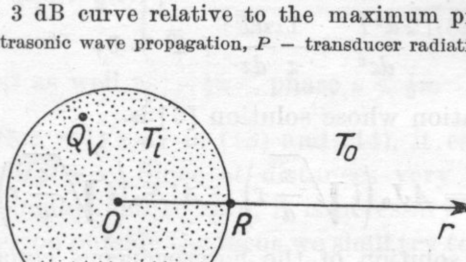

Fig. 3. Dotted focal region with temperature T_i in which heat sources with thermal effect $\dot{Q}_v$ are evenly distributed, and the surrounding region of the medium, $r > R$, with temperature T_0

Thus the problem will involve solving (with given initial and boundary conditions) the differential equation of thermal conductivity [8],

$$\frac{\partial T}{\partial t} = a\nabla^2 T + \frac{\dot{Q}_v}{\varrho c_w}, \tag{1}$$

where, generally, $T = T(r, \psi, x, t)$ denotes the temperature $a = \lambda/\varrho c_w$, λ — coefficient of thermal conductivity $[\mathrm{cal \cdot cm^{-1} \cdot s^{-1} \cdot {}^\circ C^{-1}}]$, ϱ — density, c_w — specific heat $[\mathrm{cal \cdot g^{-1} \cdot {}^\circ C^{-1}}]$, $\dot{Q}_v$ $[\mathrm{cal \cdot s^{-1} \cdot cm^{-3}}]$; r, ψ, x — coordinates of the cylindrical system, and t — time.

The equation, with the assumed rotational symmetry of the problem, takes the following form in a cylindrical coordinate system:

$$\frac{\partial T(r,t)}{\partial t} = a\left[\frac{\partial^2 T(r,t)}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}(r,t)\right] + \frac{\dot{Q}_v}{\varrho c_w}. \tag{2}$$

Applying to equation (2) the Laplace transformation described by the formula

$$L[f(t)] = \int_0^\infty f(t)e^{-st}\,dt, \tag{3}$$

we obtain

$$s\overline{T}(r,s) - T(r,t=0) = a\left[\frac{d^2\overline{T}(r,s)}{dr^2} + \frac{1}{r}\frac{d\overline{T}(r,s)}{dr}\right] + \frac{\dot{Q}_v}{s\varrho c_w}, \tag{4}$$

where $\overline{T}$ is the transform $\overline{T}(r,s) = L[T(r,t)]$.

Let us assume that the initial temperature is equal to zero. Then the other component of the left-hand side of equation (4) disappears.

Equation (4) is reduced by the substitution of $z = ir\sqrt{s/a}$ in the equation

$$\frac{d^2\overline{T}}{dz^2} + \frac{1}{z}\frac{d\overline{T}}{dz} + \overline{T} = \frac{\dot{Q}_v}{s^2\varrho c_w}. \tag{5}$$

The solution of this equation will comprise the general homogeneous solution in the form

$$\frac{d^2\overline{T}}{dz^2} + \frac{1}{z}\frac{d\overline{T}}{dz} + \overline{T} = 0, \tag{6}$$

which is a Bessel equation whose solution [6] is

$$\overline{T}_1 = AJ_0\left(i\sqrt{\frac{s}{a}}\,r\right) + A'N_0\left(i\sqrt{\frac{s}{a}}\,r\right); \tag{7}$$

and also a particular solution of the heterogeneous equation whose solution is the expression

$$\overline{T}_2 = \frac{T(r,t=0)}{s} + \frac{\dot{Q}_v}{s^2\varrho c_w}, \tag{8}$$

where J_0 and N_0 are, respectively, Bessel and Neuman functions of zero order, A and A' – constants, and $\dot{Q}_v/\varrho c_w$ is an assumed magnitude.

For r tending to zero (in the middle of the focus) the temperature has to assume limited values. However, since the function N_0 tends to infinity for $r \to 0$, the constant $A' = 0$.

A general form for the solution of equation (4), taking into consideration the zero initial condition, is the following:

$$\overline{T}_i = AJ_0\left(i\sqrt{\frac{s}{a}}\,r\right) + \frac{\dot{Q}_v}{s^2 \varrho c_w}. \tag{9}$$

Beyond the focal region, i.e. for $r > R$, there are no sources of heat, so that $\dot{Q}_v = 0$ and equation (4) becomes homogeneous. A general solution of the homogeneous equation (4) will take the form

$$\overline{T}_3 = BH_0^{(1)}\left(i\sqrt{\frac{s}{a}}\,r\right) + B' H_0^{(2)}\left(i\sqrt{\frac{s}{a}}\,r\right), \tag{10}$$

where

$$H_0^{(1)}\left(i\sqrt{\frac{s}{a}}\,r\right) = J_0\left(i\sqrt{\frac{s}{a}}\,r\right) + iN_0\left(i\sqrt{\frac{s}{a}}\,r\right), \tag{11}$$

$$H_0^{(2)} = J_0\left(i\sqrt{\frac{s}{a}}\,r\right) - iN_0\left(i\sqrt{\frac{s}{a}}\,r\right). \tag{12}$$

The functions $H_0^{(1)}$ and $H_0^{(2)}$ are Hankel's functions of the first and second kind, respectively. The subscript 0 denotes the zero order of the function.

Asymptotic expressions for Hankel functions have [6] the form

$$H_v^{(1)}(z) \cong \sqrt{\frac{2}{\pi z}}\; e^{i(z-\pi/4-v\pi/2)}\left[1 - \frac{4v^2-1}{1!8zi} + \frac{(4v^2-1^2)(4v^2-3^2)}{2!(8zi)^2} - \ldots\right], \tag{13}$$

$$H_v^{(2)}(z) \cong \sqrt{\frac{2}{\pi z}}\; e^{-i(z-\pi/4-v\pi/2)}\left[1 + \frac{4v^2-1}{1!8zi} + \frac{(4v^2-1^2)(4v^2-3^2)}{2!(8zi)^2} + \ldots\right] \tag{14}$$

for $|z| \gg 1$, $|z| \gg |v|^2$ as well as $-\frac{1}{2}\pi \leqslant \text{phase } z \leqslant \frac{1}{2}\pi$.

If we substitute $z = i\sqrt{s/a}\,r$ in (13) and (14), it can be seen that $H_0^{(1)} \to 0$ for $r \to \infty$, whereas $H_0^{(2)} \to \infty$. Since at distances very remote from the focus the temperature T should tend to zero, it is necessary to assume that $B' = 0$.

Lastly, in the region outside the focus we shall try to solve the homogeneous equation (4) in the form

$$\overline{T}_0 = BH_0^{(1)}\left(i\sqrt{\frac{s}{a}}\,r\right). \tag{15}$$

The constants A and B which occur in equations (9) and (15) are determined from two boundary conditions which should be satisfied on the boundary surface that divides the considered tissue into the focal region and the outer region.

The first condition of the continuity of the heat flux $\vec{q}$,

$$\vec{q} = -\lambda \operatorname{grad} T, \tag{16}$$

takes the form

$$\frac{d\overline{T}_i}{dr} = \frac{d\overline{T}_0}{dr} \quad \text{for } r = R. \tag{17}$$

Substituting in (17) the expressions for $\overline{T}_i$ from (9) and $\overline{T}_0$ from (15) we obtain

$$\frac{A}{B} = \frac{H_1^{(1)}\left(i\sqrt{\frac{s}{a}}R\right)}{J_1\left(i\sqrt{\frac{s}{a}}R\right)}. \tag{18}$$

In this relation Bessel and Hankel functions of the first order occur as a result of the differentiation of the functions of zero order.

The other condition is the equality of temperatures on each side of the boundary surface, thus

$$\overline{T}_i = \overline{T}_0 \quad \text{for } r = R. \tag{19}$$

Inserting into (19) expressions (9) and (15), we obtain

$$AJ_0\left(i\sqrt{\frac{s}{a}}\,R\right) + \frac{\dot{Q}_v}{s^2\varrho c_w} = BH_0^{(1)}\left(i\sqrt{\frac{s}{a}}\,R\right). \tag{20}$$

From relations (18) and (20) we determine the constant A, namely

$$A = \frac{\dot{Q}_v}{s^2\varrho c_w}\,\frac{H_1^{(1)}(i\sqrt{s/a}R)}{J_1(i\sqrt{s/a}\,R)H_0^{(1)}(i\sqrt{s/a}\,R) - J_0(i\sqrt{s/a}\,R)H_1^{(1)}(i\sqrt{s/a}\,R)}. \tag{21}$$

Now we can finally calculate the transform of the required temperature $\overline{T}_i$ of the focus from relations (9) and (21) and obtain

$$\overline{T}_i = \frac{\dot{Q}_v}{s^2\varrho c_w}\left[1 + \frac{\pi}{2}\sqrt{\frac{s}{a}}\,RH_1^{(1)}\left(i\sqrt{\frac{s}{a}}\,R\right)J_0\left(i\sqrt{\frac{s}{a}}\,r\right)\right]. \tag{22}$$

4. The temperature in the focal region of the ultrasonic field

The inverse transform of expression (22) can be determined by virtue of the theorem of the homology between the transformed function and its transform [8]:

$$L[f(at)] = \frac{1}{a}\bar{f}\left(\frac{s}{a}\right). \tag{23}$$

From the foregoing it can be concluded that high values of the argument $i\sqrt{s/a}\,R$ will correspond to small values of at/R^2. Small values of at/R^2 are the subject of our interest since the heat conductivity of soft tissues is insignificant. Furthermore, we are interested in ultrasonic impulse of short duration.

Therefore, when evaluating the inverse transform, we expand expression (22) into a series by taking advantage of the asymptotic expressions (13) and (14) for Hankel functions of large arguments, and the following expressions [8] for Bessel functions:

$$J_0(iz) \cong \frac{e^z}{\sqrt{2\pi z}}\left(1+\frac{1^2}{1!(8z)}+\frac{1^2\cdot 3^2}{2!(8z)^2}+\frac{1^2\cdot 3^2\cdot 5^2}{3!(8z)^3}+\ldots\right), \tag{24}$$

$$J_1(iz) \cong \frac{ie^z}{\sqrt{2\pi a}}\left(1-\frac{1\cdot 3}{1!8z}-\frac{1\cdot 3\cdot 5}{2!(8z)^2}-\frac{1^2\cdot 3^2\cdot 5\cdot 7}{3!(8z)^3}-\ldots\right). \tag{25}$$

Substituting (13), (14), (24) and (25) into (22) we obtain

$$\bar{T}_i = \frac{Q_v}{s^2\varrho c_w}\left\{1-\frac{\exp[-\sqrt{s/a}\,(R-r)]}{2}\sqrt{\frac{R}{r}}\left[1+\frac{1}{\sqrt{s/a}}\left(\frac{1}{8R}+\frac{3}{8R}\right)+\right.\right.$$
$$\left.\left.+\frac{1}{s/a}\left(\frac{9}{128r^2}+\frac{3}{64Rr}-\frac{15}{128R^2}\right)+\ldots\right]\right\}. \tag{26}$$

Having taken advantage of the relations (see [8])

$$L^{-1}\left[\frac{\exp[-\sqrt{s/a}\,x]}{s^2}\right] = 4t\,\mathrm{int}^2\mathrm{erfc}\,\frac{x}{2\sqrt{at}}, \tag{27}$$

$$L^{-1}\left[\frac{[\exp[-\sqrt{s/a}\,x]}{s^2\sqrt{\frac{s}{a}}}\right] = 8t\sqrt{at}\,\mathrm{int}^3\mathrm{erfc}\,\frac{x}{2\sqrt{at}}, \tag{28}$$

$$L^{-1}\left[\frac{\exp\left[-\sqrt{\frac{s}{a}}\,x\right]}{s^3}\right] = 16t^2\,\mathrm{int}^4\mathrm{erfc}\,\frac{x}{2\sqrt{at}}, \tag{29}$$

where

$$\operatorname{erfc} a = 1 - \operatorname{erf} a = \frac{2}{\sqrt{\pi}} \int_a^\infty e^{-u^2} du \tag{29a}$$

as well as

$$\operatorname{int} \operatorname{erfc} a = \int_a^\infty \operatorname{erfc} u du, \tag{29b}$$

we can calculate the inverse transformation of expression (26). As result we obtain an expression for the temperature T_i at focal region ($r \leqslant R$):

$$T_i = \frac{\dot{Q}_v t}{\varrho c_w} \left\{ 1 \frac{1}{2} \sqrt{\frac{R}{r}} \left[4 \operatorname{int}^2 \operatorname{erfc} \frac{(R-r)}{2\sqrt{at}} + \sqrt{at} \operatorname{int}^3 \operatorname{erfc} \frac{(R-r)}{2\sqrt{at}} \times \left(\frac{1}{r} + \frac{3}{R} \right) + \right.\right.$$
$$\left.\left. + \left(at \operatorname{int}^4 \operatorname{erfc} \frac{(R-r)}{2\sqrt{at}} \right) \left(\frac{9}{8r^2} + \frac{3}{4Rr} - \frac{15}{8R^2} \right) + \ldots \right] \right\}. \tag{30}$$

This formula is not valid for very small values of r, since in this case the expansion (24) is not valid.

When the heat conductivity of a biological medium tends to zero ($a \to 0$), and the boundary surface ($r \neq R$) is neglected, square bracket of expression (29) disappears, since then we have $\operatorname{erfc} \infty = 0$ whence likewise

$$\operatorname{int}^n \operatorname{erfc} \infty = 0. \tag{31}$$

A similar result is obtained from expression (30) at a finite value of heat conductivity ($a \neq 0$), when the time tends to zero. In this case the temperature of the medium is

$$T_i = \frac{\dot{Q}_v t}{\varrho c_w} \quad (a = 0) \tag{32}$$

and

$$\frac{dT_i}{dt} = \frac{\dot{Q}_v}{\varrho c_w} \quad (a \neq 0, t = 0). \tag{33}$$

When we substitute the magnitude of $\dot{Q}_v$ from (37) into (33), we get the expression quoted by FREY [4].

On the boundary surface of the focus ($r = R$) we obtain from formula (30) a value of temperature equal to

$$T_i = \frac{\dot{Q}_v}{\varrho c_w} t \left[\frac{1}{2} - \frac{1}{3R} \sqrt{\frac{at}{\pi}} \right]. \tag{34}$$

In view of the fact that water accounts for 75 % of soft tissue content, we may approximate by assuming such parameters for soft tissue as are encountered in water. Thus, we have at a temperature of 30°C the value of the heat conductivity factor equal to $\lambda = 0.00038$ cal/cm.s °C [7]. In this case the value of the coefficient a is

$$a = \frac{\lambda}{\varrho c_w} = 0.00038 \text{ cm}^2/\text{s}. \tag{35}$$

Fig. 4 shows the temperature distribution in the region of the focus, as evaluated from formula (30), as a function of radius $r (r \leqslant R = 1.25$ mm) for various durations of the impulse t and also with the assumption of a zero value of the coefficient a, using expression (32).

To determine the value of the temperature T_i in the case of impulse ultrasonography considered by us, we calculate the rate of heat generation per unit volume $\dot{Q}_v$ in the focal region. Let us consider an infinitesimal focal region in the form of a cylinder of length Δx (Fig. 5). The intensity of a plane ultrasonic wave I_x [W/cm²] propagating along the x-axis decreases exponentially according to the relation

$$I_{x+\Delta x} = I_x e^{-2\alpha\Delta x}, \tag{36}$$

where α denotes the pressure coefficient of the attenuation of ultrasound. The quantity $\dot{Q}_v$ is defined as the ratio of the amount of heat released per unit time to the volume of the region $A\Delta x$

$$\dot{Q}_v = k\,\frac{E_x - E_{x+\Delta x}}{\Delta t \cdot A\Delta x} = k\,\frac{I_x - I_{x+\Delta x}}{\Delta x}, \tag{37}$$

where A denotes the cross-section of the focus, E_x – the energy of the progressive wave, and $k = 0.24$ cal/W_s.

Substituting (36) into (37) we finally obtain

$$\dot{Q}_v = kI_x \frac{(1 - e^{-2\alpha\Delta x})}{\Delta x} = 2k\alpha I_x \quad \text{for } \Delta x \to 0. \tag{38}$$

From this it can be seen that the thermal effect of the heat source is dependent on I_x. We will thus consider the end of the focal region closest to the transducer shown in Fig. 2, where the value of the intensity of ultrasound is assumed to be equal to 20 W/cm². Assuming an attenuation coefficient in soft tissues of 3 dB/cm, we have $\alpha = (3 \text{ dB/cm})(8.67 \text{ dB})^{-1} = 0.34 \text{ cm}^{-1}$. Hence we obtain the maximum value $\dot{Q}_v = 14k$ W/cm³, and also $\dot{Q}_v/\varrho c_w = 3.3$°C/s.

For ultrasonic impulses of duration 1 μs we obtain, from formula (32), a maximum temperature increase in the focus barely equal to $T_i = 3.3 \cdot 10^{-6}$ °C.

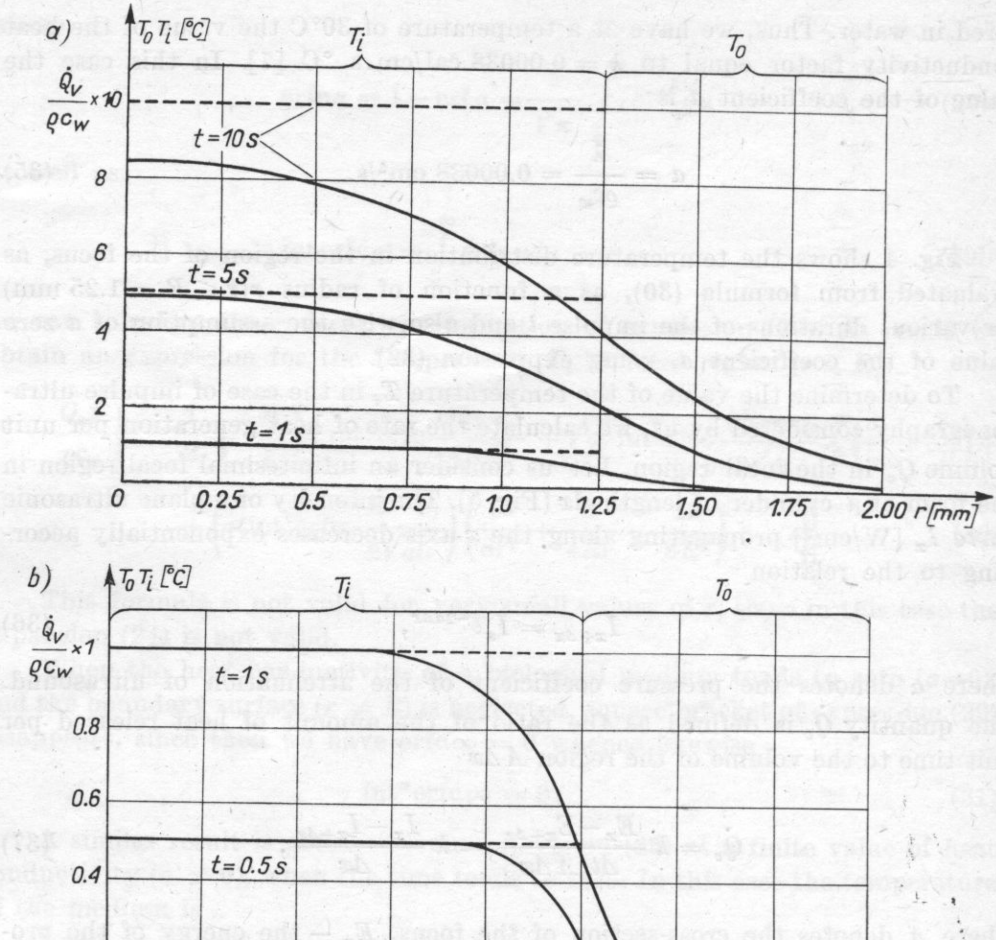

Fig. 4. The temperature distribution in the focal region $r < 1.25$ mm and beyond the focus $r > 1.25$ mm computed from formulae (30) and (41) as a function of the radius r at various durations t of ultrasonic irradiation for $a = 0.00038$ cm^2/s (continuous curve) and for $a = 0$ (broken curves)

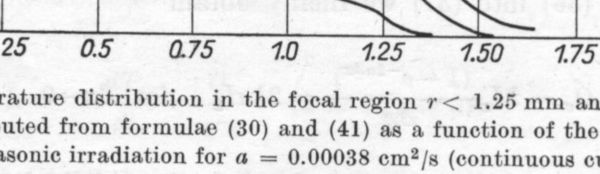

Fig. 5. The intensity of the wave passing through an element the focal region, with length Δx and cross-sectional area A

5. The temperature beyond the region of the ultrasonic focus

This temperature is determined on the basis of formula (15) by evaluating the constant B from expressions (18) and (30). Then we have

$$\overline{T}_0 = \frac{\dot{Q}_v}{s^2 \varrho c_w} \frac{\pi}{2} \sqrt{\frac{s}{a}}\, R J_1\left(i\sqrt{\frac{s}{a}}\, R\right) H_0^{(1)}\left(i\sqrt{\frac{s}{a}}\, r\right). \tag{39}$$

The inverse transform of this expression is determined in a similar manner as before by using series expansions (13), (14), (24), (25) of Bessel and Hankel functions and relations (27)-(29). Then we have

$$\overline{T}_0 = \frac{\dot{Q}_v}{s^2 \varrho c_w} \frac{1}{2} \sqrt{\frac{R}{r}} \exp\left[-\sqrt{\frac{s}{a}}\,(r-R)\right]\left[1 - \frac{1}{\sqrt{\frac{s}{a}}}\left(\frac{1}{8r} + \frac{3}{8R}\right) + \right.$$

$$\left. + \frac{1}{\frac{s}{a}}\left(\frac{9}{128r^2} + \frac{3}{64rR} - \frac{15}{128R^2}\right) - \ldots\right], \tag{40}$$

whence we obtain the final temperature beyond the region of the focus ($r \geqslant R$) equal to

$$T_0 = \frac{\dot{Q}_v t}{\varrho c_w} \frac{1}{2} \sqrt{\frac{R}{r}} \left[4\, \mathrm{int}^2 \mathrm{erfc} \frac{r-R}{2\sqrt{at}} - \left(\frac{1}{r} + \frac{3}{R}\right)\sqrt{at}\, \mathrm{int}^3 \mathrm{erfc} \frac{r-R}{2\sqrt{at}} + \right.$$

$$\left. + \left(\frac{9}{r^2} + \frac{6}{rR} - \frac{15}{R^2}\right) \frac{at}{8}\, \mathrm{int}^4 \mathrm{erfc} \frac{r-R}{2\sqrt{at}} - \ldots\right]. \tag{41}$$

It can be easily seen that on the boundary surface of the focal region ($r = R$) we obtain from formula (41), as expected, the same value as expressed already by formula (34).

Fig. 4 also shows the temperature distribution calculated beyond the focal region ($r \geqslant R = 1.25$ mm) as a function of the radius r for various durations t of the ultrasonic impulse.

6. Conclusions

In the range of impulse ultrasonography, in which the impulse duration is about 1 μs, the temperature increase of the medium during the pulse duration is of the order of 10^{-6}°C. For an intensity $I = 20$ W/cm² the temperature calculated by us was barely $3.3 \cdot 10^{-6}$°C. This increase is entirely negligible.

At the intensities assumed and times of ultrasonic radiation equal to 1 s the temperature increases can attain a value of 3.3°C, thus at the times of this order they may cause irreversible changes in the tissues irradiated by the ultrasound.

The temperature increase in the middle of the focus does not depend on the thermal conductivity for short duration of ultrasonication (in our case for $t < 5$ s, Fig. 4a). The thermal conductivity then influences only the temperature distribution in the proximity of the focal boundary.

The temperature distribution in the focus shown in Fig. 4 is an approximated distribution by virtue of the assumed stability of the intensity of ultrasound in the focus for $0 \leqslant r \leqslant R$. In reality, because of decreasing intensity in the focus along the coordinate r, the temperature will decrease for considerably smaller values of r and this decrease will occur more evenly. The distribution can also be modified by heat flow along the x-axis of the focus and this has not been considered in this paper.

In the case of tissues well supplied with blood one should consider the additional factor of heat transfer by the blood causing a temperature decrease of the tissues. This problem was also not considered in this paper.

A number of simplifications assumed in the paper, chiefly in the range of the field distribution, is not of any great importance in view of the purpose of the paper, namely the evaluation of the thermal effect encountered in tissues.

The estimates obtained are approximately in agreement with the experimental results obtained in the tissues of mammalian muscles by Frey [3], where, with the aid of thermocouple probes, a temperature increase of 2.9°C has been measured at an intensity of 64 W/cm² for 1 s.

It is also of interest to compare the results obtained with the threshold curves for ultrasonic doses causing irreversible structural changes in mammalian brain [1]. From the curves presented in the cited paper, there is, at a frequency of 3 MHz, a threshold value at an intensity of 200 W/cm² with ultrasonic impulses of duration 1 s. On the basis of formulae (32) and (38) we would then obtain a temperature increase in the focus of 33°C, a temperature that would doubtless cause damage to the tissues.

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